THE INTERNATIONAL JOURNAL OF SCIENCE & TECHNOLEDGE

Investigation of non-Newtonian Fluid in the Unsteady Laminar Free Convective Flow by *AGM Method*

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Abstract:
In this research, the two-dimensional unsteady laminar free convective flow of a non-Newtonian fluid adjacent to a vertical plate with mass and heat transfer is investigated. The fundamental partial differential equations are reduced to the nonlinear ordinary differential equations which are solved using Akbari-Ganji's Method (AGM). Comparison between the Numerical Method (4th-5th -order Runge–Kutta–Fehlberg) and analytical conclusions of the issue, illustrates excellent complying in solving this nonlinear differential equation. As well as, in the present perusal, the influence the active parameters like: Schmidt number (Sc)*,Prandtl number* (Pr)*,power law index* (n)*,Richardson number andbuoyancy ratio* (N) *on the dimensionless velocity, concentration and temperature profiles plus skin friction factor, Sherwood number and Nusselt number are examined. Conclusions indicates that power law index* (n) *has direct relationship with dimensionless concentration profiles, but buoyancy ratio* (N) *has reverse relationship with it. Also, by increasing of buoyancy ratio* (N) *the amount of the thermal boundary-layer decreases. In addition, this study shows AGM is strong manner to solve nonlinear differential equations.*

Keywords: *Non-Newtonian fluid flow, Akbari-Ganji's Method (AGM), Sherwood number*

1. Introduction

A Non-Newtonian fluid is defined as a fluid that does not follow the relationship of viscosity and its shear stress rate is not fix, or as a fluid whose relationship between shear stress and rate of change of angular shape is not linear. Non-Newtonian fluids are named after the famous physicist and mathematician Sir Isaac Newton. It should be noted that Non-Newtonian fluids have opened the way to another world in the science of matter and its applications and they are waiting for our technological wonders. Many natural events prove how Non-Newtonian fluids are useful for mankind. Inspired by this, researchers have conducted many researches to explore the applications of non-Newtonian fluids in life. Some of these applications include: liquid soaps and cosmetics, foods such as butter and cheese, natural materials such as magma and volcanic lava, biological fluids such as blood, saliva and joint fluid and many others. Moreover, behavior of Non-Newtonian fluid flow in Industry has become a noteworthy issue lately, because it has sundry applications in various engineering fields. This fluid issues can be examined in various applications instance filtration practice, ceramic processing, paints, electronic packing, hydrocarbon oils, polymer processing, drilling silt, drag decline, greases and lubricant, cooling issues and coal slurries etc. [1-9]. However, in Specifically, the interest in heat transfer and boundary layer flow issue of non-Newtonian fluids flow past a stretching sheet has grown considerable. Therefore, understanding the nature of related the non-Newtonian fluid flow via mathematical modeling with a view to predict the associated behavior of fluid flow and the have been done by number of scholars under various assumptions. For example, early studies in the non-Newtonian fluids past a stretching sheet are the works of Nath and Kumari [10]. Nath and Kumari studied the magnetic hydro dynamic boundary layer flow of a non-Newtonian fluid over a continuously moving surface with a parallel free stream in the attendance of a magnetic field. Their outcomes illustrate that the heat-transfer and skin friction coefficients increase with the magnetic parameter and these are more for the pseudo plastic fluid than for the dilatant fluid. Also, the impact of slip condition on magnetic hydro dynamic free convective flow of non-Newtonian fluid over a nonlinearly stretching plate saturated with Newtonian heating in poriferous medium is analyzed by Shafie et al. [11]. The similarity solutions of magneto hydro dynamic flows of power law fluids over a stretching plate studied by Zhang and Wang [12]. As well as, the magneto hydro dynamic flow of the third grade fluid with heat transfer between two permeable discs analyzed by Hayat et al. [13]. They applied the (HAM) Method to solve governing equations of MHD flow with heat transfer. Jayachandra Babu and Sandeep. [14]

investigated the cross-propagation influence on the MHD Williamson fluid flow by viewing velocity slip across a variable density stretching plate. They showed that the rate of heat transfer efficiency is much lower in non-Newtonian fluid compared with Newtonian fluid. The effect of radiation and viscous dissipation on unsteady magneto hydro dynamic free-convection flow with time-dependent suction in a poriferous medium past an infinite heated vertical sheet examined by Israel-Cookey et al. [15]. Plus, the steady MHD flow of an incompressible viscose non-Newtonian power law fluid with heat transfer of above an infinite gyrating porous disc investigated by Attiaa et al. [16]. recent years some scholars investigated the issue of heat transfer and steady/unsteady flow in porous media on unstructured grids. Samples of the most related research in this field are as follows: The resemblance equations for blended convection boundary-layer flow over a vertical semi-infinite at sheet in which the wall temperature is inversely and the free stream velocity is uniform proportional to the interval along the sheet investigated by Merkin and Pop [17]. Sanatan et al. [18] discussed the impacts of radiation on unsteady free convection flow of a viscose incompressible fluid embedded in a porous medium with viscose dissipation past a moving vertical sheet. Their outcomes show that with an increase in radiation parameter the rate of heat transfers at the sheet increases. Also, the transient MHD laminar free convection flow of Nano-fluid stretched under acceleration and past a vertical surface porous studied by Freidoonimehr et al. [19]. Bhattacharyya et al. [20] have considered impacts of slip at the boundary in the mixed convective boundary layer flow over a flat sheet. Their reviews show that temperature and velocity are found to increase with the increasing thermic slip parameter. In addition, several authors analyzed about convective flow [21-23]. Because of the nonlinear essence of Non-Newtonian fluid, we need a strong analytical tool.Akbari-Ganji's Method (AGM) is a modern manner that be applied for study of nonlinear issues. A synopsis of AGM advantages compared to other manners is as follow: Boundary conditions are needed in accordance with the order of differential equations in the solution procedure however when the number of boundary conditions is minor than the order of the differential equation, this approach can create additional new boundary conditions in regard to the derivatives and its own differential equation. Therefore, AGM is a strong manner for solving the nonlinear differential equations like presented equation in this article.In this paper, we have used AGM to identify the solutions of nonlinear differential equations governing the unsteady laminar free convective flow of a non-Newtonian fluid adjacent to a vertical plate with mass and heat transfer. As well as, the impact of Schmidt number (Sc),Prandtl number (Pr),power law index (n),Richardson number andbuoyancy ratio (N) on the dimensionless velocity, concentration and temperature profiles are studied. The comparison of the outcome of Akbari–Ganji's method (AGM) and Numerical Method (4th-5th -order Runge–Kutta–Fehlberg) outcome illustrate excellent complying in solving this nonlinear issue.

