THE INTERNATIONAL JOURNAL OF SCIENCE & TECHNOLEDGE

Lattice Boltzmann Method in Pore Scale Porous Medium: Application in Three-dimensional Domain

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Abstract:
Pore scale porous media analysis currently commonly conducted due to availability of deduced pore structure from micro-computed tomography of core rock samples and alternative fluid flow numerical simulation based on Boltzmann transport equation that modeled the micro state particles behavior structurally developed in Lattice Boltzmann Method. This work is conducted to verify the applicability of the fluid flow numerical simulation based on Lattice Boltzmann Method in several three dimensional(3D) samples of pore scale porous media geometry. Besides to verify the method, this work also providing the general algorithm to determine the matrix properties mainly in calculating the single-phase permeability of the samples that easily conducted. The method provided is less expensive compare to the commonly conducted in core analysis during obtaining the rock properties, and also open up possibility to use simple drill cutting during well drilling to be scanned with micro-computed tomography rather than rock sample from coring that very expensive to obtain. Therefore, commonly macroscopic parameters could be determined such as permeability and average velocity of the fluid inside the pore structure due to pressure different. In this respect, the Lattice Boltzmann Method may be considered to replace standard physical macroscopic scale experiments to determine such macroscopic parameters. The result of the macro properties obtained are compared to the pore network model and permeability-porosity correlation.

Keyword: *Complex geometric domain, digitally pore scale sample, drilling cutting, fluid flow, lattice boltzmann method, pore scale porous media*

1. Introduction

Reviewing the fluid flow in porous media in term of pore scale recently transform attractively due to availability of the tools such as a scanned sample of rocks with micro tomography that can be converted to digital as well as the particle based fluid flow equation based on Lattice Boltzmann Method. Therefore, physical properties analysis of fluids that flows in pore scale porous media as well as rocks properties could be conducted in relatively simple.

Many experiments were designed and done in order to understand how the fluid flows through the matrix or rock in macro scales. Macroscopic parameters could be measured such as permeability, average velocity, but we cannot understand the detail fluid flow phenomena inside the matrix. With current development of the computer-simulation of fluid dynamics using Lattice Boltzmann Method (LBM) which increasing to be applied, now the fluid flow in micro scale could be explored deeper.

In term of the applicability, especially during obtaining rock properties such as permeability of sandstone or limestone rocks, the small rock sample from the oil or gas reservoir from drilling cutting is adequate to be micro-tomography scanned and these segmented images are converted into binary image and formed the 3D image, compare to obtaining the expensive core sample during drilling. These small samples are adequate as representative to obtain its properties as a domain for Lattice Boltzmann Method numerical simulation.

2. Background

The advance of the micro-CT technology makes possible to describe in more detail of the porous medium structure. Micro-CT allows samples a few mm across to be imaged non-destructively in 3D at a resolution of a few micron using a laboratory micro-CT scanner. The method developed generally can be used to determine a quick look of the physical properties of the porous media with the availability of the thin slices of the rock sample from coring or even if only from the drilling cutting.

Generally, the rock sample images are converted into binary image. In this work, the rock images sample obtained with consist of sandstone and limestone core rock samples were used for fluid flow simulation. Each sample consist of 300 slices of digital image and formed the 3D geometry porous media with dimensions 300 x 300 x 300 lattices. The detailed discretization of the porous geometry allows the simulation of the transport of mass and momentum without any homogenization models, that generally used in engineering application.

The Lattice Boltzmann Method is used in this study to simulate the fluid flow based on transport of mass and momentum, that very compatible with the complex geometry such as porous medium. Using this method combining with the pore scale porous medium structure, the measurement of the macroscopic of core sample or rock cutting could be done easily and faster. The advantage of the Lattice Boltzmann method (LBM) is that, the LBM modeled the fluids as consisting of fictitious particles performing consecutive propagation and collision processes on the discrete lattice mesh rather than solving numerically for the conservation of equation of mass and momentum. In other word, the LBM captures the essential physics at micro or meso-scopic scale in order to satisfy the macroscopic dynamics from averaged particles behaviour. In this respect, the Lattice Boltzmann Method may be considered to replace standard experiments to determine such as the permeability.

