ISSN 2278 – 0211 (Online)

Gender Geometric Reasoning Stages and Gender Differences in Achievements of Preservice Teachers of E. P. College of Education, Bimbilla, Ghana

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Abstract:

The purpose of this study was to investigate gender differences in geometric reasoning levels and gender differences in achievement scores of Preservice Teachers (PSTs) Van Hiele theory levels of Geometry thinking in E.P. College of Education, Bimbilla-Ghana. Six null hypotheses and one research question were formulated to guide the study. Three hundred and fifty-one (351) 74.2% of the population was used as the sample for the study. The sample composed of 133 (37.89%) female PST and 218 (62.11%) male PST. Both purposive and convenient samplings were used in this study. The instrument used was the 25 Van Hiele Geometry Test (VHGT) items. The combined results from the independent sample t-test revealed that there was no significant difference in all the VHGT levels between the male and female PSTs, even though the male performed slightly better than their female counterparts except only in level 4 which favored the female PST. Also, the findings revealed the following levels of reasoning stages for both sexes. For level 0, (Female = 42.1% and Male = 34.4 %); level 1, (Female = 30.1% and Male = 33.9%); level 2, (Female = 19.5% and Male = 21.6%) ; level 3 (Female = 6.8% and Male = 8.7 %); level 4 (Female = 1.5% and Male = 1.4 %) and level 5 both had 0 (0 %). It was recommended that Geometry Course outline/syllabi for Colleges of Education in Ghana should be revised to create room for Van Hiele phase-based approach.

Keywords: Geometry, Preservice teachers (PSTs), Van hiele Geometry Test (VHGT), Significant Difference, Independent Sample t-test

1. Introduction

Geometry is one of the most significant topics in Mathematics. Geometry is a branch of mathematics and one of the basic skills to be mastered (Hoffer and Hoffer, 1992; Hong, 2005; NCTM, 2000). According to Olkun and Aydogdu (2003) stated that geometry is so important not only as a subject area but a conceptual building blocks of Mathematics. According to van de Walle, (2001), geometry is applied in so many fields like architecture, geology, astronomy, science, and engineering. Students gain basic skills in geometry which help them to make a comparison, creative thinking, criticizing, investigation, analysis, and generalization in mathematics classrooms. Geometry help students develop cognitive and mental skills necessary for solving daily problems they encounter. Teaching and learning geometry is emphasized in all mathematics curriculums all over the world because of its importance in our daily lives. According to Noraini (2005) because of the importance of geometry, its process of teaching and learning is different from the other mathematics topics such as arithmetic, algebra, and probability.

"Primary school geometry in Ghana is treated as shapes and space and is approximately 17% of the major content area in the Mathematics teaching syllabus" Akayuure, Asiedu-Addo, Alebna (2016). Teaching space and shape rationale is to give pupils early development of spatial visualization and mental rotation abilities and to enable them "organize and use spatial relationship in two or three dimensions, particularly in solving problems" (MOESS, 2007, p.ii) and for progress in learning higher Mathematics. The main objective of teaching geometry in the Senior High Schools is to teach students to (i) Think clearly and logically and (ii) To be critical and generalize correctly. In other words, it is to train students to seek and discover the truth and to understand the meaning of exactness and precision in Mathematics reasoning. The objectives of teaching geometry in the Ghanaian Colleges of Education are; (i) Review and consolidate the concepts and skills related to Geometry. (ii) Discover relations involving shapes, perimeters, areas, and volumes and use these to solve problems. (iii) Relate and apply Mathematics knowledge to solve problems in Geometry (Institute of Education, Ghana, draft syllabus, 2014). In Ghana, geometry in the Colleges of Education is divided into two Mathematics content and Mathematics methodology courses which is compulsory for every preservice teacher. The Mathematics content is studied in the first year while the Mathematics methodology is studied in the second year for all the programmes. Geometry in the first year content comprises of lines and angles, polygons and congruent and similar triangles, geometrical construction including loci, circle theorem, 2D and 3D

shapes, movement geometry and coordinate geometry. At the Colleges of Education in Ghana, "the inclusion of geometry in both content and methodology is not only to equip pre-service teachers with subject matter, but more especially to expose them to more pedagogy on how to teach it effectively at the basic level of Education (Institute of Education, 2005, Acquah, 2011, P.i).

Gender differences in geometrical thinking levels and achievements scores of level 200 general programme preservice teachers from E.P. College of Education, Bimbilla-Ghana are what this paper reports on.

1.2. Statement of Problem

This research arises from hitches the researcher encountered in the 20 years of his teaching of mathematics and 13 years of teaching geometry at E.P. College of education, Bimbilla. The researcher and some of his colleagues at the College made several efforts in teaching geometry to mathematics/science and general programme PSTs. Nonetheless, the semester by semester results from the Institute of education, UCC always revealed that PSTs are not performing well in geometry as expected. Similarly to support the issue of the poor performance Halat (2008) cited some studies by (Gutierrez, Jaime, & Fottuny, 1991; Yetkin, 2003) indicated that learning of geometry is difficult for many students which manifest in their poor performance in mathematics classrooms. Also, as early as in the 1950's, similar problem led to Pierre van Hiele and Dina van Hiele-Geldof to probe the way learners' reason and think about geometry (Van Hiele, 1986). According to Van De Walle (2004), their study led to the development of their different theory levels of geometric reasoning.