2. Description of the Problem

Unsteady two dimensional laminar free convective mass and heat transfer flow of a Non-Newtonian fluid adjacent to a vertical plate is assumed. The physical model for Perception is shown in Figure 1. the governing boundary layer equations acquired by [24], as systems of nonlinear ordinary differential equation are given by:

$$n\left|f''\right|^{n-1} f''' + f' + \left(\frac{2-n}{n+1}\right).\eta f'' - f'^{2} + \left(\frac{2n}{n+1}\right) f f'' + \theta + N\phi = 0, \quad (1)$$

$$\frac{1}{\Pr}\left[(n-1)\left|f''\right|^{n-2} f'''\theta' + \left|f''\right|^{n-1}\theta''\right] + 2\theta + \left(\frac{2-n}{n+1}\right).\eta\theta' - f'\theta + \left(\frac{2n}{n+1}\right) f\theta' = 0, \quad (2)$$

$$\frac{1}{Sc}\phi'' + 2\phi + \left(\frac{2-n}{n+1}\right).\eta\phi' - f'\phi + \left(\frac{2n}{n+1}\right) f\phi' = 0. \quad (3)$$

The boundary conditions are [24]:

$$\begin{aligned} &\eta = 0: \quad f = 0, \quad f' = 0, \quad \theta = 1, \quad \phi = 1, \\ &\eta = \infty: \quad f' = 0, \quad \theta = 0, \quad \phi = 0. \end{aligned} \quad (4)$$

Where 'prime' denotes differentiation with respect to $\eta.N, Pr, Sc$and$n$ Parameters arebuoyancy ratio, Prandtl number,Schmidt number and power law index, respectively.

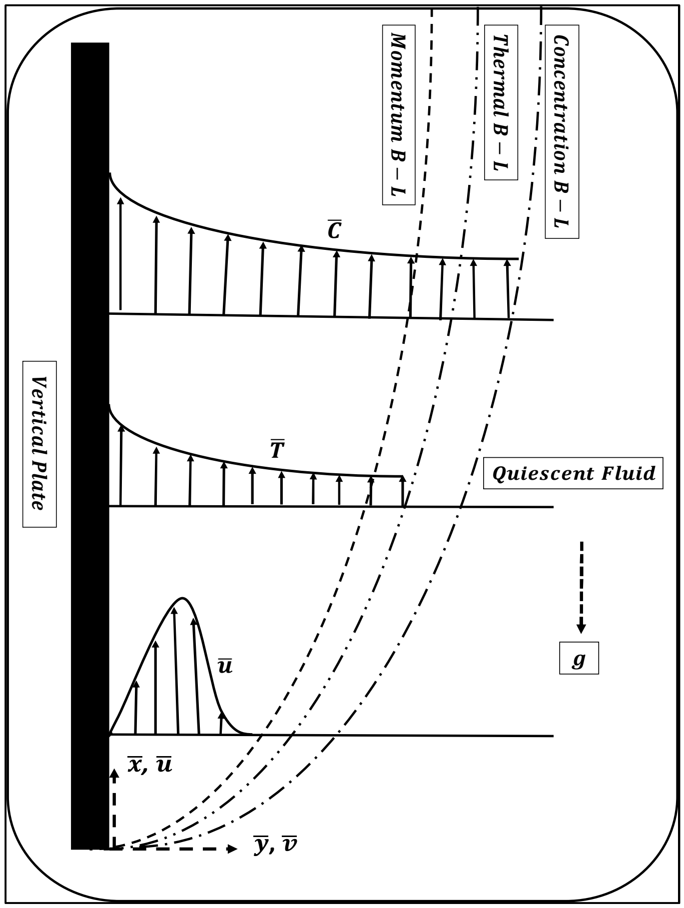

Figure 1: Schematic representation of the considered issue

Moreover, the quantities of interest are the local Sherwood number $Sh_{\bar{x}}$, the local Nusselt number $Nu_{\bar{x}}$ and the skin friction factor $Cf_{\bar{x}}$ can be found from the following definition [24]:

$$C_{f\bar{x}} = \frac{2\tau_w}{\rho U^2},\ N_{u\bar{x}} = \frac{\bar{x} q_w}{k(T_w - T_\infty)},\ Sh_{\bar{x}} = \frac{\bar{x} J_w}{D(C_w - C_\infty)} \quad (5)$$

3. Mathematical Procedures

In this section AGM method have been investigated:

3.1. Akbari-Ganji's Method (AGM)

Primary conditions and Boundary conditions are needed differential equation conforming to the physic of the moot point. Therefore, we can solve every differential equation with any degree. In order to understand the given manner in this research, two differential equations ruling on engineering operations will be solved in this new method. The nonlinear differential equation of $p$which is a function of$u$(which is a function of$x$), and their derivatives are considered as follows:

$$p_k : f(u, u', u'', \ldots u^m) = 0 \ ; u = u(x), \quad (6)$$

Boundary conditions:

$$\begin{cases} u(0) = u_0, u'(0) = u_1, \ldots u^{(m-1)}(0) = u_{m-1} \\ u(L) = u_{L0}, u'(L) = u_{L1}, \ldots u^{(m-1)}(L) = u_{Lm-1} \end{cases} \quad (7)$$