3. Lattice Boltzmann Method

The Lattice Boltzmann Method (LBM) is an outcome of the improvement to the Lattice Gas Cellular Automata (LGCA). However, the LBM can be derived directly from the Boltzmann equation formulated by Ludwig Boltzmann uses classical mechanics and statistical physics to describe the evolution of a particle distribution function. The LBM solves the Boltzmann equation in a fixed lattice. Instead of taking into consideration every individual particles' position and velocity as in classical microscopic model (molecular dynamics), the particle distribution function in the LBM gives the probability of finding a fluid particle located at the location x, with velocity v at time t. The statistical treatment in the LBM is necessary because of the large number of particles interacting in a fluid; however, it leads to substantial gain in computational efficiency.

In the LBM, fluid flows are simulated by calculating the streaming and collision of particles within the lattices, often together with some boundary condition that must be fulfilled for each time step. The discrete lattice locations correspond to volume elements that contain a collection of particles, and represent a position in space that holds either fluid or solid. In the streaming phase, particles move to nearest neighbor along their path of motion, where they collide with other arrived particles. The outcome of the collision is designed to be consistent with conservation of mass, energy and momentum. After each iteration, only the particle distribution changes, while the particle distribution function in the center of each lattice location remains unchanged and the underlying lattice must have enough symmetry to ensure isotropy.

This work describing the application of the Lattice Boltzmann Method for single phase fluid flow application in three dimensional(3D) of pore scale samples of porous media, which consist of 300 sheets of digital image and formed the dimension in 300 x 300 x 300 lattices. Started with porous media structures deduced from slices image of core rocks that converted into digital image as the domain of the fluid flow evaluation. The single-phase fluid flow evaluation is conducted to deduce the absolute permeability of the sample of the core rocks and compare with other methods of permeability evaluation such as extracted pore network model initiated by Bryant et al (1992; 1993a; 1993b) and permeability with porosity correlation from experiment of Fontainebleau sandstone by (Bourbie and Zinszner, 1985).

The hydrodynamics or fluid transport equations such as Euler equations for inviscid flow and Navier-Stokes equations for viscous fluid can be derived with kinetic theory and continuum method. Individual particles are ignored in continuum method and fluid is represented as continuum substance, so at each point the value of parameters such as density, pressure, temperature and velocity is existing and governed by a set of nonlinear partial differential equation with commonly solution spatial discretization and time evolution of the equations. The obtained value of the parameters stated as macro variables are an averaging property of the substances.

Kinetic theory as a foundation of Boltzmann equation in contrast to the continuum approach, is governed by fluid consisting of molecules that the movement controlled and obey the Newtonian dynamics law. Essentially, the objective is to describe the collective behavior and represented statistically. In the form of statistic that formulated in distribution function equation, solving the transport equation with collision process in the interaction between particles. The moments of the distributions function equations give the solution to the transport equation such as Navier-Stokes equations with generated transport coefficients such bulk and shear viscosity and thermal conductivity.

The Boltzmann equation, with collision term that considering the binary elastic collision, in original form between binary particles:

$$\frac{\partial f_1}{\partial r}\mathbf{v_1} + \frac{\mathbf{F}}{\mathbf{m}}\frac{\partial f_1}{\partial \mathbf{v_1}} + \frac{\partial f_1}{\partial t} = \int \mathbf{d}^3\mathbf{p_2}\mathbf{d}^3\mathbf{p}'_1\mathbf{d}^3\mathbf{p}'_2\delta^4(\mathbf{P_f} - \mathbf{P_i})|\mathbf{T_{fi}}|^2\left(f'_2 f^1_1 - f_2 f_1\right) \tag{1}$$

Pf and Pi are total momentum of final and initial respectively, is delta function, Tfi is transition matrix as basic quantity for scattering problem in quantum mechanics.

It is difficult to solve analytically the nonlinear integro-differential of distribution function in Boltzmann equation as described in Equation (1). An alternative of the solution is the linear Boltzmann Equation, which simple to implement, compatible to the two and three dimensions, susceptible to the statistical noise. The Lattice Boltzmann was introduced by McNamara & Zanetti in 1988, it used the continuous single-particle distribution which interacts locally and propagates after collision to the next neighbor node with linearized of the collision operator with such head-on rule.
Higuera and Jimenez in 1989 introduced the linearized of collision operator in Boltzmann equation by assuming the distribution function close to its equilibrium. Then the next development for the collision operator is single relaxation time approximation by BGK (Bhatnagar, Gross, Krook), in this model, collisions are not defined explicitly anymore. In this study, the author proposed to modified single relaxation time approximation of Lattice Boltzmann BGK model which function of the temperature. Hence, the single relaxation time that relate to the viscosity will depend on the temperature of the system.