Learning of geometry is perceived to be difficult hence a large percentage of PSTs failed to develop the adequate understanding of the geometry topics. According to Cangelosi (1996), ineffective instruction, geometry language, and visualization abilities are the factors that have been put forward why geometry learning is difficult. Many PSTs are unable to extract necessary information from questions to solve, interpret and draw conclusions. The lack of understanding of geometry concepts, geometry problem-solving skills, and geometry reasoning often cause discouragement among PSTs, which invariably lead to poor performance in geometry. The trend of the poor performance is worrying and has serious implication for geometry teaching as students move to higher areas of mathematics such as engineering will require strong geometric reasoning, (Akayure et al, 2016).

The poor performance of PSTs in geometry has been a concern to college Mathematics tutors, parents, and government. The Institute of education chief examiners reports on geometry for 2013, 2015 and 2017 academic years are good testimonies of those facts. Extensive analyses of written papers for 2013, 2015 and 2017 academic years suggest that student fail geometry and those who even pass the geometry paper in the Colleges of Education get weak passes. For example, in 2013 academic year, 38.8% of the 7, 449 PSTs either fail or get a weaker grade of D+ or D. Again, in 2015 academic year, 28.8% of PSTs failed while 42.3% got weak passes in the geometry paper. In 2017 academic year, 3,132 (23.2%) out of the total 13,513 PSTs either failed or obtained a weaker of D+ or D at the end of the semester exam per the Institute of education, professional board report UCC – Ghana, 2013,2015, and 2017).

Recent studies (Alex & Mammen, 2016; Vojkuvkova, 2012; Yegambaram & Naidoo, 2009; Armah, Cofie, & Okpoti, 2017; Anas, 2018; Armah, Cofie, & Okpoti, 2018) suggest the adoption of Van Hiele theory model for teaching geometry which has the potential to increase performance of geometry at all levels of the educational leather including the Colleges of Education in Ghana. However, there is limited literature in Ghana, regarding the introduction of Van Hiele model of teaching geometry. The present study sort to examine gender differences among E.P. College of Education–Bimbilla general programme PSTs in Geometry.

1.3. Purpose of the Study

The purpose of this study was to investigate gender differences in geometric reasoning levels and gender differences in achievement scores of PSTs Van Hiele theory levels of Geometry thinking in E.P. College of Education, Bimbilla-Ghana.

1.4. Research Hypothesis

The following hypotheses were formulated to guide the study.

- Ho: There is no statistically significant difference in achievement scores in VHGT level 1 among male and female PSTs.
- Ho: There is no statistically significant difference in achievement scores in VHGT level 2 among male and female PSTs.
- Ho: There is no statistically significant difference in achievement scores in VHGT level 3 among male and female PSTs.
- Ho: There is no statistically significant difference in achievement scores in VHGT level 4 among male and female PSTs.
- Ho: There is no statistically significant difference in achievement scores in VHGT level 5 among male and female PSTs.
- Ho: There is no overall statistically significant difference in achievement scores in VHGT levels 1 to 5 among male and Female PSTs.

1.5. Research Question 1

What reasoning stages do male and female PSTs reach in VHGT?

2. Theoretical Framework

According to (Alex & Mammen, 2016; Vojkuvkova, 2012; Yegambaram & Naidoo, 2009), the most popular established theory for students' levels of geometric thinking is based on van Hiele theory. The theory emerged as a product of the two mathematics educators completing their doctoral studies at the University of Utrecht, the Netherlands in 1957 (Usiskin, 1982). The theory was developed by Pierre Marie van Hiele and his wife Dina van Hiele Geldof (Armah, et al, 2017; Armah, et al, 2018; Essays, 2013). Van Hiele geometry theory is the most important of the studies conducted on geometric thinking and development of geometric thinking (Alex & Mammen, 2016; Armah, et al, 2017; Abdullah & Zakaria, 2013; Essays, 2013). The theory attracted attention in the 1970s and has been widely used since 1984 across the world to improve geometry instruction (Dindyal, 2007; Yazdani, 2007; Breyfogle & Lynch, 2010; Alex & Mammen 2012; Siew, Chong & Abdullah, 2013). The Van Hiele theory enumerated three segments, as (i) Levels of geometric thinking. (ii) Properties of the levels and (iii) Phases of learning which defined teaching processes that teachers could apply to improve learners' levels of understanding geometry (Vojkuvkova, 2012; Crowley, 1987; Howse & Howse, 2015; Van Hiele, 1986). The theory covered five sequentially and hierarchical distinct levels of geometric thought, Van Hiele (1986), listed as follows (1) Recognition / Visual, (2) Analysis, (3) Order, (4) Deduction, and (5) Rigor (Usiskin, 1982; Alex & Mammen, 2016) in ascending order of difficulty.