To solver the first differential equation, with respect to the boundary conditions in$x = L$ in Eq. (7), the series of letters in the nth order by constant coefficients, which is the reply of the first differential equation, is considered as follows:

$$u(x) = \lim_{n\to\infty} \sum_{i=0}^{n} a_i x^i = \lim_{n\to\infty} (a_0 + a_1 x^1 + a_2 x^2 + \ldots a_n x^n), \quad (8)$$

- The boundary conditions are applied to the function as follows:

a) The application of the boundary conditions for the reply of differential Eq. (8) is in the form of

If $x = 0$

$$\begin{cases} u(0) = a_0 = u_0 \\ u'(0) = a_1 = u_1 \\ u''(0) = a_2 = u_2 \\ \vdots \quad \vdots \quad \vdots \end{cases} \tag{9}$$

and when $x = L$

$$\begin{cases} u(0) = a_0 + a_1 L + a_2 L^2 + \ldots + a_n L^n = u_{L_0} \\ u'(0) = a_1 + 2a_2 L + 3a_3 L^2 + \ldots + n a_n L^{n-1} = u_{L_1} \\ u''(0) = 2a_2 + 6a_3 L + 12 a_4 L^2 + \ldots + n(n-1) a_n L^{n-2} = u_{L_{m-1}} \\ \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \end{cases} \tag{10}$$

b) After substituting Eq. (10) into Eq. (6), the application of the boundary conditions on differential Eq. (6) is done according to the following procedure:

$$\begin{aligned} & p_0 : f\,(u(0), u'(0), u''(0), \ldots u^{(m)}(0)) \\ & p_1 : f\,(u(L), u'(L), u''(L), \ldots u^{(m)}(L)) \\ & \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \end{aligned} \tag{11}$$

With attention to the selection of n; $(n < m)$ sentences from Eq. (8) and in order to make a set of equations which is consisted of $(n + 1)$ equations and $(n + 1)$ unknowns, we confront with a number of additional unknowns which are indeed the same coefficients of Eq. (8). So, to delete this issue, we should derive m times from Eq. (6) according to the additional unknowns in the afore-mentioned set differential equations and then this is the time to apply the boundary conditions of Eq. (7) on them.

$$\begin{aligned} & p_k{}' : f'(u', u'', u''', \ldots u^{(m+1)}) \\ & p_k{}'' : f''(u'', u''', u^{IV}, \ldots u^{(m+2)}) \\ & \vdots \quad \vdots \quad \vdots \quad \vdots \end{aligned} \tag{12}$$

c) Usage of the boundary conditions on the derivatives of the differential equation P_k in Eq. (12) is done in the form of

$$p_k{}' : \begin{cases} f'(u'(0), u''(0), u'''(0), \ldots, u^{(m+1)}(0)) \\ \\ f'(u'(L), u''(L), u'''(L), \ldots, u^{(m+1)}(L)) \end{cases} \tag{13}$$

$$p_k{}'' : \begin{cases} f''(u''(0), u'''(0), \ldots, u^{(m+2)}(0)) \\ \\ f''(u''(L), u'''(L), \ldots, u^{(m+2)}(L)) \end{cases} \tag{14}$$

The $(n + 1)$ equations can be created from Eq. (9) to Eq. (14) so that $(n + 1)$ unknown coefficients of Eq. (8) for instance,$a_0 + a_1 + a_2 + a_3 + \cdots a_n$can be calculated. The reply of the nonlinear differential Eq. (6) will be achieved by defining coefficients of Eq. (8).

4. Application of Described Manners in the Issue

4.1. Akbari-Ganji's Method (AGM)

by considering the fundamental idea of the manner, we rewrite the issueEqs. (1-3) in the following order:

$$\begin{aligned} & F(\eta) = n\left|f''\right|^{n-1} f''' + f' + \left(\frac{2-n}{n+1}\right).\eta f'' - f'^2 + \left(\frac{2n}{n+1}\right) f f'' + \theta + N\phi = 0, \\ & \Theta(\eta) = \frac{1}{\Pr}\left[(n-1)\left|f''\right|^{n-2} f'''\theta' + \left|f''\right|^{n-1}\theta''\right] + 2\theta + \left(\frac{2-n}{n+1}\right).\eta\theta' - f'\theta + \left(\frac{2n}{n+1}\right) f\theta' = 0, \\ & \Phi(\eta) = \frac{1}{Sc}\phi'' + 2\phi + \left(\frac{2-n}{n+1}\right).\eta\phi' - f'\phi + \left(\frac{2n}{n+1}\right) f\phi' = 0, \end{aligned} \tag{15}$$

In AGM, the reply of the nonlinear differential equations equation is considered as a confined series of polynomials via constant coefficients, as follows:

$$f(\eta)=\sum_{k=0}^{4}a_k\eta^k=a_0+a_1\eta+a_2\eta^2+a_3\eta^3+a_4\eta^4, \quad (16)$$

$$\theta(\eta)=\sum_{k=0}^{3}c_k\eta^k=c_0+c_1\eta+c_2\eta^2+c_3\eta^3, \quad (17)$$

$$\phi(\eta)=\sum_{k=0}^{3}v_k\eta^k=v_0+v_1\eta+v_2\eta^2+v_3\eta^3, \quad (18)$$

In the proposed issue we have engaged with three trial functions which contain 13 constant coefficients. The constant coefficients a_0 to$a_4$and $c_0$toc_5 and v_0 tov_4, which can easily be calculated by using the boundary conditions and initial conditions [25].

