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A Review of Malmquist Productivity Index and Data Envelopment Analysis

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Abstract:

As an important method of measuring distance function, Data Envelopment Analysis has greatly promoted the related research of Malmquist Productivity Index. The combination of the two is one of the mainstream methods for calculating total factor productivity, which has been used by Chinese and foreign scholars in many fields such as economics, medical care, policy, and education. This article reviews and sorts out related literature of Malmquist Productivity Index and Data Envelopment Analysis.

Keywords: Malmquist productivity index, data envelopment analysis, review

1. Malmquist Productivity Index

Swedish economist Malmquist (1953) first proposed the method of constructing a consumption quantity index using the ratio of distance.^[i]Inspired by this, Caves et al., (1982a, b) introduced their consumption quantity index into the production field and derived the Malmquist Productivity Index (MPI) ¹. And pointed out that if the market is perfectly competitive, the geometric mean of the MPI of adjacent periods is equal to the Tornqvist index, which greatly simplifies the calculation process of total factor productivity in theory. ^{[ii],[iii]}In the same period, Nishimizu & Page (1982) used parametric methods to decompose total factor productivity into two parts: technical change (TC) and efficiency change (EC).

However, due to the lack of a method to measure the distance function, the Malmquist index before this has not attracted widespread attention and has only been regarded as a theoretical index. Until Färe et al. (1994) used data envelopment analysis to analyze the Malmquist index proposed by Caves et al. (1982a, b) according to Nishimizu & Page (1982) method is decomposed^[iv], For the first time, the Malmquist index was used for empirical analysis in the field of technological progress, and related research was prosperous.

Subsequently, Ray & Desli² (1997) corrected the problems in the decomposition of returns to scale in the FGNZ model ^[v], But Färe et al. (1997) quickly responded and rejected its amendment.^[vi]In this debate, both sides have no shortage of supporters. Grifell and Lovell (1999) ^[vii], Lovell (2003) ^[viii] demonstrated the correctness of the RD model, and Groskov (2003) verified the FGNZ model.^[ix]Chinese scholars Zhang Xiangsun and Gui Binwei (2008) elaborated on the difference between the FGNZ model and the RD model, and explained the rationality of the RD model. They believed that the RD model had already won the debate and used the RD model for China 1979-2005 The total factor productivity of the year and its decomposition are analyzed.^[x]But in fact, the research using FGNZ model is more extensive.

At that time, MPI was only suitable for comparison within the same technology set. Due to the lack of a common benchmark, cross-group analysis of different production boundaries was not possible. In fact, Battese et al. (2004) have used linear programming to construct a common boundary production function of the parametric method and realize cross-group analysis.^[xi]And Rao (2006) put forward the estimation framework of the Common Frontier Malmquist Productivity Index³ (Metafrontier Malmquist Productivity Index,MMPI) at the productivity and efficiency seminar held in Taiwan in 2006. ^[xii]Subsequently, O'Donnell et al. (2008) combined their research, based on non-parametric data envelopment analysis (DEA) and used MMPI to conduct an empirical analysis on the technological progress of agricultural sectors in multiple countries.^[xiii]This study intends to use this method to measure technological progress. Taiwanese scholars Chen Guzhen and Yang Haoyan (2008) also perfected MMPI. They used the parameter method to further decompose the technological catch-up in MMPI into pure technological catch-up and relative technological changes, and used 26 high-income countries from 1980 to 2003. And data from 40 low- and middle-income countries, decomposing their total factor productivity.^[xiv]

In addition, scholars have also proposed some new Malmquist indexes and applied them in empirical analysis. For example, Pastor and Lovell (Pastor & Lovell, 2005) proposed the global Malmquist index^[xv], and Gaoqiang (Kao, 2010)

¹ Hereinafter referred to as Malmquist Index or MPI.

² The abbreviation for the model in the original text of Ray & Desli (1997), namely the RD model, is used below. ³ Hereinafter referred to as MMPI.

constructed a global Malmquist index based on common weights^[xvi], Wang Yingming and Lan Yixin (2011) considered both the optimistic and pessimistic frontiers, and constructed the dual frontier Malmquist index^[xvii], Kao & Hwang, (2014) used a multi-period two-stage KH model to measure the overall efficiency and period efficiency at the same time, and used this method to measure the Malmquist index^[xvii]

2. Data Envelopment Analysis

Data Envelopment Analysis (DEA), as an effective frontier estimation method of mathematical programming, was developed by famous American operations researchers and professors Charnes, Cooper and Rhodes of the University of Texas. Formally proposed in 1978, and established the first DEA model, the CCR (C²R) model. ^[xix]Subsequently, Banker et al. (1984) removed the restriction of fixed returns to scale in the C²R model, and proposed the BCC (BC²) ^[xx]model based on variable returns to scale. C²R and BC² models are the two most influential models in the DEA field.

Since then, the DEA field has experienced vigorous development, and scholars have continuously constructed new DEA models, in chronological order, mainly include: C²GS²model for evaluating the relative effectiveness of departments^[xxii]; FG model under constant returns to scale or diminishing returns to scale^[xxii]; C²W model with infinitely many decision-making units ^[xxiii]; C²WHmodel that can reflect the preferences of decision makers^[xxiv]; Symmetrical with the FG model, the ST model with constant returns to scale or increasing returns to scale^[xxv]; WY model that can measure the phenomenon of 'crowding' (overinvestment)^[xxvi]; A two-stage network KH model that can open the 'black box' of the DEA model^[xxviii] and composite network WYP model^[xxviii], etc.

Chinese scholar Wei Quanling and his collaborators conducted a lot of research from 1986 to 2016, made a detailed analysis of some important DEA models, continued to advance the theory, and gave some empirical examples For example, in 1986, the relevant definitions and derivation steps of C²R_× C²GS²_× C²W_× C²WHmodels were elaborated ^[xxix]; Based on the detailed description of multiple models, the series of papers in 1989 carried out empirical analysis with cases from the perspectives of physics, economics, statistics and other disciplines ^{[xxxi],[xxxx}

In terms of empirical applications, the research using DEA methods is not only numerous, but also covers a wide range of fields. As pointed out by Li Meijuan and Chen Guohong (2003), the DEA method has produced important research results in areas such as productivity and technological innovation, cost and benefit issues, resource allocation, financial investment, and department management.^[xli]Taiwanese scholar Sun Xun (2004) reviewed the literature and found that among the literature on DEA published at that time, the main application fields of Chinese literature were medical, banking, transportation, efficiency (theory), education, agriculture, and government. Insurance, environmental protection, securities, etc. The English literature is in order of method, finance, medical treatment, service industry, agriculture, animal husbandry and fishery, industry, government, education, mining, etc.^[xli]Ali Emrouznejad & Yang (2018) analyzed the literature on DEA from 1978 to 2016 and pointed out that the paper on DEA has experienced 'exponential' growth in the past 40 years, and by the end of 2016, <code>[xliii]I</code> can be seen that the DEA method is not only improving in theory, it has also become a powerful tool for researchers in practice.

3. References

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