Originally, the van Hiele levels were from Level 0 to Level 4. However, the following researchers (Alex & Mammen, 2016; Howse & Howse, 2015; Siew & Chong, 2014) studies have altered it from Level 0 to Level 4, to the levels 1 to 5. The modification allows for a sixth level named pre-recognition for students who are beneath level 1, Mason (1998). That level also recommended by Clements and Battista (1990). Students at this level are unable to distinguish between figures but perceive geometric shapes. According to (Karakus & Peker, 2015, p.339) cited by Armah, et al (2018), students "may recognize the difference between triangles and rectangles, but may not be able to distinguish between a rhombus and a square".

2.1. Van Hiele Theory Levels

Levels	Characteristics
Level 1 (Visualization)	Students recognize figures by their appearance. They make decision based on intuition not reasoning.
Level 2 (Analysis)	Students recognize figures by their properties. They can analyze and name properties of figures, but they cannot make relationships between these properties.
Level 3 (Informal deduction)	Students can distinguish between necessary and sufficient conditions for a concept. They can form meaningful definitions and give informal arguments to justify their reasoning.
Level 4 (Deduction)	Students can construct theorems within an axiomatic system. They know the meaning of necessary and sufficient conditions of a theorem
Level 5 (Rigor)	Students understand the relationship between various systems of geometry. They can compare, analyze and create proofs under different geometric systems.

Table 1: Characteristics of Van Hiele Geometry Levels

Source: Karakus & Peker, 2015, p. 339 as Cited by Armah, et al 2018

2.2. Studies on Van Hiele Theory in Ghana

A study by Anas (2018) was carried out in five Mathematics/Science Colleges of Educations. The purpose of the study was to investigate Van Hiele levels of geometric thinking among Mathematics Pre-service Teachers in the three Northern regions of Ghana. The population was 412 Mathematics Pre-service Teachers. Two hundred and ninety- eight (298) Mathematics Pre-service Teachers were used as the sample. The sample was drawn from Upper West, Upper East, and Northern regions of Ghana. The detailed sample distribution was fifty 50 (16.8%) female and two hundred and forty-eight 248 (83.2%) male. Sampling techniques employed was purposive and simply random. The Van Hiele Geometry Test was administered to Mathematics Pre-service Teachers during their second year second semester. Results from his study indicated that majority of 50.3% reached Van Hiele Level 0. Another, 23.5% reached Van Hiele Level 1. Also, 14.8% and 9.1% reached Van Hiele Levels 2 and 3 respectively. Finally, 2.34% and 0 % attained VHL 4 and 5 respectively. His conclusion revealed that only 11.44% were eligible to teach at the basic school in Ghana. Also, a majority of 264 (88.56%) attained between levels 0 to

2. Anas (2018) recommended that Colleges of Education should be encouraged to adopt Van Hiele phase-based learning of Geometry.

Also, Armah, et al (2017), their results implication based on Van Hiele geometry test levels was that 75.33% of pre-service teachers were lower than their expected future JHS 3 students. Their study concentrated on Van Hiele geometric thinking of pre-service teachers' in Ghana prior to their embarking on teaching practice. A sample of 300 students was drawn from 4 Colleges in Ghana from three (3) regions second-year students participated in the study. From the results analysis, it was clear that 16.33% of the pre-service teachers were at level 0 based on Van Hiele levels. Also, 27%, 32 %, 17.67% of the pre-service teachers' attained levels 1, 2 and 3 respectively. Further analysis revealed that only 6% and 1% of the pre-service teachers achieved level 4 and 5 respectively.

Furthermore, a quantitative research approach to measure Van Hiele level of geometric thinking was done by Asemanni, et al (2017) on Ghana's Senior High School final year students. A sample of 200 was randomly selected from three (3) municipalities in the Central Region for the study. The sample composed of 44% males and 56% females. The minimum age was 14 years while the maximum age was 20 years in that study. It was clear from the analysis that 42.5% of the students did not meet any Van Hiele Geometric thinking level. Also, 33% of the final year students attained Van Hiele level 1, while 22.5% reached level 2. The rests are 1.5% and 0.5% for levels 3 and 4 respectively. The said publication conclusion of Van Hiele Geometric thinking revealed that nearly 43% of Ghana's final year Secondary School students did not attain any Van Hiele Geometric thinking level.

In Baffoe & Mereku (2010), a sample size of 188 from Winneba Senior High School and Zion Girls schools were used. Both schools were from the Winneba metropolis. The study sought to measure the Van Hiele levels of geometric thinking of Ghana's SHS 1 students. As soon as the students entered the SHS and stayed on campus for four weeks, the study was conducted. The study revealed that Ghana's SHS 1 students performance were lower than their colleagues from other countries when the analysis of Van Hiele geometric thinking was done. The results analysis showed that 59%, 11%, 1% of Ghana's SHS 1 students attained Van Hiele levels 1, 2 and 3 respectively.