- In AGM, the boundary conditions are used in two ways:

a) Applying the boundary conditions on Eqs. (16-18) is expressed as follows:

$$f=f(BC), \qquad \theta=\theta(BC), \qquad \phi=\phi(BC). \quad (19)$$

Therefore, the boundary conditions are used with according to the Eq. (19) as follows:

$$\begin{aligned}
&f(0)=0\rightarrow a_0=0,\\
&f'(0)=0\rightarrow a_1=0,\\
&\theta(0)=1\rightarrow c_0=1,\\
&\phi(0)=1\rightarrow v_0=1,\\
&f'(\infty)=0\rightarrow a_1+20a_2+300a_3+4000a_4=0,\\
&\theta(\infty)=0\rightarrow 1000c_3+100c_2+10c_1+c_0=0,\\
&\phi(\infty)=0\rightarrow 1000v_3+100v_2+10v_1+v_0=0.
\end{aligned} \quad (20)$$

b) Boundary conditions are applied on Eq. (15), shown by F (η),Φ (η) and$\Theta(\eta)$ also on their derivatives as

$$\begin{aligned}
&F(f(\eta))\rightarrow F(f(BC))=0,\ F'(f(BC))=0,\ldots\\
&\Theta(\theta(\eta))\rightarrow \Theta(\theta(BC))=0,\ \ \Theta'(\theta(BC))=0,\ldots\\
&\Phi(\phi(\eta))\rightarrow \Phi(\phi(BC))=0,\ \Phi'(\phi(BC))=0,\ldots
\end{aligned} \quad (21)$$

We have to produce 6 additional equations from Eq. (21) in order to achieve a set of polynomials which contains of 13 equations and 13 constants. According to the above descriptions we have produced additional equations Eq. (21) in the following order:

- 2 equations have been produced by computing acquired equations from:

$$F(f(0))=0, \quad F(f(\infty))=0,$$

- 2 equations have been produced by computing acquired equations from:

$$\Theta(\theta(0))=0, \quad \Theta(\theta(\infty))=0,$$

- 2 equations have been produced by computing acquired equations from:

$$\Phi(\phi(0))=0, \quad \Phi(\phi(\infty))=0.$$

The mentioned equations in upper are too big to be illustrate graphically. For instance, solving to differential Eqs. (1-3) by used Akbari-Ganji's Method with ($N=0, n=2, Sc=0.22, Pr=0.2$). The constant coefficients of Eqs. (16-18) can easily be gained. by using of above explanations, constant coefficients as follows:

$$a_0=0, a_1=0, a_2=0.55901, a_3=-0.07453, a_4=0.00279, c_0=1, c_1=0.14860,$$

$$c_2=-0.05972, c_3=0.00348, v_0=1, v_1=0.39214, v_2=-0.10010, v_3=0.00507.$$

by replacing acquired constant coefficients from above-mentioned manner Eqs. (16-18) could easily be yielded as follows:

$$\begin{aligned}
f(\eta)&=0.0027950\eta^4-0.0745355\eta^3+0.559016\eta^2,\\
\theta(\eta)&=0.00348606\eta^3-0.0597213\eta^2+0.148606\eta+1,\\
\phi(\eta)&=0.00507854\eta^3-0.100100\eta^2+0.3921459\eta+1.
\end{aligned}$$

5. Results and Discussion

The comparison of the outcomeof AGM with the outcomeof the Numerical Method (4th-5th -order Runge–Kutta–Fehlberg) was done. Also, this investigation shows that AGM is strong manner to dissolve nonlinear differential equations.

5.1. Comparison between AGM, Numerical Method

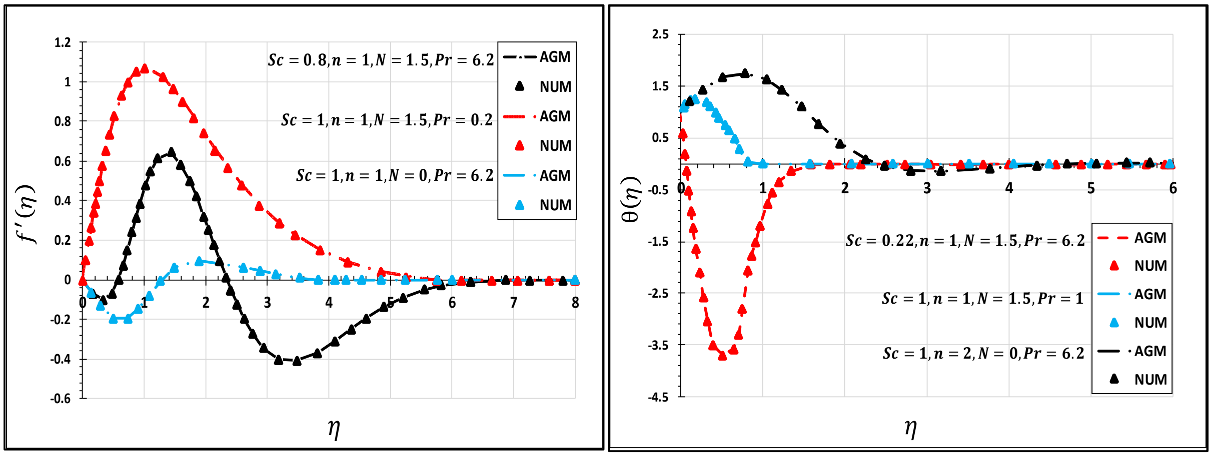

Figure 2(a): Comparison between AGM, NUM results of $f'(\eta)$and$\theta(\eta)$

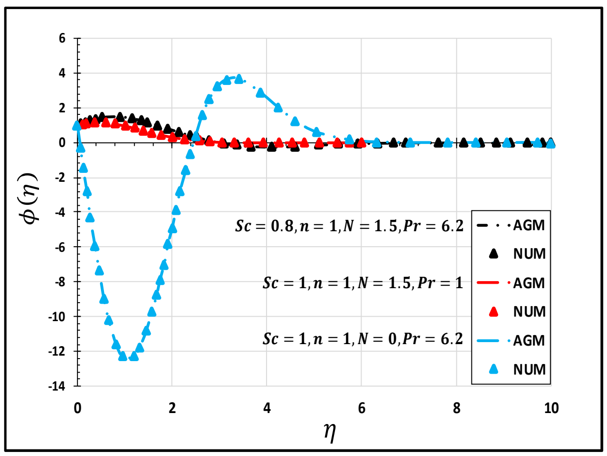

Figure 2(b): Comparison between AGM, NUM results of $\phi(\eta)$.

