INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH & DEVELOPMENT

ISSN 2278 – 0211 (Online)

Colleges of Education Students' Mathematics Proficiency: Assessing Strategic Competency and Adaptive Reasoning during Supported Teaching in Schools

Samuel Baah-Duodu
Lecturer, Department of Science Education,
AAMUSTED-Mampong Campus, Ghana
Ekow Hegan John
Tutor, Department of Mathematics/ICT,
Methodist College of Education Akim Oda, Ghana
Solomon Amoaddai
Tutor, Department of Mathematics/ICT,
Methodist College of Education Akim Oda, Ghana
Maxwel Adu Gyamfi
Tutor, Department of Mathematics/ICT,
Agogo Presbyterian Women's College of Education, Ghana
Douglas Koranteng Ndamenenu
Tutor, Department of Mathematics/ICT,
Mampong Technical College of Education, Ghana

Abstract:
Researchers in mathematics education have established the distinction between mathematical understanding and mathematical thinking and that has resulted on the call for comprehensive assessment practices that focuses on both content and process. While it has been established that teacher knowledge is vital in teaching basic school pupils, less is known about colleges of education students' proficiency levels at point of exit from the maiden B.Ed program in colleges of education in Ghana. This study reports how mathematics proficiency is partially used as mathematics assessment towards colleges of education students' strategic competency and adaptive reasoning during supported teaching in schools. Mixed methods involving descriptive and explanatory designs were employed to gather quantitative and qualitative data for analysis. Results revealed that pre-service teachers are approaching proficiency whereby exhibiting more knowledge in strategic competency than adaptive reasoning in the mathematics content they will be teaching. Factors including program structure, sustained lesson reflections with mentors and quality of teaching experiences at partner schools aided in developing pre-service teachers' proficiency. Although males attained higher proficiency scores than females, there was no statistically significant difference in proficiency levels between males and females. Recommendations suggest to further studies to explore the relationship between pre-service teachers' mathematics proficiency and their mathematics self-efficacy.

Keywords: *Mathematics proficiency, strategic competency, adaptive reasoning, Supported Teaching in Schools (STS), Student Reflective Journal (SRJ)*

1. Introduction

Teacher training has always been the heartbeat and passion of stakeholders in teacher education. As part of ensuring quality standards of teaching at the basic school level, the Government of Ghana introduced a four year Bachelor of Education program which started in 2018. This policy and initiative required all colleges of education to adhere to the National Teaching Standards (NTS) and the National Teacher Education Curriculum Framework (NTECF) in training prospective basic school teachers. These policy documents directs teacher educators to train colleges of education students (who are usually referred to as *pre-service teachers*) along a threshold which is premised on subject matter towards the domains; Literacy, Numeracy and Supported Teaching in Schools (STS).

Supported Teaching in School (STS) is one of the four pillars of the NTECF which set out the knowledge, skills and understanding necessary for effective teaching. STS lies at the heart of the B.Ed program. It is through STS that the student teachers apply and develop the skills, knowledge and understanding acquired in their college-based training in partner basic schools with the support of mentors and link tutors. STS is integrated into colleges of education students' training across the four years. Teacher education should be viewed as an applied professional qualification that requires student teachers to apply the concepts and strategies they are simultaneously learning about in their coursework within practical settings (MOE, 2017).

This requires that there are trained mentors in all partnership schools who are able to support and assess student teachers' progress to meeting the NTS. The Curriculum also emphasizes the importance of college tutors preparing student teachers for their placements and integrating their school-based learning into the overall training. The Curriculum stresses the importance of assessed, supported placements being used to develop student teachers' competence over time.

In order to improve the quality of teaching, the NTECF (2019) outlined three general intersecting areas of knowledge that beginning teachers must acquire, and this has implications for what is included in initial teacher education programmes. Firstly, knowledge of learners and how they learn and develop within a social context; secondly, understanding the subject matter and curriculum goals (skill to be taught) in light of the social purposes of education; and thirdly, understanding the teaching in light of the content and learners to be taught, as informed by assessment and supported by a productive classroom environment (p. 21, 22).

1.1. Statement of Problem

The Ghana Accountability for Learning Outcome Project (GALOP, 2019) reports that despite huge expenditure and initiatives through curriculum reforms by Government to make Ghana a mathematics friendly nation, teaching and learning mathematics has attracted questionable efficiency especially at the basic school level. Although the proportion of professionally trained teachers who are assumed to implement basic school mathematics curriculum has increased by 65%, there exist poor national learning outcomes from national assessments. Early Grade Mathematics Assessment (EGMA), National Education Assessment (NEA), Basic Education Certificate Examination (BECE) and West African Senior Secondary Certificate Examination (WASSCE) are evident to this assertion (MOE, 2017).

Although the Ministry of Education (2017) reports that teachers are resourced with the capacity to deliver but on the contrary results consistently paints a blue picture of students' performances. The teaching and learning of mathematics in schools urgently need improvement. Ghana needs a mathematically literate citizenry, but most graduates from colleges of education lack adequate mathematical competence and proficiency. Achievement gaps have persisted between teachers of mathematics and their colleagues in other subject areas. As a matter of national interest, the overall level of mathematical proficiency must be raised, and the differences in proficiency among student teachers must be curbed. Improving proficiency in mathematics and eliminating the gaps in proficiency among students has been the goal of the National Teaching Council (NTC). Developing national teaching standards and assessments strategies by the NTC is intended to measure the degree to which pre-service teachers attain proficiency. Policies which include the introduction of the Ghana Teacher Licensure Exams have been instituted to attract and retain more effective teachers of mathematics. This is because; 'success in mathematics is a predictor of success in college' (Gervasi, 2004, p. 3). New curricular materials have been developed along with training and coaching programs intended to provide teachers with the knowledge and skills needed to teach effectively. However, these efforts have been supported by limited research which is part of the reason for the limited success.