2.3. Gender Difference in Mathematics

According to (Doris, O'Neill and Sweetman, 2013; Kim and Law, 2012) there is growing interest around the world on studies on gender differences in mathematics performance. Research studies notable (Hyde, Geiringer, and Yen, 1975; Mann, Sasanuma, Sakum and Masaki, 1990) have shown that gender differences in Mathematics learning are not clear during elementary school years but female begins to fall behind male during the intermediate levels. The females fall further, behind during high school years (Fennema, 1980; Leder, 1985). Many of the research findings showed that sex differences in Mathematics are varied. Forgasiz (2005) insist that gender should be a concern in mathematics education because of its importance. He also argued that it is significant to include gender as a variable in research studies analysis even if it is not the main focus of the study concerning mathematics education. Moreover, Armstrong (1981), Lloyd, Walsh, and Yailagh (2005) stress that gender is an important factor in mathematics learning as cited by Halat (2008). This argument has motivated the researcher to assess this variable. As cited by Halat (2008), research works by the following researchers (Grossman & Grossman 1994; Lloyd et al, 2005) has shown that there are differences between achievement of male and female students in different content areas of mathematics such as computation, measurements, problem solving and spatial visualization.

A study by Armstrong (1981) showed that female students perform better than their male counterparts in computation and spatial visualization. Also, literature elsewhere indicates that female students' performance has been better than their male counterparts (Arnot, David & Weiner 1999; Hydea & Mertz, 2009). Ezech (2005) reported that Male students lagged behind their female colleagues in terms of delayed formalization approach when senior secondary school students were tested in sequences and series. Ezech also, reported delayed formalization approach enhanced students achievement hence is good in teaching and learning of mathematics. The purpose of the said research was to study the effect of delayed formalization approach on senior secondary school students' achievement in sequences and series. That study took place in Obollo Education Zone of Enugu State in Nigeria. A quasi-experimental design was used in which the 240 senior secondary form two students comprising 130 males and 110 females were used as the subjects. Furthermore, another study was conducted by Ogbonna (2007), where female students performed better than male counterparts. The study area was in Abia State in Nigeria. The purpose of the study was to investigate the effect of achievement and retention in Number and Numeration where two constructivist instructional models were implemented. A quasi-experimental design was used in which 290 JSIII students were used as the sample. The detailed of his findings discovered that students who went through the two constructivist instructional models (IEPT and TLC) accomplished and retained higher than students who were taught the conventional method.

According to Fox and Cohn (1980), male students' performance was better than their female colleagues when they both took the Scholastic Aptitude test at the high school level. Also, Smith and Walker (1988) found significant sex-related differences in favor of the male students in geometry at the tenth-grade level. Randhawa (1994) also found that males outperformed females in Mathematics during high school years. Studies conducted by the following researchers (Fennema, 2000; Kaiser-Messmer, 1994) have shown that male students performed better than female in mathematics. Similarly, studies by (Fox, Brody & Tobin, 1980; Hedges & Nowell, 1995; Peterson & Fennema, 1985; Randhawa, 1994) on standardized math tests as cited by Asante (2010) showed that male students generally achieved higher results than their female counterparts.

According to Halat (2006), there was no statistical difference between male and female students in the acquisition of their geometric levels when VHGT was employed to collect data. Again, Halat (2008), concluded that he did not find a statistical difference in geometric reasoning levels and mean among male and female students when VHGT was used. Also, Arhim and Offoe (2015), concluded that there were no gender differences among the males and females in Senior High School form one when they were tested on problem-solving abilities. Armstrong (1981), again also expresses his view that there was no statistical difference between boys and girls on achievement at sixth grade when their skills were tested on measurement application, geometry application, and statistics and probability.

3. Methodology

3.1. Research Design

The survey approach was used to investigate gender differences in geometric reasoning levels and gender differences in achievement scores of PSTs Van Hiele theory levels of Geometry thinking in E.P. College of Education, Bimbilla-Ghana.

3.1 Population

The population of this study was four hundred and seventy-three (473) level 200 PSTs. Comprising of both Mathematics/Science 82 representing 17.3 % PST and general programmes 391 representing 82.7 % PST in E.P. College of Education-Bimbilla. The population is made up of 325 representing 68.7% male and 148 representing 31.3 % female.

3.2. Sample Size and Sampling Procedure

General programme PST of Three hundred and fifty-one (351) representing 74.2% of the population was used as the sample for the study. The sample had a minimum age of 18 a maximum of 38 years, modal age was 22 years and a mean age of 23 years. The sample composed of one hundred and thirty-three (133) representing 37.89% female PST and two hundred and eighteen (218) representing 62.11% male PST. Convenient sampling was adopted in selecting the college for the study because the researcher teaches in the college. While simple random sampling was also adapted to select the general programme PST.

3.3. Research Instrument

In order to find answers to the research question and the hypotheses for this study, the general programme PSTs were given the Van Hiele Geometry Test (VHGT) to determine their geometric thinking levels. The questions were structured sequentially into five (5) subgroups from the very easy to difficult. Each subgroup covered the Van Hiele geometric thinking levels

This study used the entire VHGT 25-item multiple choice test developed by Usiskin (1982) under the Cognitive Development and Achievement Secondary School Geometry (CDASSG) special programme. The VHGT has been used by most masters' and Ph.D. Dissertation candidates notably by (Hoffer, 1983; Usiskin, 1982; Mayberry, 1983; Fuys et al 1988) used it to assess their students Van Hiele geometric thinking levels. The following researchers (Abdullah & Zakaria, 2013; Halat, 2008; Armah et al, 2017; Asemani et al, 2017) also used it in their publications.