3.1. Distribution Function
Boltzmann postulate that the probability of being in particular state at energy E of a system large or small in thermal equilibrium is proportional to $e^{\frac{-E}{kT}}$ such:

$$f(\mathrm{E}) = \mathbf{A}\mathbf{e}^{-\mathbf{E}/\mathbf{kT}} \tag{2}$$

The Boltzmann distribution is used as the equilibrium distribution function:

$$f^{eq} \equiv \frac{\rho}{\left(2\pi RT\right)^{D/2}} \exp\left[-\frac{(\mathbf{v}-\mathbf{u})^2}{2RT}\right] \tag{3}$$

where R is the ideal gas constant and D is the dimension of the space, **v** is microscopic velocity and **u** is bulk velocity.

3.2. Single Relaxation Time Approximation
The collision term in the Boltzmann equation in Equation (1), could be further simplified by single relaxation time approximation such that:

$$\int \mathbf{d}^3\mathbf{p}_2 \mathbf{d}^3\mathbf{p}_1^{'} \mathbf{d}^3\mathbf{p}_2^{'} \delta^4\left(\mathbf{P_f}-\mathbf{P_i}\right)\left|\mathbf{T_{fi}}\right|^2\left(f_2^{'} f_1^{'} - f_2 f_1\right) = -\frac{1}{\tau^{'}}\left(f - f^{eq}\right) \tag{4}$$

$\tau^{'}$ is the characteristic relaxation time of collision processes and $\omega = 1/\tau^{'}$ is the characteristic frequency or relaxation parameter. Therefore, the Boltzmann equation become as follow:

$$\frac{\partial f}{\partial r}\mathbf{v} + \frac{\mathbf{F}}{\mathbf{m}}\frac{\partial f}{\partial \mathbf{v}} + \frac{\partial f}{\partial t} = -\frac{1}{\tau^{'}}\left(f - f^{eq}\right) \tag{5}$$

3.3. Lattice Boltzmann Bhatnagar Gross Krook Model
If there is no external force, the equation will become Boltzmann equation with BGK (P.L. Bhatnagar, E.F. Gross and M. Krook) approximation:

$$\frac{\partial f}{\partial t} + \mathbf{v}\cdot\nabla f = -\frac{1}{\tau^{'}}\left(f - f^{eq}\right) \tag{6}$$

v is the microscopic velocity and f^{eq} is the particles Boltzmann distribution function at equilibrium state as describe in Equation (3).

The link between the above distribution function equation with the macroscopic quantity such density, velocity and internal energy can be obtained from the (microscopic velocity) moments of the density distribution f :

$$\rho = \int f d\mathbf{v} = \int f^{eq} d\mathbf{v} \tag{7}$$

$$\rho u = \int \mathbf{v} f d\mathbf{v} = \int \mathbf{v} f^{eq} d\mathbf{v} \tag{8}$$

$$\rho\varepsilon = \int \frac{(\mathbf{v}-u)^2}{2} f d\mathbf{v} = \int \frac{(\mathbf{v}-u)^2}{2} f^{eq} d\mathbf{v} \tag{9}$$

The internal energy has the following relationship with the temperature T:

$$\varepsilon = \frac{D_0}{2}RT = \frac{D_0}{2}N_A k_B T \tag{10}$$

where D_o is the number of degrees of the freedom for a particle; N_A is Avogadro's number and k_B is the Boltzmann constant.