1.2. Purpose of the Study

The purpose of this study was to assess pre-service teachers' mathematics proficiency in order to determine their readiness and preparedness to teach basic school mathematics.

1.3. Research Questions

This study was guided by the following questions;

- What level of proficiency do pre-service teachers possess at the point of exiting B.Ed program?
- Is there a significant difference between pre-service teachers' strategic competency and adaptive reasoning?
- What perceived factors enhance developing pre-service teachers' mathematics proficiency?

1.4. Significance of the Study

This study intends to reveal the competence of pre-service teachers in teaching the four mathematics strands. This will also inform teacher educators the preparedness of pre-service teachers to teach mathematics at the basic school. Finally the study will contribute to mathematics education by offering practical guidance to mathematics education researchers the progress towards assessing mathematics proficiency.

1.5. Organization of the Study

This work is in five sections. Section one entails the introductory background to the topic, statement of problem, purpose of the study, significance of the study, research questions. Section two and three captured the review of related literature and methodology respectively. Section four consisted of analysis and findings while section five looked at the summary, conclusion and recommendations.

2. Literature Review

2.1. Mathematics Proficiency for Teaching

Mathematics educators aim at preparing well qualified and competent teachers who are capable of delivering subject matter as well as demonstrating pedagogical knowledge through creative and innovative approaches. Several frameworks of teacher knowledge ensures deepening understanding of knowledge in mathematics (Ball et al, 2004; 2008; 2009). These studies described teachers' knowledge for teaching as specialized content knowledge which is different from common teaching knowledge that student teachers need. It is more prudent for teachers to relate subject matter to build students' capacity in making connection of mathematics topics for conceptual power which may be referred to as mathematics proficiency. According to Wilson and Heid (2010) mathematical proficiency for teaching (MPT) can be viewed from three perspectives: mathematical proficiency, mathematical activity, and mathematical work of teaching where each perspective provides a different view of MPT as a developing quality and not an endpoint (p.3-p.4): mathematical proficiency, mathematical activity, and mathematical work of teaching. Each perspective provides a different view of MPT. MPT is a developing quality and not an endpoint.

Mathematical proficiency includes aspects of mathematical knowledge and ability, such as conceptual understanding and procedural fluency that teachers need and seek to foster in their students. The mathematical proficiency teachers need, however, goes well beyond what one might find in students. This is because students' development of mathematical proficiency usually depends heavily on how well developed their teacher's proficiency is.

Proficiency in mathematical activity can be thought of as 'doing mathematics.' Examples include representing mathematical objects and operations, connecting mathematical concepts, modeling mathematical phenomena, and justifying mathematical arguments. This facet of teachers' mathematical proficiency is on display as they engage students in the day-to-day study of mathematics. Teachers need deep knowledge, for example, of what characterizes the structure of mathematics and how to generalize mathematical findings. The more a teacher's proficiency in mathematical activity has developed, the better equipped him or she will be to facilitate the learning and doing of mathematics.

Proficiency in the mathematical work of teaching diverges sharply from the mathematical proficiency needed in other professions requiring mathematics. One of its aspects is an understanding of the mathematical thinking of students, which may include, for example, recognizing the mathematical nature of their errors and misconceptions. Another aspect of the mathematical work of teaching is knowledge of and proficiency in the mathematics that comes before and after what is being studied currently. A teacher benefits from knowing what students have learned in previous years so that he or she can help them build upon that prior knowledge. The teacher also needs to provide a foundation for the mathematics they will be learning later, which requires knowing and understanding the mathematics in the rest of the curriculum.

Wilson and Heid (2010) asserts that the three components of MPT which are mathematical proficiency, mathematical activity, and mathematical work of teaching together form a full picture of the mathematics required of a teacher of mathematics. It is not enough to know the mathematics that students are learning. Teachers must also possess in-depth and extent of mathematical proficiency that will equip them to foster their students' mathematical proficiency. Mathematical proficiency informs the other two perspectives on MPT: Mathematical activity and the mathematical work of teaching emerge from, and depend upon, the teacher's mathematical proficiency (p.6).

Kilpatrick, Swafford and Findell (2001) outlines five areas of mathematics proficiency; Conceptual Understanding, Procedural Fluency, Strategic Competence, Adaptive Reasoning and Productive Disposition as premises to unravel the meaning of mathematics proficiency. Conceptual understanding refers to understanding of concepts, operations and relations. This frequently results in students comprehending connections and similarities between interrelated facts. Procedural fluency describes the flexibility, accuracy and efficiency in implementing appropriate procedures. Skill in proficiency includes the knowledge of when and how to use procedures. This includes efficiency and accuracy in basic computations. Strategic competence shows the ability to formulate, represent and solve mathematical problems. This is similar to problem solving. Strategic competence is mutually supportive with conceptual understanding and procedural fluency. Adaptive reasoning involves the capacity to think logically about concepts and conceptual relationships. Reasoning is needed to navigate through the various procedures, facts and concepts to arrive at solutions. Finally productive disposition relates positive perceptions about mathematics.

Kilpatrick et al. (2001) reveals that the five stands of mathematics proficiency are closely related to each other bearing an organic correlation where by one relies on the others. These strands are interwoven as well as independent. For one to focus on strategic competency and adaptive reasoning, it is assumed to have naturally developed conceptual understanding and procedural fluency portraying an attempt to cover any mathematics strands. This develops as students gain more mathematical understanding and become capable of learning and doing mathematics. The development of mathematical proficiency takes time. At each level of learning mathematics, students need to make progress along every strand mentioned above because each strand is important and interwoven with the others.