3.4. Validity and Reliability

The following researchers (Usiskin, 1982; Atebe, & Schafer, 2008; Baffoe & Mereku, 2010), have used the VHGT and acknowledged that it is reliable and valid for use by incoming researchers. Both face and content validity of the VHGT was conducted by two experienced mathematics tutors of the researcher's department to ensure it meets the Colleges of Education Geometry course outline. The tutors concluded that it met the pre-service teachers' standard and hence cleared it for administering.

According to Armah et al, (2018) testified estimations of the reliability of the VHGT ranged from 0.69 to 0.78. In order to state the reliability of the test items, Thirty (30) PST were engaged to write the pilot test. Those Thirty-five (35) PSTs were not part of the main sample for the actual study. To determine the reliability coefficient of the instruments, the Kuder-Richardson formula 20 methods were utilized. Analysis of the PSTs' pilot test score of reliability coefficient of 0.72 was realized. This value indicates a high degree of reliability of the instrument. The following reliability coefficients were recorded for the various levels. Level 1, 2, 3, 4 and 5 had 0.48, 0.40, 0.30, 0.43 and 0.10 respectively. From the pilot test, 3 (10%) out of the 30 PSTs did not attain any van Hiele level or precisely attained level 0. Six (6) PST representing 20%, thirteen (13) PST representing 43.3%, and six (6) PST representing 20% attained van Hiele levels 1, 2 and 3 respectively. Also, two (2) PST representing 6.7% and zero (0) MPST representing 0 % achieved van Hiele levels 4 and 5 respectively. The pilot test analysis indicated 7, 18 and 12.1 as the minimum, maximum and average marks respectively.

3.5. Nature of the Questions

Questions	Levels	Features
1-5	1	It is about visual form. It aims to determine whether the students recognize the shape by looking at the shape of the figure.
6-10	2	It is concerned with the Characteristics of the forms and on the one hand it aims to show that the students do not know the forms and on the other hand they do not know the Characteristics of the forms.
11-15	3	It determines whether students can recognize the relationships between forms. They identify students who respond correctly to questions in this group and have proven that they have knowledge of axioms.
16-20	4	It is the question of reasoning and logical deduction. In these questions, it is determined whether the students are at a level of understanding and writing.
21-25	5	The questions at this level are used to determine whether students can reason in Euclidean and Euclidean geometries.

Table 2: Characteristics of Van Hiele Geometry Thinking Test

Source: (Hurma, 2011, P.60)

3.6. Administration of the VHGT

Prof. Zalman Usiskin gave his approval and advice through email before the administering of the VHGT. Prior to his approval, a letter was written to him seeking his permission to use his test items. After submitting his demands, the VHGT was administered to the general programme PSTs in the 2017/2018 academic year, second semester to level 200 PST in the Colleges on 9th June 2018. The PST answered the questions on the question paper, the duration was 45 minutes.

3.7. Grading Systems and Level Assignment

1 mark each was allocated to correct answer to the 25 items multiple choice test. The possible minimum was expected to be 0 and expected maximum was 25 marks. For the level assignment, '3 out of the 5' correct success criterion were employed as recommended by Usiskin (1982). Meaning if a pre-service teacher correctly answered at least '3 out of the 5' items in any of the 5 subgroup correctly, then such a student was considered to have mastered that VHGT level. The PSTs were assigned a weighted sum as follows.

- 1 mark for attaining the standard on items 1-5 (Level 1, Recognition)
- 2 marks for attaining the standard on items 6-10 (Level 2, Analysis)
- 4 marks for attaining the standard on items 11-15 (Level 3, Ordering)
- 8 marks for attaining the standard on items 16-20 (Level 4, Deduction)
- 16 marks for attaining the standard on items 21-25 (Level 5, Rigor) proposed by Usiskin (1982).

According to Usiskin (1982), Classical Case and Modified Case are the two main ways that can be used in assigning levels to students. This study employed the modified case in assigning the PST. Usiskin (1982) emphasized that in assigning levels in either the Classical or Modified Case requires that a student's responses must satisfy property 1 of the levels that is students at level n satisfy the criterion not only at that level but also at all preceding levels. Meaning, a student who satisfies the criterion at levels 1, 2 and 5 would be assigned the Modified Van Hiele Level 2. A student who satisfies the criterion at level 3 only would not be assigned either classical or modified Van Hiele level. This study adopted the Modified Case because it fits more students consistently than the classical Van Hiele level Usiskin (1982).

3.8. Data Analysis

Quantitative data analysis was used which is consistent with Usiskin (1982), MayBerry (1983), Atebe (2008), Armah et al (2017), Asemanni et al (2017), Anas (2018) and Baffoe & Mereku (2010) researchers studies on Van Hiele theory of geometry learning. Microsoft Excel 2013 and SPSS Version 23 were employed to do both Descriptive and inferential statistics.