5.2. Results of AGM

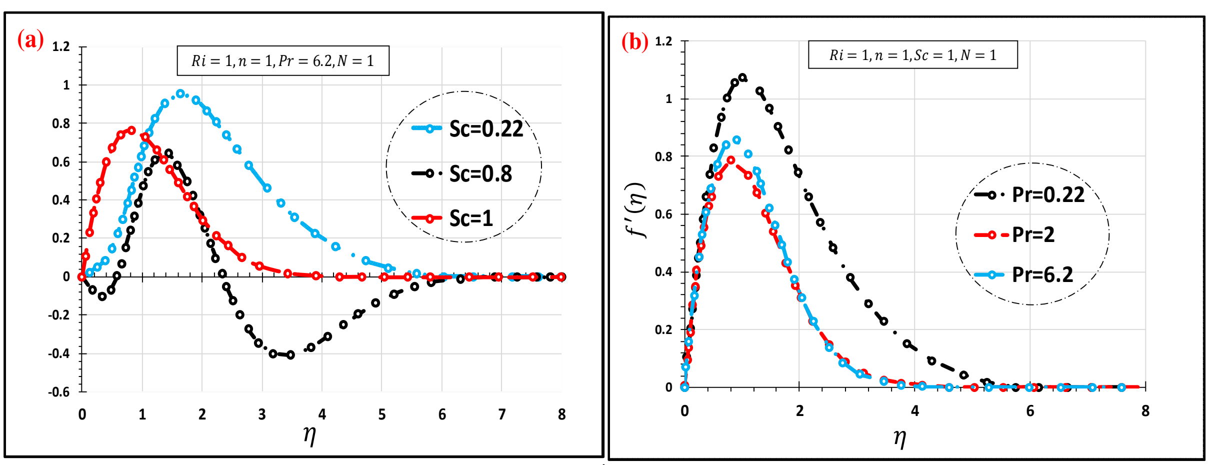

Figure 3(a-b): Velocity profiles $f'(\eta)$ for various amounts of Sc, Pr.

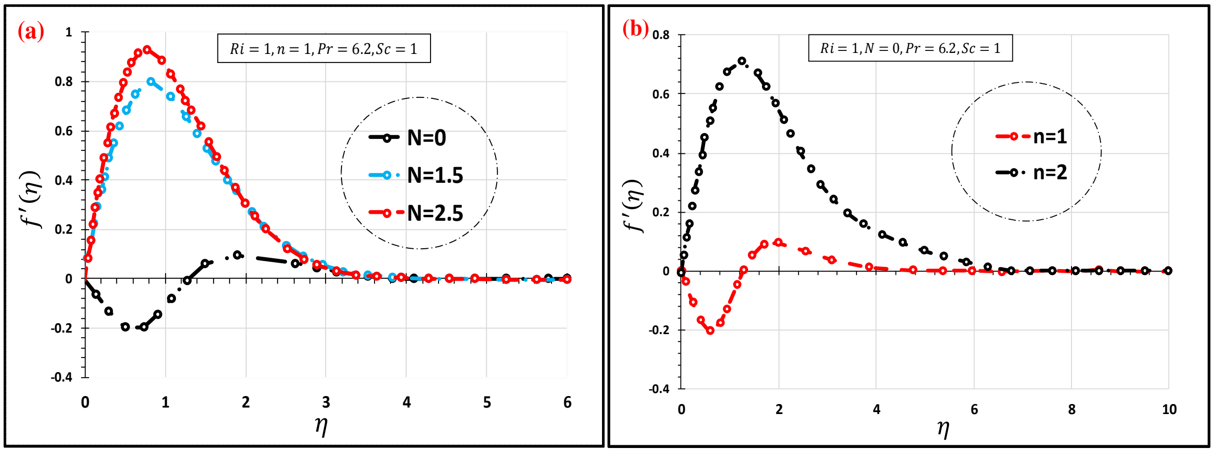

Figure 4(a-b): Velocity profiles $f'(\eta)$ for various amounts of N, n.

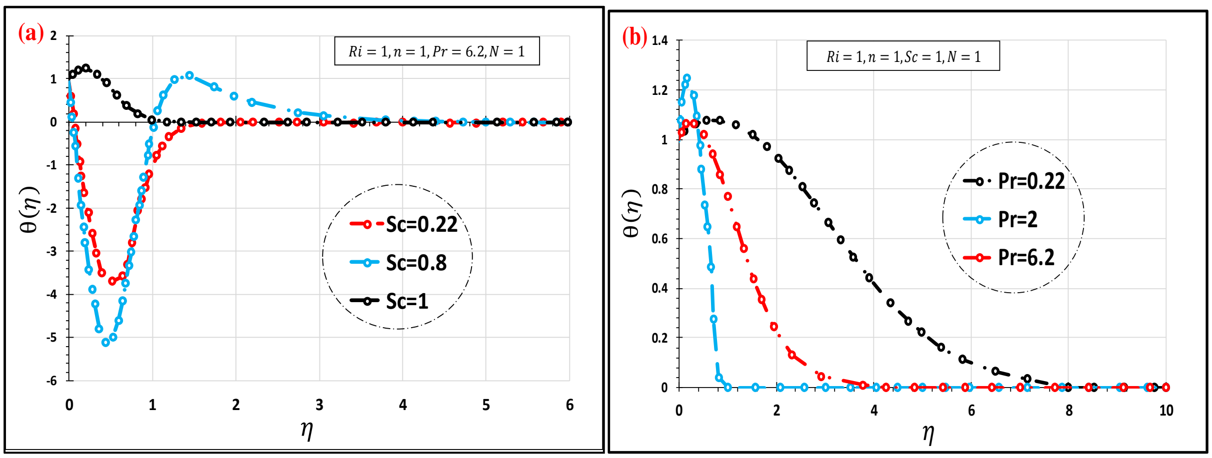

Figure 5(a-b): Dimensionless temperature profiles $\theta(\eta)$ for various amounts of Sc, Pr.