2.2. Basic School Mathematics in Ghana

The introduction of new and modern mathematics in Ghanaian schools aimed at ensuring that schools are reformed and foster the development of mathematical competences. Examples of topics that pupils are expected to learn include; numbers and numerals, number bases, sets of numbers, vectors, clock arithmetic, points in a number plane, measurements of two dimensional and three dimensional figures, use of arbitrary and standard units, transformational geometry, collecting, representing and interpreting data and chances. The learners need to be mathematical literates being conversant with terms like addend, sum, minuend, subtrahend, dividend, divisor, quotient, commutative, associative, distributive, place value, ray, intersection, line segment, mode, rational numbers, integers, to mention only a few. Number, algebra, geometry & measurement and data handling & chance are the four main mathematics strands. These strands are unifying mathematical concepts across all levels of basic education.

The intention of government to prioritize success of mathematics at the basic school level is in line with the goal for quality education, socio-economic development and the skills requirement for global thinking. The National Pre-tertiary Education Curriculum Framework (NPECF) is the policy guideline against which the basic school curriculum is positioned to develop graduates who are problem solvers, have the ability to think creatively and have both the confidence and competence to participate fully to meet global standards. This sought to address the inherent challenges of subjective exam focused approach by ensuring that the content of the national curriculum for change and sustainable development can be internationally benchmarked. Through this, the aim of transforming Ghana into a Mathematics friendly nation within an environment of science and technology will be achieved. Also producing graduates who can engage in life-long learning and can apply the knowledge innovatively and making the people of Ghana more mathematics inclined (NaCCA, 2019).

Pre-service teachers of colleges of education are trained be proficient in teaching mathematics at the basic school level. Therefore teacher education in Ghana aims to prepare teachers imbued with professional skills, attitudes and values as well as the spirit of inquiry, innovation and creativity. This will enable them to adopt to changing conditions; use inclusion strategies and engage lifelong learning. These prospective teachers who are either generalist or specialist are required to have a passion to develop pupils' mathematical and scientific literacy since confidence in numeracy and other scientific skills is a precondition to success across the national curriculum. (MoE, 2017, p.10).

2.3. Essential Learnings and Mathematics Proficiency Scale

The basic school curriculum captures all subjects by identifying the Essential Learning Areas. Teachers and pupils are provided with an Essential Learnings Chart which outlines the focus areas for the term. Each Essential Learning has a Proficiency Scale that enables students and staff to visualize the level of performance necessary to demonstrate Proficiency at each required level. The learning outcome and learning indicators specified in the curriculum are evident to this assertion. Assessment completed at the beginning of each unit of work allows a teacher to place a student at a particular level on the Proficiency Scale. Throughout the unit, ongoing assessment should ideally show students moving up the ladder on each Proficiency Scale. By sharing the Proficiency Scales with students, students build a comprehensive knowledge and can visualize their next level of learning. This empowers students to manage their own learning and to set appropriate goals. Proficiency scale provides a detailed breakdown of the performance levels for students, to communicate expectations of quality around a specific task. When Students are provided with feedback regularly it enables them to determine their growth towards achieving each Essential Learning outcome.

In this study, participants were given several opportunities to demonstrate their ability of each Essential Learning in order to track their progression up the ladder of the Proficiency Scale at the end of the research.

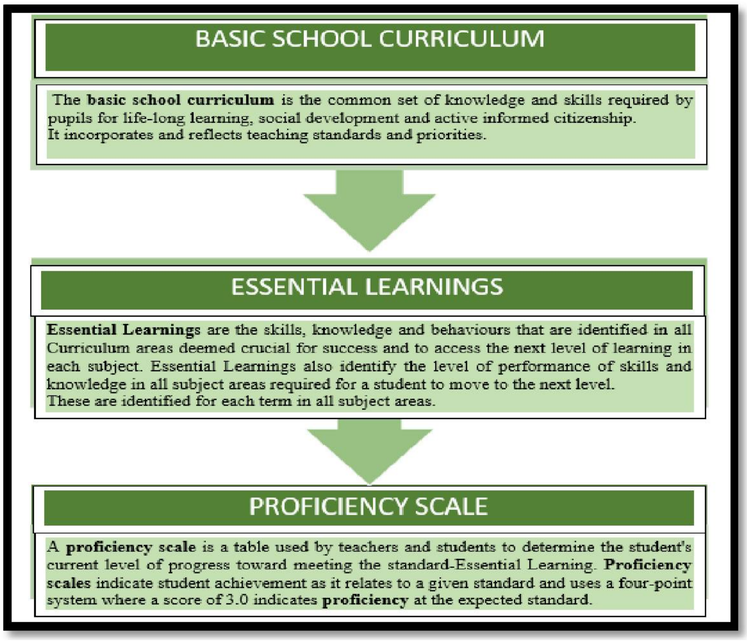

Figure 1: Attaining Proficiency via Essential Learning in the Basic School Curriculum

2.4. Assessing Mathematics Proficiency

Although many researchers do not directly address the five strands of mathematical proficiency in terms of assessment, a few highlights important teaching and assessing strategies towards this goal (Pai, 2018; Straumberger, 2018; Swan & Foster, 2018). Assessing pre-service teachers' mathematics proficiency is an extremely powerful role in teacher education. But testing proficiency raises myriad of questions that may include; what kind understanding do the test assess? Do the test capture all kind of mathematical thinking? Are the test focus area equitable? Can these tests raise the attention towards improved standards of mathematics instruction?

This is because different stakeholders including researchers, educators, test consumers and policy makers have great interest in information obtained from these assessments for diverse needs and decision making. Many of these stakeholders may have little or no knowledge about the need of the other. For instance although reliability and validity are key elements in test construction but will the issue of reliability and validity be a matter of concern to all stakeholders or worthless?

In the same manner policy makers may be more interested in high standards reflected in high average scores or percentages may not understand teachers view on teachers' need for formative diagnostic test or how interruptive teaching towards test which does not align to national curriculum. It is therefore necessary to focus on teachers' need to develop strategic competency and adaptive reasoning to provide the opportunity to 'do' what has been learnt. What to assess in a general thinking for learning may differ from a refined mathematics thinking in which mathematical competency is a goal for mathematics instruction. According to the National Council of Teachers of Mathematics (NCTM, 2000) *Curriculum and Evaluation Standards for School Mathematics*, the general basis for assessing strategic competency and adaptive reasoning focus on problem solving, reasoning, making mathematical connections and communicating mathematics orally and in writing.