4. Results

The purpose of this study was to use Van Hiele Geometric thinking test to classify general programme PST geometric thinking levels and to find out whether gender differences exist in achievements scores along the levels 1 to 5 in E.P. Colleges of Education, Bimbilla. The results of the study are organized by means of descriptive statistics and inferential statistics.

4.1. Comparison of Performance between Male and Female PSTs in VHGT from Level 1 to Level 5

4.1.1. Research Hypothesis 1

- Ho: There is no statistically significant difference in achievement scores in VHGT level 1 between male and female PSTs. Table 3 shows the descriptive statistics of the male and female result of the independent samples t-test.

Group	N	Mean	Std. Dev.	DF	t	significance
Female	133	2.74	.984	349	-.694	.488
Male	218	2.81	.982			
Total	351					

Table 3: Descriptive Statistics and Independent Samples T-Test for Female and Male on Achievement Scores in VHGT Level 1 among the PSTs

The results of the independent sample t-test shows that there is no difference in VHGT achievement between male ($M=2.81$, $SD=0.9842$) and female ($M=2.74$, $SD=0.9840$, ($t(349) = -0.694$, $p = .488 > .05$). The mean difference was 0.7 was recorded slightly in favor of the male PSTs. The performance of the male and female was not statistically significant in the VHGT in level 1 among the PSTs. The null hypothesis is therefore upheld at $\alpha=0.05$.

4.1.2. Research Hypothesis 2

- Ho: There is no statistically significant difference in achievement scores in VHGT level 2 between male and female PSTs. Table 4 shows the descriptive statistics of the male and female result of the independent samples t-test.

Group	N	Mean	Std. Dev.	DF	t	significance
Female	133	2.39	1.167	349	-.548	.584
Male	218	2.46	1.095			
Total	351					

Table 4: Descriptive Statistics and Independent Samples T-Test for Female and Male on Achievement Scores in VHGT Level 2 among the PSTs

Table 4 displays the mean of male ($M=2.46$, $SD=1.095$) and female ($M=2.39$, $SD=1.167$), with mean difference of 0.7 in favor of the male PST as in level 1. The result of the independent test sample also indicates that there is no statistically significant difference on PSTs reasoning at VHGT level 2 with ($t(349) = -0.548$, $p = .584 > .05$). The performance between the male and female PSTs was not statistically significant hence they were reasoning equally at VHGT level 2, hence the null hypothesis cannot be rejected at $\alpha=0.05$.

4.1.3. Research Hypothesis 3

- Ho: There is no statistically significant difference in achievement scores in VHGT level 3 between male and female PSTs. The result of analyses is presented in Table 5.

Group	N	Mean	Std. Dev.	DF	t	significance
Female	133	1.76	.872	349	-.341	.733
Male	218	1.79	.935			
Total	351					

Table 5: Descriptive Statistics and Independent Samples T-Test for Female and Male on Achievement Scores in VHGT Level 3 (Informal Deduction) among the PSTs

The results of the independent t-test samples in table 5 showed that there is no statistically significant difference between male ($M=1.79$, $SD=0.935$) and female ($M=1.76$, $SD=0.872$) when they were tested with the VHGT. The independent sample t-test recorded the results as ($t(349) = -0.341$, $p=0.733>0.05$). The mean difference was 0.03 in favor of the male PSTs but was not statistically significant at VHGT level 3. Hence PSTs reasoning capabilities among male and female are the same. By these results the null hypothesis cannot be rejected at $\alpha=0.05$.

4.1.4. Research Hypothesis 4

- Ho: There is no statistically significant difference in achievement scores in VHGT level 1 between male and female PSTs. The result of analyses is presented in Table 6.

Group	N	Mean	Std. Dev.	DF	t	significance
Female	133	1.01	1.019	349	-.147	.883
Male	218	1.02	.908			
Total	351					

Table 6: Descriptive Statistics and Independent Samples T-Test for Female and Male on Achievement Scores in VHGT Level 4 among the PSTs

From table 6 the results of the independent sample t-test showed that ($t(349) = -0.149$, $P = 0.883 > 0.05$) between the male and female with small mean difference of 0.1 in favor of the male. The descriptive statistically further indicates that the male ($M=1.02$, $SD=0.908$) and female ($M=1.01$, $SD=1.019$) was not statistically significant when the VHGT level 4 was tested. Therefore, the null hypothesis is upheld for VHGT level 4.

4.1.5. Research Hypothesis 5

- Ho: There is no statistically significant difference in achievement scores in VHGT level 5 between male and female PSTs. The result of analyses is presented in Table 7.

Group	N	Mean	Std. Dev.	DF	t	significance
Female	133	.75	.848	349	-.518	.605
Male	218	.80	.789			
Total	351					

Table 7: Descriptive Statistics and Independent Samples T-Test for Female and Male on Achievement Scores in VHGT Level 5 among the PSTs

The results from the independent sample t-test shows that there is no statistically significant difference in the scores of male ($M=0.80$, $SD=0.789$) and female ($M=0.75$, $SD=0.848$), ($t(349) = -0.518$, $p=0.605 > 0.05$). From table 6 the p-value (0.605) is greater than the significance level of 0.05. Therefore, I failed to reject the null hypothesis. A mean difference of 0.5 was recorded in favor of the male but was not significant. Hence male and female PSTs reasoning VHGT 5 was the same.