The Lattice Boltzmann Equation is derived initiated with Boltzmann Equation with BGK approximation as described in the Equation (5). Started with discretization of velocity space e, into finite set of velocity {ci} without loss of conservation law. Therefore, the f^{eq} is need to be expanded into Taylor series in term of fluid velocity,

$$f^{eq} = \frac{\rho}{(2\pi RT)^{D/2}}\exp\left(\frac{-\mathrm{v}^2}{2RT}\right) * \exp\left\{\frac{(\mathrm{v}.u)}{RT} - \frac{u^2}{2RT}\right\}$$
$$= \frac{\rho}{(2\pi RT)^{D/2}}\exp\left(\frac{-v^2}{2RT}\right) * \left[\left\{1 + \frac{(\mathrm{v}.u)}{RT} + \frac{(\mathrm{v}.u)^2}{2(RT)^2} - \frac{u^2}{2RT}\right\}\right] \quad (11)$$

where $\frac{\rho}{(2\pi RT)^{D/2}}\exp\left(\frac{-\mathrm{v}^2}{2RT}\right)$ is called $\mathbf{w_B(v)}$ the Boltzmann equilibrium distribution function for fluid at rest.

The expansion only valid for Mach number flow i.e. $|\mathbf{u}|/\sqrt{\mathbf{RT}} << 1$. In systematically obtaining Navier-Stokes equations with range of the low Mach number, discrete velocity is chosen that the quadratures expanded distribution function meet exactly as follows:

$$\int \mathbf{v}^{\mathbf{k}} f^{eq} \mathbf{dv} = \sum_i \mathbf{w}_i \mathbf{c}_{\mathbf{i}}^{\mathbf{k}} f^{eq}(\mathbf{c}_{\mathrm{i}}) \;,\; 0 \le \mathbf{k} \le \mathbf{3} \quad (12)$$

where $\mathbf{w_i}$ and $\mathbf{c_i}$ are the weights and finite set velocity points in numerical quadrature rule.

To define the discrete distribution function $f_{\boldsymbol{i}}(x,t) = \mathrm{w}_{\mathbf{i}} f_{\boldsymbol{i}}(x,\mathrm{c}_{\mathbf{i}},t)$ and $f_i^{eq}(x,t) = \mathrm{w}_{\mathbf{i}} f_i^{eq}(x,\mathrm{c}_{\mathbf{i}},t)$, Therefore, fluid density and velocity can be obtained from discrete distribution function,

$$\rho = \sum f_{\mathrm{i}} \quad (13)$$

$$\rho u = \sum \mathrm{c}_{\mathrm{i}} f_{\mathrm{i}} \quad (14)$$

By integrating Equation (13) from t to $t + \delta t$, and assuming the collision term is constant during time interval, could obtain

$$f_{\mathrm{i}}(x + \mathrm{c}_{\mathrm{i}}\delta t, t + \delta t) - f_{\mathrm{i}}(x,t) = -\frac{1}{\tau}\left[f_{\mathrm{i}}(x,t) - f_{\mathrm{i}}^{\mathbf{eq}}(x,t)\right] \quad (15)$$

where $\tau = \tau'/\delta t$ is the dimensionless relaxation time.

3.4. Lattice Model

The Lattice Boltzmann Method with Lattice Boltzmann BGK model is commonly used and applied widely in variety of complex flow. The discrete equilibrium distribution function can be written as:

$$f_{\mathrm{i}}^{\mathrm{eq}} = \mathrm{w}_{\mathrm{i}}\rho\left[1 + \frac{\mathrm{c}_{\mathrm{i}}.u}{c_s^2} + \frac{(\mathrm{c}_{\mathrm{i}}.u)^2}{2c_s^4} - \frac{u^2}{2c_s^2}\right] \quad (16)$$

The commonly used three-dimensional models is D3Q19 as shown in Figure 1.

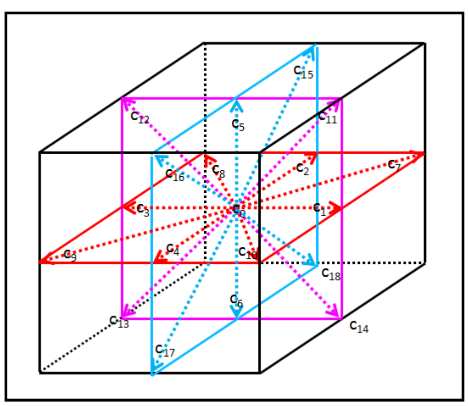

Figure 1: D3Q19 lattice model with discrete particle velocity $\mathbf{c}_{1 \ldots}\mathbf{c}_{18}$ *and* $\mathbf{c}_0$ *as reside velocity assigned to each distribution function*

The Table 1 describe the parameters of each lattice link component for D3Q19 model.