It may seem straightforward to assess students' proficiency in number, algebra, or geometry, statistics or probability but the complexities involved in obtaining accurate measurements of students' content understandings are actually complex. Measuring students' abilities to solve problems, reason, and make mathematical connections needs to be more focused as to what to assess. Which mathematics will be assessed when assessing mathematical thinking proficiency and competency: strategic competency and adaptive reasoning? One of the complexities is the issue of assessment in general, and mathematics assessment in particular as well as the degree of attention given to a particular topic or strand. Focusing specifically on assessments that capture students' mathematical understandings in accurate ways point to assessing, mathematical proficiency.

According to NCTM's *Principles and Standards* (NCTM 2000), a consequence of the 'standards movement' has been the establishment of high-stakes accountability measures tests designed to see whether students, schools, districts, and states are meeting the standards that have been defined. The consequences of meeting or not meeting those standards have been enormous. For students, failing a test may mean being denied promotion to the next level. Consequences for schools are complex. As has been well documented, students who score much worse on mathematical assessments have higher dropout rates. One mechanism for compelling schools to focus their attention on traditionally lower-performing groups has been to disaggregate scores by groups. No longer can a school declare that it is performing well because its average score is good; all subgroups must score well. This policy raises new equity issues: schools with diverse populations now face more stringent requirements than those with homogeneous populations, and dire consequences if they fail to meet the requirements [NCTM 2006]. Therefore this study is cautioned about the use of a single test score as the sole determinant of student success or failure.

Assessing pre-service teachers via traditional and procedural approach when it comes to mathematics may create the impression that a procedure based course is enough to build students' mathematical knowledge. When students mindlessly practice mathematical procedures, they are becoming proficient at utilizing only procedures without understanding the ideas that underline the procedure. Students may think they are competent at that specific skill, they might fail if they have to deal with a slightly different problem or procedure. Schoenfeld (2007A) calls that an 'Illusion of Competence' (p.10).

Although classroom assessments should focus on what is valued than what is easy to measure, share a common model of student learning, signal to teachers and students what is important for them to teach and learn, Burkhardt (2007) argues that assessing mathematics proficiency should foster developing thinkers by emphasizing on mathematics that often focus on the 'solve phase,' 'formulate,' 'evaluate' or 'interpret' phases (p.86). When working with students and assessing their mathematical proficiency, teachers often focus on the final product of the students' work rather than the complex mathematical thinking that leads to that product (the process). Suurtamm (2018) adds to the discussion, claiming that mathematics assessment intentions need to be visible in all aspects of teaching (planning, instruction, and assessment). The practices that teachers engage in while teaching and assessing provide students with the grounds needed to deepen their mathematics understanding. Mathematics assessment practices must have a relationship with the tasks used to evaluate student learning.

Khairani and Nordin (2011) used test items to assess conceptual understanding, procedural fluency, and strategic competence of 588 14-year-old students. According to their study these three strands were chosen because the excluded strands (productive disposition and adaptive reasoning) are not yet mature enough to assess at that level. In this current study participants already exhibit moderate performance on conceptual understanding and procedural fluency via standardized tests constructed, organized and conducted by Affiliate University. Therefore much attention is given to assessing participants' strategic competency and adaptive reasoning at this time of teaching practice during supported teaching in schools.

According to Ozdemir and Pape (2012), there is a connection between self-regulated learning, problem based learning and the strands strategic competence and adaptive reasoning in that students are meant to reflect on the strategies to use when given a problem while using their autonomy to carry out their plan. In their observations of problem-based lessons, Ozdemir and Pape identified four categories that support the development of students' strategic competence and adaptive reasoning: '(a) the nature of tasks and activities, (b) practices supporting understanding, (c) practices supporting strategic knowledge and skills, and (d) practices supporting motivation to reason' (p.160). Freund (2011) sought to understand teachers' approaches when teaching for mathematical proficiency in an urban school context. Freund exemplified how teachers engaged in problem based teaching to develop mathematical proficiency by filming teaching lessons about algebraic thinking. In his study he asserted productive disposition as a difficult strand to assess.

Groth (2017) ascertained that productive disposition is the most challenging strand to be assessed because teaching approaches has important role to play especially when it comes to creating appropriate mathematical tasks to promote mathematics proficiency. He therefore suggested that teachers can develop fundamental habit of mind by basing daily instructional decisions on observations of their students' strengths. This current study intends to extend Groth's proposition by offering an analysis of mathematics assessment practices, instead of mathematics instructional practices. For this research study, the goal is to work with colleges of education students to provide enriched mathematical tasks combined with a holistic mathematical assessment that evaluates strategic competency and adaptive reasoning. Indeed, mathematics education research indicates that teachers and students can benefit from holistic assessment practices that go beyond the reproduction of procedures and is not solely product-based.

2.5. Potential Factors That Influence Pre-Service Teachers' Mathematics Proficiency

Literature in studies by Rubie-Davies et al (2012) and Bullock (2011) revealed potential factors that affect pre-service teachers' mathematics proficiency resulting on their preparedness to teach. According to these studies it was revealed that four factors including; University experiences, teaching experiences, mathematical knowledge for teaching (MKT) and World experiences influence pre-service teachers' proficiency in mathematics. University experiences refer to the nature of program and courses offered at the university. Teaching experience refers to practicum during field experiences in schools of practice as well as collaboration with practicing teachers in a regular classroom. Mathematics content and pedagogical knowledge forms part of pre-service teachers' mathematical knowledge for teaching. World experiences refer to their race, gender and real life experiences with mathematics. Drawing upon these perspectives, the following potential factors were carved out to suit the setting and purpose of this study after reviewing entries in participants' SRJ; Gender, Program offered at College and Field experience. These factors were agreed on because at level 400, pre-service teachers are expected to undergo these experiences during initial teacher preparation.