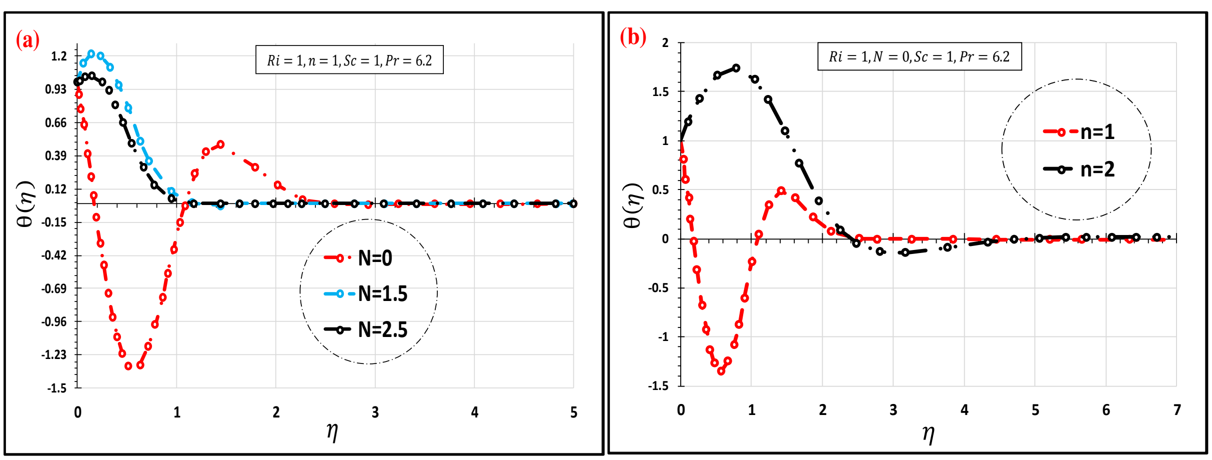

Figure 6(a-b): Dimensionless temperature profiles $\theta(\eta)$ for various amounts of N, n.

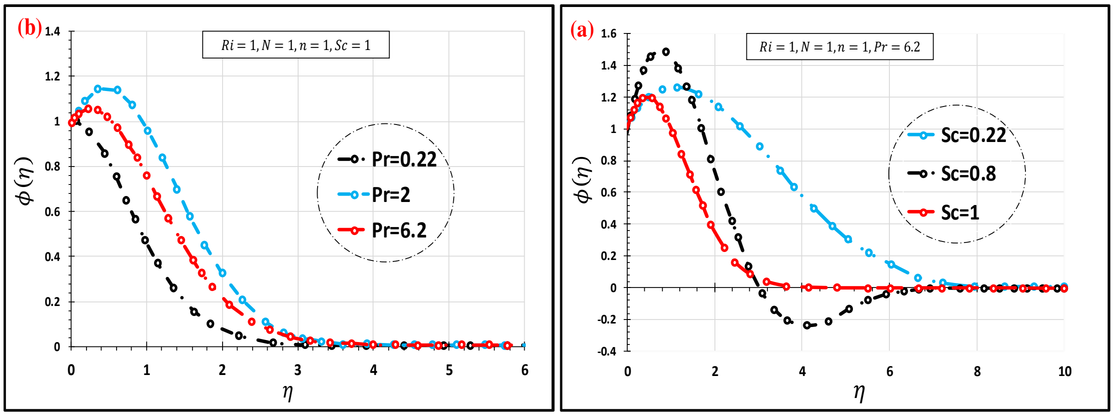

Figure 7(a-b): Dimensionless concentration profiles$\phi(\eta)$ for various amounts of Sc, Pr.

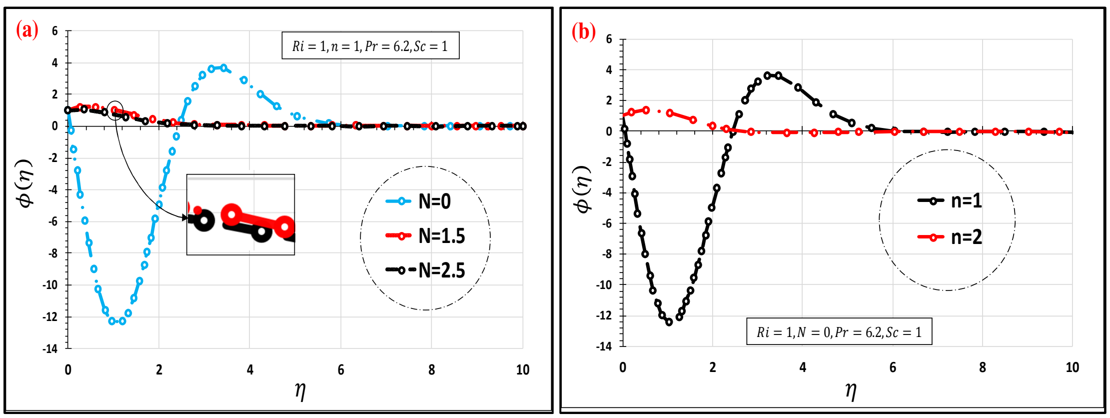

Figure 8(a-b): Dimensionless concentration profiles$\phi(\eta)$ for various amounts of N, n.

