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Measures of Directed Divergence

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Abstract:
The measures of directed divergence ofparametric entropy have been obtained which are generalizations of Shannon's Kapur's, Bose Einstein, Fermi-Dirac, and Havrda-Charvat's measures of Entropy. We have also examined its concavity property and some special cases.

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1. Measures of Directed Divergence Corresponding to the Measure of Entropy

$H_{\frac{b}{a}}(P) = -\sum_{i=1}^{n} p_i ln p_i + \frac{a}{b}\sum_{i=1}^{n}\left(1+\frac{b}{a}p_i\right) ln\left(1+\frac{b}{a}p_i\right) - \frac{a}{b}\left(1+\frac{b}{a}\right) ln(1+\frac{b}{a})$, b> -1 a>0 (1)

The proposed measure of directed divergence

D (P: Q) = $\sum_{i=1}^{n} p_i ln\frac{p_i}{q_i} - \frac{a}{b}\sum_{i=1}^{n}(1+\frac{b}{a}p_i) ln\frac{(1+\frac{b}{a}p_i)}{(1+\frac{b}{a}p_i)}$ b> -1, a>0 (2)

This measure holds all the properties

This is permutationally symmetric, Continuous convex function of $p_1, p_2, p_3, \dots, p_n$ and vanishes iff $p_i = q_i \quad \forall i$

However, it is not in general a convex function of $q_1, q_2 \dots q_n$

Now to generalized equation (2)

To consider the measure

D (P: Q) = $\sum_{i=1}^{n} p_i ln\frac{p_i}{q_i} + A\sum_{i=1}^{n}(i+\frac{b}{a}p_i) ln\frac{(1+\frac{b}{a}p_i)}{1+\frac{b}{a}q_i}$ (3)

This is convex function of $p_1, p_2, \dots p_n$

If, D'(P: Q) = $ln\frac{P_i}{q_i} + 1 + A.\frac{b}{a} ln\frac{1+\frac{b}{a}P_i}{1+\frac{b}{a}q_i} + A.\frac{b}{a}$

D"(P: Q) = $\frac{1}{P_i} + A\frac{(\frac{b}{a})^2}{1+\frac{b}{a}P_i}$

$$\frac{1}{p_i} + \frac{A(\frac{b}{a})^2}{1+\frac{b}{a}p_i} > 0$$

This will be always satisfied if A> 0, if A is negative, it will still be satisfied

A (b/a) ²> - $\left(\frac{1}{P_i} + \frac{b}{a}\right)$

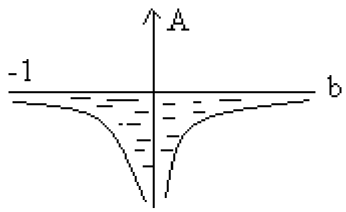
i.e. - $\frac{1}{P_i} < A\left(\frac{b}{a}\right)^2 + \frac{b}{a}$ (4)

Now, $\frac{1}{p_i}$ varies from 1 to ∞ , so that (4) will satisfied if

$A\left(\frac{b}{a}\right)^2 + \frac{b}{a} > -1$or $A > -\left(\frac{a}{b}\right) - \left(\frac{a}{b}\right)^2$ (5)

The graph of A = - $\left(\frac{a}{b}\right) - \left(\frac{a}{b}\right)^2$

When b> -1 a>0, a$\neq b$



Where all points inside the shaded region give permissible values of A, b

Now, $\lim_{b \to 0} \sum_{i=1}^{n}(1 + \frac{b}{a}p_i) ln \frac{(1+\frac{b}{a}p_i)}{(1+\frac{b}{a}q_i)} = 0$ (6)

$\lim_{b \to 0} \frac{a}{b}\sum_{i=1}^{n}(1 + \frac{b}{a}p_i) ln \frac{(1+\frac{b}{a})p_i}{1+\frac{b}{a}q_i}$ (7)

$\sum_{i=1}^{n}(P_i - q_i) = 0$(8)

So that for all finite values of A, positive or negative which are independent of b (3) approaches K.L. measures [7] as b→0

Also we can use,

D (P: Q) =$\sum_{i=1}^{n} P_i ln \frac{P_i}{q_i} - \left(\frac{ac}{b} + \frac{a^2 d}{b^2}\right)\sum_{i=1}^{n}(1 + \frac{b}{a}P_i) ln \frac{(1+\frac{b}{a}P_i)}{(1+\frac{b}{a}q_i)}$ (9)

Where c and d are any positive number less than unity.