The authors perceived that pre-service teachers' MKT could be embedded in their field experience during teaching practice especially when assessing mathematics proficiency. Pre-service teachers develop minimal pedagogical content knowledge in mathematics until they enter a regular classroom to observe practicing teachers and well teaching children in the regular classroom (Baah-Duodu et al, 2019). Therefore MKT (pedagogical knowledge and content knowledge) were excluded from the potential factors for this study.

3. Methods

The study employed a mixed method and descriptive design to ascertain pre-service teachers' mathematics proficiency. To draw interpretation from both quantitative and qualitative data, Creswell (2015) suggests that researchers need to identify and consider a fixed or an emergent design approach that matches the study's problem, purpose and questions in order to be explicit for mixing methods. Therefore the rationale for considering mixed method was that neither qualitative nor quantitative data were sufficient to achieve the purpose of this study. A partial explanatory design was adapted to gather in-depth description of factors that enhanced participants' proficiency in mathematics. The explanatory phase sought for qualitative responses from participants in SRJ to help ascertain and follow up on factors that influence pre-service teachers' mathematics proficiency for teaching.

According to Creswell (2015), explanatory design occurs in in two distinct phases where the collection and analysis of quantitative data is subsequently followed up by a collection and analysis of qualitative data. Although quantitative data was the priority for answering first two research questions, the second phase required qualitative responses from participants' SRJ to answer research question three.

3.1. Sampling Techniques

A purposeful and convenient sampling was considered to narrow the selection of participating institutions to Agogo Presbyterian Women's College of Education, Methodist College of Education and Mampong Technical College of Education. The reason for this approach was the researchers were directly affiliated to these institutions and could easily participate, monitor and supervise the conduct of the study including data collection processes as expected.

3.2. Participants

A total of 354 final year pre-service teachers who have successfully completed three year mathematics content and methodology courses participated in the study. These students were in fourth year teaching practicum at partner schools for their supported teaching in schools (STS) programme. Although participants may belong to different institutions but their training adheres to the national teacher education curriculum framework (NTECF) a common framework under the supervision and mentorship of the National Teaching Council (NTC) and University of Cape Coast (UCC) respectively. Details of participants' are expressed in the table 3.1.

Colleges	Male	Female	Primary Ed	JHS Ed
Agogo		162	150	12
Mamtech	84			84
Methco	88	20	80	28
Total	172	182	230	124

Table 1: Number of Participants from Colleges

3.3. Data Collection and Analysis

A mathematics proficiency test (MPT) was conducted and participants' performances were rated with a mathematics proficiency scale (MPS) adapted from Victorian curriculum (see appendix). The test items were carefully selected from psychometrically validated assessment items from the question bank of National Teaching Council for the conduct of Ghana Teacher Licensure Examination. The intention of the proficiency test was not to assess all mathematics skills but rather a minimum mathematics competency assessment of strategic competency and adaptive reasoning on; number sense, relationship and proportional reasoning, measurement, collecting data, interpreting data and chance. The content distribution of the test items are shown in Table 2 below.

Content	Distribution
Number Sense	32%
Relationship and Proportional Reasoning	40%
Measurement	14%
Data & Chance	14%

Table 2: Distribution of Mathematics Content Dimensions

The 3-hour test comprised of a total of 71 calculator required and non-calculator required items of which 50 aimed at assessing mathematics content items whiles 21 items assessed pedagogy and curricular knowledge. Approximately equal proportion of the MPT items; 36 and 35were mapped to strategic competency and adaptive reasoning categories respectively. The MPT focused on mathematics content and subject matter covering the four strands in the basic school mathematics curriculum. Table 3below shows the distribution of test items for mathematics proficiency domains.

Content	Strategic Competency	Adaptive Reasoning
Number Sense	• 15%	• 15%
Relationship and Proportional Reasoning	• 15%	• 15%
Measurement	• 15%	• 15%
Data & Chance	• 10%	• 10%

Table 3: Distribution of Mathematics Content Dimensions

For the purpose of participants' teaching experiences, the test covered knowledge on pedagogy and curriculum where participants' foundational understanding of the basic school mathematics curriculum, knowledge on assessments and evaluation of students learning were focused. The framework of the pedagogy and curriculum knowledge was such that participants were to identify and interpret pupils' mathematical errors and misconceptions as well as devising proper procedures in correcting the wrongs. Pedagogy section also explored underlying principles of teaching, learning skills, performance standards, core competencies and knowledge in assessment (for, as and of learning). Participants were also expected to exhibit knowledge of growing success of students through feedback and evaluation via professional ethics as enshrined in the national teaching standards (NTS). The component on curriculum focused on participants' knowledge of the learning outcomes and performance indicators of strands and sub-strands, outline, connotations, underlying concepts and ideas of the basic school mathematics curriculum.

Content	Percentage
Identifying and interpreting errors	40%
Correcting errors	40%
Knowledge on curriculum	20%

Table 4: Distribution of items for knowledge on Pedagogy and Curriculum

Participants also responded to a questionnaire items in their students' reflective journal (SRJ) with the purpose of gathering perceptions on mathematics proficiency. The questionnaire sought for biographic and demographic data, mathematics background, and their readiness to attempt MPT, mathematics proficiency ratings as well as factors that enhances their mathematics proficiency. These items were adapted from an already designed questionnaires template by transforming teacher education and learning (T-Tel) for evaluating colleges of education supported teaching in school (STS) programme.