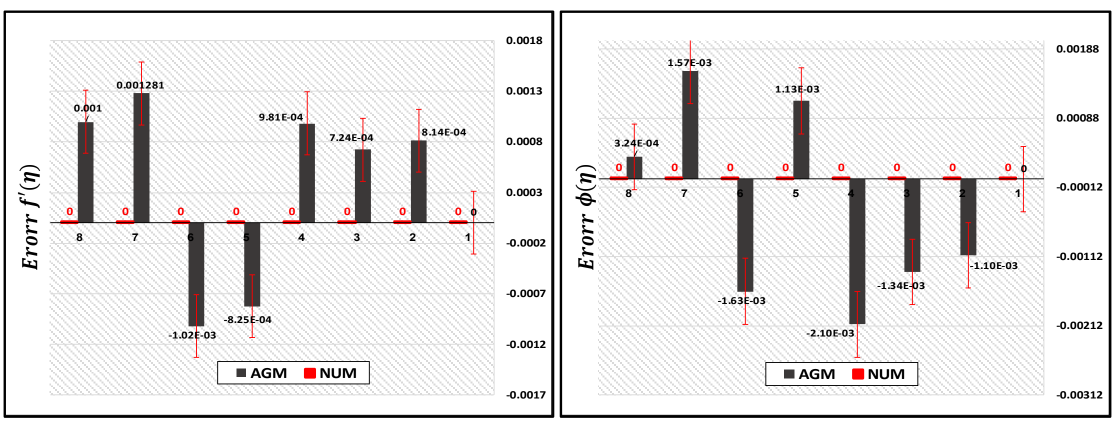

Figure 9: The comparison between Errors of AGM and NUM for$f'(\eta)$and$\phi(\eta)$ when$Sc = 0.8, Pr = 2, N = 1.5$ and$n = 1$.

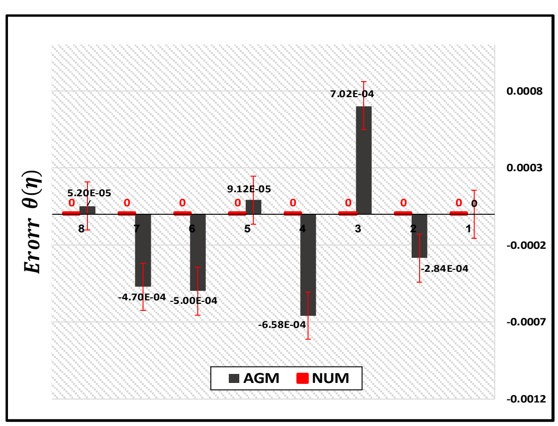

Figure 10: The comparison between Errors of AGM and NUM for$\theta(\eta)$ when$Sc = 0.8, Pr = 2, N = 1.5$ and$n = 1$.

η	Nu	AGM	$Error$	Nu	AGM	$Error$
0	0.000000	0.000000	0.000000	1.000000	1.000000	0.000000
1.0	0.685700	0.684787	0.000913	-0.127143	-0.126394	-7.49e-04
2.0	0.866531	0.865638	0.000893	0.603330	0.602230	1.10e-03
3.0	0.462265	0.461383	0.000882	0.145436	0.146840	-1.40e-03
4.0	0.203965	0.217340	-0.001338	0.029833	0.029739	9.40e-05
5.0	0.044962	0.046489	-0.001530	-0.009343	-0.009293	-5.00e-05
6.0	0.026180	0.025000	0.001181	0.000711	0.000729	-1.80e-05
7.0	0.014829	0.013829	0.001000	0.000201	0.000229	-2.80e-05
8.0	0.004153	0.004025	0.000128	0.000047	0.000042	5.00e-06

Table 1: Comparison between the AGM solution and Numerical outcomes for $f'(left)and\theta$ $(Right)$.
$N = 1.5, n = 1, Pr = 6.2, Sc = 0.2$ $N = 1, n = 1, Pr = 6.2, Sc = 0.8$

η	Nu	AGM	$Error$
0	1.000000	1.000000	0.000000
1.0	1.260432	1.261730	-0.001300
2.0	1.135236	1.136410	-0.001170
3.0	0.884123	0.886064	-0.001940
4.0	0.499902	0.489368	0.010534
5.0	0.313198	0.327361	-0.014160
6.0	0.144275	0.145701	-0.001430
7.0	0.025143	0.028848	-0.003710
8.0	0.006254	0.007163	-0.000910

Table 2: Comparison between the AGM solution and Numerical outcomes for$\phi(\eta)$.
$N = 1.5, n = 1, Pr = 0.2, Sc = 1$

Sc	Pr	Ri	N	n	$Nu_{\bar{x}}$	$Cf_{\bar{x}}$	$Sh_{\bar{x}}$
1	6.2	1	1.5	1	3.75851	2.08443	3.17458
0.8	6.2	1	1.5	1	-4.8657	-1.2651	4.02548
0.22	6.2	1	1.5	1	-4.2463	1.02686	1.94423
1	0.22	1	1.5	1	0.85971	2.90431	0.02314
1	2	1	1.5	1	1.86994	2.26024	2.55984
1	6.2	1	0	1	-3.9804	-1.9408	-7.4908
1	6.2	1	2.5	1	2.59021	2.37458	1.84755
1	6.2	1	1.5	2	3.75422	2.08312	3.16582

Table 3: Amounts of Nusselt number, skin friction factor and Sherwood number for various amounts of the parameters$N, n, PrandSc$.