4.1.6. Research Hypothesis 6

- Ho: There is no statistically significant difference in achievement scores in VHGT levels 1 to 5 between male and female PSTs. The result of analyses is presented in Table 8.

Group	N	Mean	Std. Dev.	DF	t	significance
Female	133	8.65	2.478	349	-.870	.385
Male	218	8.89	2.502			
Total	351					

Table 8: What Is the Overall Gender Difference in Terms of Achievement Scores in VHGT among the PSTs from Level 1 to Level 5

The independent sample t-test results indicate that there is no statistically significant difference in VGHT for all the levels between the male and female PSTs. The mean score of the male is slightly better than the female ($8.89 > 8.65$). The mean difference was 0.24. The standard deviation for the male and female is recorded as 2.502 and 2.478 respectively. Table 8 further indicates that, ($t(349) = -0.870$, $p=0.05$). Hence the null hypothesis is upheld.

Further analysis for all the levels, for level 1 the combine mean, standard deviation, median, and mode are 2.78, 0.98, 3.00 and 3.00 respectively. Also, the minimum and maximum score for level 1 is 0 and 5 respectively. Level 2, analysis has the combine scores as follows (Mean = 2.43, $SD=1.12$, Mode=2.00, Median = 2.00, Minimum = 0, and Maximum = 5). For level 3 (informal deduction stage), the following were recorded for both male and female PSTs (Mean = 1.78, $SD=0.91$, Mode=2.00, Median = 2.00, Minimum = 0, and Maximum = 5). The following were subsequently recoded for the formal deduction stage of Van Hiele level 4 for both male and female PSTs (Mean = 1.02, $SD=0.95$, Mode=1.00, Median = 1.00, Minimum = 0, and Maximum = 5). The rigor stage (level 5) range was 3 which indicated a minimum score of 0 and maximum score of 3. Further analysis from the rigor stage shows 0.78, 0.81, and 1.00 for mean, standard deviation and median respectively. Majority of the students scored zero at this level.

Further analysis for all the levels combined also revealed that, 20 (5.7%) PSTs got the mean mark of 12. As much as 298 (84.9%), PSTs scored marks below the average mark of 12, while 33 (9.5%) scored marks above the mean mark 12.

4.1.7. Research Question 1

Sought to find out, what reasoning stages do male and female PSTs reach in VHGT.

Gender	No Level Reached	Visualization Level 1	Analysis Level 2	Order Level 3	Deduction Level 4	Rigor Level 5
Female	56 (42.1%)	40 (30.1%)	26 (19.5%)	9 (6.8%)	2 (1.5%)	0 (0 %)
Male	75 (34.4 %)	74 (33.9%)	47 (21.6%)	19 (8.7 %)	3 (1.4 %)	0 (0 %)
Total	131 (37.3%)	114 (32.5 %)	73 (20.8%)	28 (8.0%)	5 (1.4 %)	0 (0 %)

Table 9: Overall Gender Van Hiele Levels Attained by the PST Represented by Number and Percentage Levels

In determining the PSTs reasoning levels with the VHGT. Table 9 provides a summary of the distribution of VHGT level 0 to level 5. The most frequent level or stage apart from the no level reached (pre-visualization) for female is 30.1% while that of the male is 33.9%. The male PSTs at level 2 obtained 21.6% which was better than the female value of 19.5%. The female teachers obtained levels 3, 4 and 5 as 6.8%, 1.5% and 0% respectively. Their male counterparts also obtained 8.7%, 1.4% and 0% for level 3, 4 and 5 respectively. Strangely, majority at any of the levels were found in level 0 that is the pre-visualization. The pre-visualization for the male is 34.4% while pre visualization for the female is 42.1%. A close look at all the levels the male PSTs geometric thinking levels was higher than their female counterpart except level 4 where it favored the female PST. The closest level was level 4 that is at logical deduction level. For the percentage differences in the levels, 0.38%, 2.1%, and 1.9% representing levels 1, 2, and 3 respectively all favored the male PSTs over their female colleagues. However, the female PSTs gained a percentage of 0.1 over their male counterparts in level 4. Level 5 was equal for both sexes. Finally, the pre-visualization was in favor of the male PSTs with a difference of -7.7%.

Also, the bar chart numbered figure 1 below is showing the numerical frequency pictorial comparison presentation of the male against the female PSTs.

Figure 1: Overall Van Hiele Levels Attained by the PST

4.2. Discussion

The purpose of this study was to investigate General programmes male and female PST Van Hiele levels of geometric thinking and also Sought to find out whether gender differences exist between the male and female PST VHGT of E.P. College of Education, Bimbilla-Ghana.