Conceptual Framework	Research Question	Sources of Data
Proficiency Level	1. What level of proficiency do pre-service teachers possess at the point of exiting B.Ed program? 2. Are there differences in pre-service teachers' strategic competency and adaptive reasoning?	Test Scores Mathematics Proficiency & Ability Scale (MPAS)
Factors that enhance mathematics Proficiency	3. What perceived factors enhance developing pre-service teachers' mathematics proficiency?	Demographic Responses Student Reflective Journal (SRJ)

Table 5: Conceptual Framework, Research Questions and Data Collected

Method	Data Collected	Analysis
Quantitative Qualitative	Test Scores Entries of Student Reflective Journal (SRJ)	• Test Scores • Mathematics Proficiency & Ability Scale (MPAS) • Responses in SRJ

Table 6: Data Collected and Analysis

4. Results and Discussion

SPSS was used for the descriptive analysis of the MPT scores. Due to independent and identical characteristics, paired sample t-test was appropriate to be used for testing the significance of mathematics proficiency between participants' gender and program offered at college. To the extent of this study, research surrounding the five strands of mathematics proficiency in the mathematics classroom was limited.

Participants	Mean	Std. Deviation
Overall MPT	70.01	14.78
Strategic Competency	72.02	17.71
Adaptive Reasoning	67.78	14.32
Content strands		
Number Sense	80.23	22.01
Relationship & Proportional Reasoning	64.43	21.21
Measurement	71.70	20.11
Data & Chance	63.12	16.03

Table 7: Summary of Participants' Scores on Mathematics Proficiency Scale (MPS)

Table 7 shows an overall performance (mean = 70%, SD=14.78) which suggests that pre-service teachers demonstrate a moderate proficiency. Participants attained higher scores in strategic competency than in adaptive reasoning ($t=5.839$, $p<0.001$). It was revealed that participants' performances in number sense was higher followed by measurement. Data & chance and Relationship & proportional reasoning recorded a closely least mean score because of the nature of their test items. These items were interrelated to other strands well as demanded more interpretations and analysis. The above depicts that pre-service teachers were approaching proficiency in regards to the observed domains.

Participants		Strategic Competency	Adaptive Reasoning	Total
Male	Mean	3.05	2.86	2.96
	Std. Deviation	1.068	0.947	1.007
Female	Mean	2.96	2.71	2.84
	Std. Deviation	0.959	0.981	0.970
Total	Mean	3.00	2.79	2.90
	Std. Deviation	1.014	0.964	0.990

Table 8: Summary of Participants' Gender Scores on Mathematics Proficiency Scale (MPS)

Table 8 reveals the total scores, mean and standard deviation obtained by participants in the mathematics proficiency test. Strategic competency (mean = 3.00, SD=1.014) was better than that of their adaptive reasoning (mean = 2.79, SD= 0.964) for all gender. Males obtained higher means scores (mean = 2.96, SD= 1.007) as compared to females (mean = 2.84, SD= 0.970).

	Participants	Strategic Competency	Adaptive Reasoning	Total
JHS	Mean	2.89	3.18	3.04
	Std. Deviation	0.868	1.099	0.983
Primary	Mean	2.80	2.72	2.76
	Std. Deviation	0.873	0.992	0.828
Total	Mean	2.85	2.95	2.90
	Std. Deviation	0.871	1.056	0.900

Table 9: Summary of Participants' Program Offered Scores on Mathematics Proficiency Scale (MPS)

Table 9reveals that participants who offered JHS education program had high proficiency scores (mean = 3.04, SD= 0.983) as compared to those offering primary education programme (mean = 2.76, SD= 0.828).

The following were obtained as summary of the results for this study;

- What level of proficiency do pre-service teachers possess at the point of exiting B.Ed program?

The study revealed that pre-service teachers at the colleges of education were moderate and approaching proficiency at the point of exiting their bachelor of education programme.

- Are there differences in pre-service teachers' strategic competency and adaptive reasoning?

Pre-service teachers exhibited higher strategic competency than adaptive reasoning.

- What perceived factors enhance developing pre-service teachers' mathematics proficiency?

Review of SRJ entries revealed pre-service teachers' perceived factors that could influence their mathematical proficiency. A summary of the review revealed that factors included; mathematics background as to mathematics courses offered at Senior High School, program offered at College, teaching experience, curriculum knowledge and self-efficacy. Other factors including program structure, sustained lesson reflections with mentors and quality of teaching experiences at partner schools aided in developing pre-service teachers' proficiency. Out of these perceived factors identified by participants, this study explored whether gender and program offered at college may have influence on pre-service teachers' mathematics proficiency. Although there wasn't a significant difference based on gender, results revealed that males had slightly higher proficiency than females. It was also revealed that participants who offered JHS program attained higher mathematics proficiency scores than their colleagues offering primary education.

5. Summary, Conclusion and Recommendations

Much of the relevant research for this study partially considered strategic competency and adaptive reasoning. Besides, most of the research literature identified related to mathematics instructional practices. This study discusses the ways in which these mathematics proficiency strands have been used and how the findings relates to mathematics assessment and instruction.

The findings of this study are relevant for mathematics education in Ghana. Since the basic education put premium on mathematics and numeracy, pre-service teachers who are being trained to teach in basic schools must be proficient. The current basic school mathematics curriculum emphasizes on the need for teachers to have in-depth knowledge about the four strands of basic school mathematics in order to teach efficiently. Pre-service teachers performed better in number sense and measurement. Although at final year pre-service teachers at colleges of education are perceived to be proficient in conceptual understanding and procedural fluency this study revealed that their strategic competency and adaptive reasoning were moderate and approaching proficiency.

The implication is that, though pre-service teachers understand basic concepts and procedures in performing mathematical task but their knowledge in fostering learners' mathematical thinking is insufficient. Although these participants have successfully completed methodology and pedagogical content knowledge for courses in mathematics there seems be an insufficient proficiency to identify and correct errors in pupils' solutions. Pre-service teachers had difficulties in analyzing students' thinking which exhibit less knowledge about interpreting, analyzing and making conclusive judgment on tasks of students they are about to teach. Previous studies (Jacobbe, 2015; Jacobbe et al, 2014) revealed similar trend for both prospective and practicing teachers in area of teaching statistics and the reason was attributed to participants' low mathematics self-efficacy. Jacobbe (2015) opines that even in-service teachers do not feel prepared to teach concepts in statistics. They struggle due to lack of content and pedagogical knowledge. Strategic competency and adaptive reasoning are higher domains of mathematics proficiency which develops through varied teaching experiences in a regular classroom by in-service teachers (Pai, 2018).