We consider some cases

When c=1 d=0

Or c=1 d=1

Or c=0 d=1

The measure (9) is again in general not a convex function of $q_1, q_2, \ldots \ldots q_n$

2. A measure which is a convex function of both P and Q is obtained from Csiszer's [1] measure.

$\sum_{i=1}^{n} q_i \phi(\frac{P_i}{q_i})$ (10)

Where ϕ (.) is a twice differentiable convex function with ϕ (1) =0 by taking

$\boldsymbol{\phi}$(x) = x lnx - $\frac{a}{b}\left(1 + \frac{b}{a}x\right) ln \frac{(1+\frac{b}{a}x)}{(1+\frac{b}{a})}$ b>0, a->1 (11)

This gives

D (P: Q) = $\sum_{i=1}^{n} P_i ln \frac{P_i}{q_i} - \frac{a}{b}\sum_{i=1}^{n}(q_i + \frac{b}{a}P_i) ln \frac{q_i+\frac{b}{a}p_i}{q_i(1+\frac{b}{a})}$ (12)

It can be generalized as

$\boldsymbol{\phi}$(x) = x $ln x + A\left(1 + \frac{b}{a}x\right) ln \frac{(1+\frac{b}{a}x)}{1+\frac{b}{a}}$ (13)

$\boldsymbol{\phi}$'(x) = $ln x + A\left(1 + \frac{b}{a}x\right) ln \frac{(1+\frac{b}{a}x)}{1+\frac{b}{a}} + A\left(\frac{b}{a}\right) + 1$

$\boldsymbol{\phi}$''(x) =$\frac{1}{x} + \frac{A(\frac{b}{a})^2}{(1+\frac{b}{a}x)}$

It will be convex if

$$\left.\begin{aligned} &\frac{1}{x} + \frac{A\left(\frac{b}{a}\right)^2}{\left(1+\frac{b}{a}x\right)} > 0 \\ &or\ A\left(\frac{b}{a}\right)^2 + \left(\frac{b}{a}\right) > -1/x \end{aligned}\right\} \quad (14)$$

Now x can vary from 0 to ∞ so that$-\frac{1}{x}$ can vary from - $\infty\ to\ 0$ so that the condition becomes

$A(\frac{b}{a})^2 + \left(\frac{b}{a}\right) > 0$ or A> - a/b (15)

Thus, the generalized measure of directed divergence which is a convex function of both P and Q is,

D (P: Q) = $\sum_{i=1}^{n} P_i \ln\frac{P_i}{q_i} - A\sum_{i=1}^{n}\left(q_i + \frac{b}{a}p_i\right)\ln\frac{q_i+\frac{b}{a}p_i}{q_i(1+\frac{b}{a})}$ (16)

Where A is any positive number or a negative number $\geq -b/a$

3. Now Consider

$\boldsymbol{\phi}$(x) $= \frac{x^{\propto}-x}{1-\propto} + A\frac{(1+\frac{b}{a}x)^{\propto}-(1+\frac{b}{a}x)}{\propto-1} - A\frac{(1+\frac{b}{a})^{\propto}-(1+\frac{b}{a})}{\propto-1}$ (17)

$\boldsymbol{\phi}$''(x) $= \propto x^{\propto-2} + A\propto\left(1-\frac{b}{a}x\right)^{\propto-2}(\frac{b}{a})^2$

This will be convex if

$\propto x^{\propto-2} + A\propto(1+\frac{b}{a}x)^{\propto-2} \geq 1$ (18)

If A is positive, this is always satisfied

$[\frac{x}{1+\frac{b}{a}x}]^{\propto-2} \geq -A$

If A = -B this gives,

$$\left.\begin{aligned} &\left[\frac{x}{1+\frac{b}{a}x}\right]^{\propto-2} \geq B \\ &or\ \left[\frac{1+\frac{b}{a}x}{x}\right]^{2-\propto} \geq B \end{aligned}\right] \quad (19)$$

As x goes from 0 to∞,

$\frac{x}{1+\frac{b}{a}x}$Goes from 0 to b/a

If $\propto > 2$, thus requires B ≤ 0 or B=0

If $\propto = 2$, $B \leq 1$ (20)

If $\propto < 2$ $(\frac{1+\frac{b}{a}x}{x})$ can vary from b to ∞ a→1

Expression (19) gives

B $\leq (b)^{2-\propto}$ (21)

Also

D (P: Q) = $\frac{1}{\propto-1}[(\sum_{i=1}^{n} p^{\propto}q^{1-\propto} - 1 + A\left[\left(q_i + \frac{b}{a}P_i\right)^{\propto}q_i^{1-\propto} - (q_i + ap_i)\right] - A(1+a)^{\propto} + A(1+a)]$(22)

Gives a valid measure of directed divergence for all non-negative values of A.

Also,

D (P: Q) =$\frac{1}{\propto-1}[\sum_{i=1}^{n} p^{\propto}q^{1-\propto} - 1 - B[(q_i + ap_i)^{\propto}(q_i)^{1-\propto} - (q_i + ap_i)] + B(1+a)^{\propto} - B(1+a)]$ (23)

Gives a valid measure of directed divergence if B$\leq b^{2-\propto}$ when 0$\leq b \leq 2$ and a→1

B=0 when $\propto > 2$.

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