Model	Lattice Vector $\mathrm{c_i}$	Weight $\mathrm{w_i}$	$\mathbf{c}_s^2$
D3Q19	(0,0,0), (±1,0,0), (0, ±1,0), (0,0, ±1), (±1, ±1,0), (±1,0, ±1), (0, ±1, ±1)	1/3, 1/18, 1/36	1/3

Table 1: Parameters on D3Q19 describing the lattice vector, weight $\boldsymbol{w_i}$ *is the weight associated with* $\boldsymbol{c_i}$ *of individual lattice link, and* $\mathbf{c}_s^2$ *is sound of speed square.*

The weight ($\mathbf{w_i}$) are lattice constant that depend on lattice structure and the number of finite velocities. It is evaluated such that lattice velocity moment up to fourth order respective over w_i for the isothermal models and sixth order for thermal models that should be isotropic.

An explicit of discrete equilibrium distribution function for D3Q19 lattice model is described as follow:

$$f_i = \frac{1}{3}\rho\left[1 - \frac{u^2}{2c_s^2}\right] \qquad \text{for } i = 0 \qquad (17)$$

$$f_i = \frac{1}{18}\rho\left[1 + \frac{\mathrm{c_i}.u}{c_s^2} + \frac{(\mathrm{c_i}.u)^2}{2c_s^4} - \frac{u^2}{2c_s^2}\right] \qquad \text{for } i = 1..6 \qquad (18)$$

$$f_i = \frac{1}{36}\rho\left[1 + \frac{\mathrm{c_i}.u}{c_s^2} + \frac{(\mathrm{c_i}.u)^2}{2c_s^4} - \frac{u^2}{2c_s^2}\right] \qquad \text{for } i = 7\ldots18 \qquad (19)$$

The Lattice Boltzmann Method with BGK model for this D3Q19 lattice structure describe as:

$$f_j(x+\Delta x, t+\Delta t) = f_j(x,t)[1-\omega] + \omega f_j^{eq}(x,t) \quad \text{for } j=0, ..18 \qquad (20)$$

The basic Lattice Boltzmann Method algorithm consist of two steps; particle streaming and collision. The general algorithm is summarized in Figure 2.

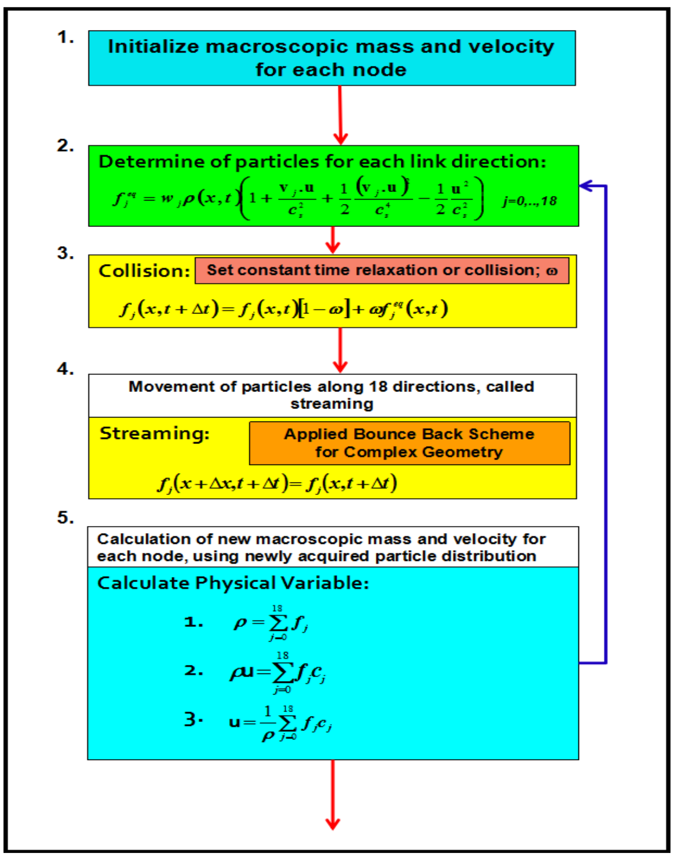

Figure 2: General algorithm particle distribution for D3Q19 LBM model

3.5. Boundary Condition for LBM

Lattice Boltzmann Method is an attractive method because the apparent ease to implement boundary conditions even in the complicated geometry like porous medium. Here, the boundary condition will be not in detail, instead summarized the concept that currently available. Two available boundary conditions are attributed in two main approaches:

Wet node approach:

- Boundary nodes are wet, i.e. they part of the fluid
- Can be split into equilibrium and off-equilibrium part
- Associated with macroscopic variables of flow

Bounce back approach:
- Boundary nodes are located outside the fluid
- The value of the known particle populations is copied to their unknown neighbor pointing in the opposite direction
- As these nodes are not part of the fluid, they follow different rules.