It is therefore recommended that pre-service teachers must be taken through rigorous mentorship and supervision by subject experts during supported teaching in schools (STS). Also more practical opportunities and a regular teaching experience needs to be ensured in order to build their competency and proficiency in teaching the strand; relationships, reasoning, data and chance. Males had slightly higher proficiency than females which may be due to what is termed as confidence/efficacy gap. Many research studies might reveal males outperforming females in mathematics assessments but in this study it is believed that even gifted female students underestimated their confidence. The aim of the NTECF envisage equity and inclusivity by ensuring low or no gender disparity in preparing teachers. Female pre-service teachers need to be encouraged in order to build their mathematics self-efficacy through innovative pedagogical approaches in order to bridge the gender gap (Samuel Baah-Duodu et al, 2021).

The authors of this study suggests for further research to consider exploring pre-service teachers' mathematics proficiency and their teaching efficacy since it is possible that similar findings could be obtained.

6. References

i. Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*, 389-407. http://dx.doi.org/10.1177/0022487108324554

ii. Ball, D. L., & Bass, H. (2003). Toward a practice-based theory of mathematics knowledge for teaching. In B. Davis & E. Simmt (Eds.), *Proceedings of the 2002 Annual Meeting of the Canadian Mathematics Education Study Group* (pp. 3-14). Edmonton: AB: CMESG/GCEDM.

iii. Ball, D. L., & Bass, H. (2009). *With an eye on the mathematical horizon: Knowing mathematics for teaching to learners' mathematical futures.* Paper presented at the 43rd Jahrestagung für Didaktik der Mathematik, Oldenburg, Germany.

iv. Ball, D. L., Bass, H., & Hill, H. C. (2004). Knowing and using mathematical knowledge in teaching: Learning what matters. In A. Buffgler & R. Lausch (Eds.), *Proceedings for the 12th Annual Conference of the South African Association for Research in Mathematics, Science and Technology Education* (pp. 51-65). Durban: SAARMSTE.

v. Bullock, S. M. (2011). Inside teacher education: Challenging prior views of teaching and learning. Rotterdam, The Netherlands: Sense Publishers.

vi. Burkhardt, H. (2007). Mathematical proficiency: What is important? How can it be measured? In H. Schoenfeld (Ed.), Assessing Mathematical Proficiency (pp.77-97). New York: Cambridge University Press.

vii. Creswell, J. W. (2015). A concise introduction to mixed methods research. Thousand Oaks, CA: SAGE.

viii. Freund, D. P. N. (2011). Opportunities to develop mathematical proficiency: how teachers structure participation in the elementary mathematics classroom (Doctoral dissertation). Retrieved from Proquest Dissertations and Theses database. (UMI No. 3452133).

ix. GALOP (2019). Ghana Accountability for Learning Outcomes Project. International Development Association Project Appraisal Document on A Proposed Credit in The Amount of SDR 107.9 Million ($150 Million Equivalent) and A Proposed Grant In The Amount Of Us$23.9 Million from the Global Partnership for Education to the Republic Of Ghana. Education Global Practice Africa Region Report No: PAD3320

x. Groth, R. E. (2017). Classroom data analysis with the five strands of mathematical proficiency. The Clearing House: A Journal of Educational Strategies, Issues and Ideas. 90(3). 103-109.

xi. Jacobbe, T. (2015). Developing K-12 teachers' understanding of statistics. Retrieved from https://www.causeweb.org/webinar/teaching/2015-04/

xii. Jacobbe, T., Case, C., Whitaker, D., & Foti, S. (2014). Establishing the validity of the LOCUS assessments through an evidenced-centered design approach. In K. Makar, B. de Sousa, & R. Gould (Eds.), *Sustainability in statistics education. Proceedings of the Ninth International Conference on Teaching Statistics (ICOTS9, July 2014) Flagstaff, Arizona, USA*. Voorburg, The Netherlands: International Statistical Institute.

xiii. Khairani, A. Z. & Nordin, M. N. (2011). The development and construct validation of the mathematics proficiency test for 14-year-old students. Asia Pacific Journal of Educators and Education, 26(1), 33-50.

xiv. Kilpatrick, Jeremy, Jane Swafford and Bradford Findell (2001). Adding It Up: Helping Children Learn Mathematics. Washington, DC: National Academy Press, 2001.

xv. Ministry of Education. (2017). National teacher education curriculum framework: The essential elements of initial teacher education. Accra: Author.

xvi. National Council for Curriculum and Assessment (2019). Mathematics Common Core Program Curriculum. Published by Ministry of Education, Republic of (Ghana), under Creative Common Attribution-Share Alike 4.0 International. Available online at www.t-tel.org/hub.html

xvii. National Teacher Education Curriculum Framework (2019). The Element of Initial Teacher Education. Published by Ministry of Education, Republic of (Ghana), under Creative Common Attribution-Share Alike 4.0 International. Available online at www.t-tel.org/hub.html

xviii. [NCTM 2000] National Council of Teachers of Mathematics, *Principles and standards for school mathematics*, Reston, VA: Author, 2000.

xix. [NCTM 2006] National Council of Teachers of Mathematics, Position statement on high stakes testing, January 2006. Available at http://www.nctm.org/about/position statements/highstakes.htm. Retrieved 24 Jun 2006.

xx. Ozdemir, I. E. Y. & Pape S. (2012). Supporting students' strategic competence: A case of a sixth grade mathematics classroom. Mathematics Education Research Journal, 24, 153-168.