In this article, Akbari-Ganji's Method (AGM) are applied to acquire a clear analytic solution of the unsteady laminar free convective mass and heat transfer fluid flow of a non-Newtonian fluid adjacent to a vertical plate (Figure 1). In order to verify the veracity and exactitude of the present conclusions, we have compared AGM conclusions with Numerical Method (NUM). Tables (1-2) andFigs. (A) shows the comparison between AGM solution and Numerical solution for various parameters of Eqs (1-3). As seen in these Tables and figures, error rate is very low. Figs. 2(a-b) indicates the comparison between the conclusions AGM method and NUM solution. In these Figs, we investigated the dimensionless velocity, concentration and temperature profiles for different values of (N, Sc, n, Pr). The conclusions illustrate excellent complying between the AGM manner and Numerical Method (NUM).Also, in this paper the impact of the embedding parameters like: Schmidt number (Sc), power law index (n), Prandtl number (Pr), Richardson number (Ri) and buoyancy ratio (N) on the dimensionless velocity, concentration and temperature profiles are examined.Figs. 3(a-b) show the effect of Schmidt number (Sc) and Prandtl number (Pr)on the dimensionless velocity profiles $\left(f'(\eta)\right)$ when the exterior mass within the boundary layer is Helium ($Sc = 0.22$ approx.), Oxygen ($Sc = 0.80$ approx.) and Carbon Dioxide ($Sc = 1.0$ approx.) and ($Pr = 0.3, 2, 6.2$)respectively.As we know, Schmidt number (Sc) is a dimensionless number specified as the ratio of momentum diffusivity (viscosity) and mass diffusivity and Prandtl number (Pr) is the ratio of momentum diffusivity to thermal diffusivity for a given Nano fluid. According to the Figure 3(a), from the bottom of the plate to its top, the velocity profile for Carbon Dioxide is greater than for Helium and the velocity profile for Helium is greater than for water vapor. On the contrary, the thickness of the boundary layer for Oxygen is greater than the two other fluids.Also, as shown in Fig 3(b), the velocity profile for Pr ranging between 0.3 and 2 has a decreasing trend, but $2 < pr < 6.2$ has an increasing trend. In addition, from the lower plate to upper one, the amount of $f'(\eta)$ increases to a maximum amount and then starts decreasing. The buoyancy ratio $N = \beta_c(C_w - C_1)/\beta_T(T_f - T_1)$ represents the relative value of species buoyancy and thermal buoyancy forces. The effect of buoyancy ratio (N) and power law index (n) on the dimensionless velocity profiles $\left(f'(\eta)\right)$ when ($N = 0, 1.5, 2.5$ and$n = 1, 2$) are shown in Figs. 4(a-b), respectively. For both cases, increases dimensionless velocity profiles with the increment of buoyancy ratio (N) and power law index (n). Plus, conclusions show that for n = 1 and $N = 0$ firstly boundary layer grows on the inverse side and then returns to real direction. Figs. 5(a-b) illustratethe impact of Schmidt number (Sc) and Prandtl number (Pr)on the dimensionless temperature $\theta(\eta)$ inthe thermal boundary layer, when $(Sc = 0.22, 0.8, 1$ and$Pr = 0.3, 2, 6.2)$, respectively. What is noteworthy in Fig 5(a), is that for Sc between $0.2 < Sc < 0.8$, the temperature gradient is negative at the beginning of the lower plate till it reaches a max point, after which the temperature profile moves toward the positive direction. In contrast, the temperature gradient is positive at the beginning of lower plate for $Sc = 1$. Plus, Figure 5(b) show that as the Prandtl number increases, the thickness of the thermic boundary-layer decreases. The influence of buoyancy ratio (N) and power law index (n) on the dimensionless temperature profiles $\theta(\eta)$ are shown in Figs. 6(a-b), respectively. This Figs shows by increasing of buoyancy ratio (N) the amount of the thermal boundary-layer decreases. But, with the increasing index power law index (n), thermal boundary-layer increases. However, for $N = 0$ and $n = 1$, the boundary layer grows in the opposite direction before moving toward the actual boundary. Figs. 7(a-b) indicates the effect of Schmidt number (Sc) and Prandtl number (Pr) on the dimensionless concentration profiles$\phi(\eta)$ graph, respectively. Conclusions show for $0.2 < Sc < 0.8$ the curves of $\phi(\eta)$ become increasingly steeper but for$Sc > 0.8$ milder $\phi(\eta)$ process. In Fig 7(b),for $0.3 < pr < 2$, the profile concentration thickness has an increasing trend near the lower wall, while it begins to decrease for $pr > 2$. As well as,Figs. 8(a-b) illustratethe impact of buoyancy ratio (N) and power law index (n) on the dimensionless concentration profiles$\phi(\eta)$, respectively.Outcome illustrate that power law index (n) has direct relationship with dimensionless concentration profiles, but buoyancy ratio (N) has reverse relationship with it. It is worth noting, similar the temperature and velocity profile, the concentration profile for $N = 0$ at the beginning of the plate has a negative movement. However, the noticeable point for the concentration profile unlike the temperature and velocity profile is that the movement of the boundary layer at the beginning of the plate is in the negative direction for $n = 2$. Moreover,in this study has focused on the physical parameters including $Sh_{\bar{x}}$, $Cf_{\bar{x}}$ and $Nu_{\bar{x}}$based on the changes of the factors N, Sc, nandPr, as shown in Table 3. The investigations show that we need more systematic studies regarding temperature/ mass transfer. Because there is no general systematic behavior for different values.

6. Conclusion

In this research, unsteady laminar free convective flow of a non-Newtonian fluid adjacent to a vertical plate is investigated. For this purpose, a new efficient method called AGM is introduced which solves nonlinear problems based on the trial Function. As well as, the influence the active parameters like:Schmidt number (Sc), power law index (n),Prandtl number (Pr),Richardson number andbuoyancy ratio (N) on the dimensionless velocity, concentration and temperature profiles are studied. Conclusions illustrate that:

- Velocity profile for Carbon Dioxide is greater than for Helium and the velocity profile for Helium is greater than for Oxygen.
- By increasing buoyancy ratio (N) and power law index (n), dimensionless velocity profiles increases.
- Sherwood number increase while mass and heat transfer rates increase as the Prandtl number increases.
- By increasing buoyancy ration while mass and heat transfer rates decrease, the Friction factor increases.

Plus, this research shows that AGM is powerful method to solve nonlinear differential equations like this problem.

7. References

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Appendix A.

Nomenclature

$\overline{C}$ … … … … concentration

$\overline{T}$ … … … temperature

$\overline{u}$ … … … velocity in the $\overline{x}$ − direction

g … … … acceleration due to gravity

$\overline{v}$ … … … velocity in the $\overline{y}$ − direction

Sc … … Schmidt number

Nu … … … … Nusselt number

Cf … … … …skin friction factor

Greek symbols

β_C … … … coefficient of mass expansion

β_T … … coefficient of thermal expansion

θ … … … dimensionless temperature

ϕ … … dimensionless concentration