For all the null hypotheses from the hypothesis 1 there was no significant difference between the male and female PSTs VHGT thinking of achievement score. The male PSTs attained a higher mean than their female counterparts. The mean difference was 0.7 hence the null hypothesis was not rejected at 0.05 significant levels. Also, analysis from hypothesis 2

showed that there was no significant difference between the male and female PSTs. It revealed a mean difference of 0.7 in favor of the male PSTs. The null hypothesis was upheld at 0.05 significant levels. Hypothesis 3 which is about the informal deduction gave a mean difference of 0.03, also in favor of the male PSTs but it was not statistically significant at 0.05 hence the null hypothesis was not rejected. Hypothesis 4 is formal deduction, it revealed a mean difference among the PSTs of 0.1 was the closest in all the levels hence it was not statistically significant to reject the null hypothesis. For the rigor stage, the null hypothesis was not rejected because there was no statistically significant difference. The mean difference was 0.5 also in favor of the male PSTs.

The analysis from all the levels combine revealed that there was no significant difference between the male and female PSTs in achievement score when the VHGT was administered. This results is related to the findings by the following researchers (Halat, 2008; Arhim and Offoe, 2015; Armstrong (1981), who have all found no statistical differences in their researches on gender differences in mathematics. The male had a mean of 8.89 and the female had a mean of 8.65 with a difference of 0.24 in favor of the male PSTs. The standard deviation for the male was 2.50 while the female value was 2.49 with a difference of 0.01.

The research question was to find out the reasoning stages of the PSTs in VGHT. For the pre-visualization level, majority of the female PSTs of 56 (42%) could not attain any of the Van Hiele levels which is similar to (Anas; 2018; Asemani, et al, 2017) papers where some students reached level 0. Further analysis shows that 40 out of the 133 PSTs used as the female sample attained 30% for the VH level 1. The female PST also attained 26 (19.5%) for level 2. The implication here is that majority of this PST cannot teach their basic school students because those students are supposed to be at level 3. These findings are in line with previous studies by (Halat & Sahin, 2008; Ndlovu, 2014; and Armah, et al, 2017; Anas, 2018) where some preservice teachers fell below that standard. Also, 9 (6.8 %) of the female PSTs reached level 3. Results from female level 4 and level 5 had 2(1.5%) and 0 (0%) respectively which is confirming Armah, et al (2017), and Anas (2018), who had similar results, where very few students attained those levels.

The male PSTs reasoning stages of VH levels revealed that 74 (33.9%) and 19 (8.7%) reached levels 1, 2 and 3 respectively. These findings are in line with previous studies by (Halat & Sahin, 2008; Ndlovu, 2014; Anas, 2018; and Armah, et al, 2017), where significant number of students fell below level 4. Level 4 had 3(1.4%) and level 5 recorded no student representing zero (0) percent. Also, 75 (34.4%) of the male PST attained level 0 pre visualization stage similar to Anas (2018) who had 50.3% mathematics PSTs at that level.

The average mark was set at 12. Both the male and female scores recorded 5.7% as the mean mark. Also, majority of 298 (84.9%) from the entire sample scored marks below the average mark of 12 consistent with Anas (2018) findings where majority 269 representing 90.3% of the Mathematics preservice teachers attained marks below 12. Also, from the analysis only a few of 33(9.4%) out of the 351 PST scored marks higher than the mean mark consistent with Anas (2018) paper where 4.3 % Mathematics pre-service teachers attain those marks.

4.3. Conclusion

The purpose of this study was to investigate General programmes male and female PST Van Hiele levels of geometric thinking in E.P. College of Education, Bimbilla-Ghana. It also sorts to find out General programme gender differences scores among PST Van Hiele theory levels of Geometry thinking in E.P. College of Education, Bimbilla-Ghana.

Three Hundred and fifty-one (351) PSTs were used for the study as sample from E.P. College of Education, Bimbilla. The Van Hiele Geometry Test (VHGT) from the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) test items were given the PSTs. From the results, only a few 33(9.4%) are qualified to teach at the basic school in Ghana. Also, majority 318 (90.6%) attained between level 0-level 2 hence are not qualified to teach at the JHS because they are below the standard for that level. To be able to handle geometry very well at the basic schools, Crowley (1987), Usiskin (1982) and Armah, et al (2017) documented that basic school teachers need to be at least level 3. The study revealed that there was no statistically significant difference in all the levels between the male and female PSTs in geometry specifically VHGT. Generally, 269 (90.3%) of the PSTs scored below the mean mark of 12 which is very poor for their status as PSTs, while 33(9.4%) managed a scored above the mean mark. The implication is that since geometry topics keeps featuring at the final basic school examinations students will continue to record low grades mathematics.

4.4. Recommendations

Based on the finding of this study, it is recommended that:

- Geometry Course outline/syllabi for Colleges of Education should be revised to create room for Van Hiele phase based approaches for the new bachelor programmes slated for September 2018.
- Mathematics Teachers at all level should be encouraged by their associations, Ghana Education Service and Ministry of Education to always use teaching and learning materials in teaching mathematics.
- The National Council for Tertiary Education, Ghana Education Service and Ministry of Education and Mathematics Teachers Associations of Ghana (MAG) should organized professional development workshops for all teachers to learn Van Hiele phase based instruction.

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