xxi. Patricia S. Wilson and M. Kathleen Heid (2010). Framework for Mathematical Proficiency for Teaching. Center for Proficiency in Teaching Mathematics; The University of Georgia, Athens and Mid-Atlantic Center for Mathematics Teaching and Learning The Pennsylvania State University, College Park.

xxii. Pai, J. (2018). Observations and conversations as assessment in secondary mathematics. In D. R. Thompson, M. Burton, A. Cusi, & D. Wright (Eds.), Classroom Assessment in Mathematics: Perspectives from Around the Globe (pp.25-44). Cham, CH: Springer International Publishing AG.

xxiii. Samuel Baah-Duodu , Seth Borbye, Ebenezer Someah-Addae, Francis Cornelius Enni, Vivian Osei-Buabeng (2021). Developing Female Pre-Service Teachers' Mathematics Self-Efficacy by Integrating History of Mathematics into Teaching During Lesson Study. Social Education Research, 3(1), 91-102.https://doi.org/10.37256/ser.3120221011

xxiv. Samuel Baah-Duodu, Francis Cornelius Ennin, Seth Borbye, Solomon Amoaddai (2019).Pre-Service Primary School Teachers' Mathematical Knowledge for Teaching Fractions through Problem Solving. International Journal of Advances in Scientific Research and Engineering (ijasre); Volume 5, Issue 3; DOI: 10.31695/IJASRE.2019.33085.

xxv. Schoenfeld, A. H. (2007A). Issues and tensions in the assessment of mathematical proficiency. In H. Schoenfeld (Ed.), Assessing Mathematical Proficiency (pp.3-15). New York: Cambridge University Press.

xxvi. Schoenfeld, A. H. (2007B). What is mathematical proficiency and how can it be assessed? In A. H. Schoenfeld (Ed.), Assessing Mathematical Proficiency (pp.59-73). New York: Cambridge University Press.

xxvii. Siegfried, J. M. (2012). The hidden strand of mathematical proficiency: Defining and assessing for productive disposition in elementary school teachers' mathematical content knowledge (Doctoral Dissertation). Retrieved from Proquest Dissertations and Theses database. (UMI No. 3526772).

xxviii. Straumberger, W. (2018). Using self-assessment for individual practice in math classes. In D. R. Thompson, M. Burton, A. Cusi, & D. Wright (Eds.), Classroom Assessment in Mathematics: Perspectives from Around the Globe (pp.45-60). Cham, CH: Springer International Publishing AG.

xxix. Suurtamm, C. (2018). Enhancing Mathematics teaching and learning through sound assessment practices. In A. Kajander, J. Holm, & E. J. Chernoff (Eds.), Teaching and Learning Secondary School Mathematics (pp.473-482). Cham, CH: Springer International Publishing AG.

xxx. Swan, M. & Foster, C. (2018) Formative assessment lessons. In D. R. Thompson, M. Burton, A. Cusi, & D. Wright (Eds.), Classroom Assessment in Mathematics: Perspectives from Around the Globe (pp.11-24). Cham, CH: Springer International Publishing AG.

xxxi. Rubie-Davies, C. M., Flint, A., & McDonald, L. G. (2012). Teacher beliefs, teacher characteristics, and school contextual factors: What are the relationships? *British Journal of Educational Psychology, 82*(2), 270-288.

Appendix

Proficiency Scale-Mathematics adapted from (http://victoriancurriculum.vcaa.vic.edu.au/) **Pre-Service Level 400**	
Scale	**Description**
Score: 3.1-4.0 Proficient	In addition to Score 3.0 and 2.0, in-depth inferences and applications of the four strands that go beyond what was taught. • Identify errors and misconceptions in reasoning by justifying computations. • transfer learning to more complex content and thinking. • including deeper conceptual understanding and applications that go beyond what is explicitly taught in class.
Score: 2.1-3.0 Approaching proficiency	In addition to Score 2.0, there are no major errors or omissions regarding the simpler details and processes as the student recognizes or recalls specific terminology, such as: • place value names from thousandths to tens of thousands; decimal point; product; quotient; power of 10; • ratios, proportions, relations, formulae, equations, inequalities, symbols and mathematical statements. • comparison, measurement of 2 & 3-dimensionl geometric figures. • average, chance, representing and interpreting data. The student exhibits adequate knowledge the essential learning for the four strands with no major errors or omissions. • create two numerical patterns using rules that are given to me. • use a coordinate plane to analyze the relationships between numerical patterns. • classify two-dimensional figures based on their properties. • convert between different-sized measurement units within a given measurement system. • construct and interpret graphs to display a data set of measurements using the fractions of a unit to solve problems. • explain my thought process in solving a problem in multiple ways and using multiple strategies/representations. can explain their own thinking and thinking of others with accurate vocabulary, and justify why their solution is correct.
Score: 1.0-2.0 Developing proficiency	Recognizes and perform basic processes under the four strands such as: • recognize and recall specific vocabulary (for example, pattern, rule, table of values, term, corresponding term, ordered pairs, x and y axis, origin, coordinate points, and sequence) • create one numerical pattern using rules that are given to me. • demonstrate an understanding that in a multi-digit number, a digit in one place represents 1/10 of what it represents in the place to its left. • read and write and compare decimals to the thousandths place using base-ten numerals, number names, and expanded form using >,=, and < symbols. • use formulas in solving problems involving angles and polygons • understand the attributes with assistance that belong to each two-dimensional and three—dimensional shapes (polygon, quadrilateral, rectangle, regular, rhombus, right angle, side square, trapezoid, triangle, two dimensional). • recognize and recall specific vocabulary for standard and arbitrary units • construct graphs to display a data set of measurements using the fractions of a unit to solve problems. I can communicate their reasoning and solution to others. • can stay with a challenging problem for more than one attempt. • can explain other students' solutions and identify strengths and weaknesses of the solution. • can understand and discuss other ideas and approaches to solving problems.
Score: <1.0 Below proficiency	With help, a partial or no success understanding of some of the simpler details of score 2 content and score 3 content

Table 10