With each of these boundary condition, it is possible to implement velocity Dirichlet boundaries (all velocity components have an imposed value) and pressure boundaries (the pressure is imposed, and tangential velocity is zero). Furthermore, various types of Neumann boundary conditions are available; in this case a zero-gradient a given variable is imposed.

4. Permeability Calculation on Three-Dimensional Pore Scale Porous Media

Darcy flow (Equation 21) is common in the bulk of the reservoir, far away from the well where the dominant viscous force produces a creeping motion of the reservoir fluid.

$$u = \frac{k}{\mu} \nabla p \qquad (21)$$

In this regime, the Reynolds number (Re), given in Equation (22), is small and the flow is dominated by fluid viscous forces, such that the pressure gradient responsible for the flow is linearly proportional to the superficial velocity.

$$\mathrm{Re} = \frac{\rho u D}{\mu} \qquad (22)$$

Based on experiments, the classification of the flow in porous media is identified as shown in Figure 3. The laminar flow in porous media is valid if the Reynolds number is up to 1. So, the Darcy's law is valid if the Reynolds number of the single-phase flow up to about 1.

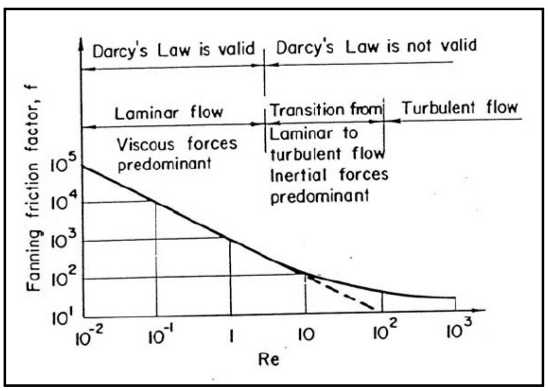

Figure 3: Classification of flow in porous media (Rose, 1945)

Permeability is depending on the size, distribution and connectivity of the pore spaces and it defines the physical relationship between porous media, the fluid that flow through it and conditions imposed by the flow process. Thus, a quantitative and qualitative prediction of this property in porous media requires an accurate microscopic model of the porous media and an understanding of the contribution of the microstructure of the medium to the flow distribution.

The permeability in the Lattice Boltzmann Method could be obtained after calculating the average lattice velocity where the pressure gradient is assigned, and for this work, the average velocity is calculated from LBM simulation through Equation-37. All parameters involved are in lattice unit.

$$k = \frac{u.\mu}{\nabla p} \qquad (23)$$

4.1. Workflow Permeability Calculation

The work flow in this work is quite straight forward as described in the following flowchart (Figure-4). There are generally two basic workflows, first is the converting the real object images from core plug or rock cutting become 3D binary, which converted as complex geometric boundary condition.

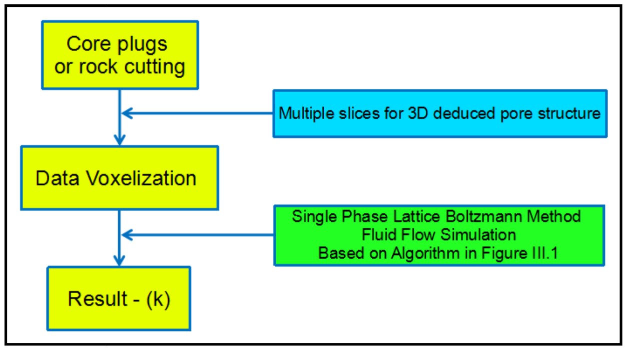

Figure 4: Summary diagram the process of voxel-mesh generation and Lattice Boltzmann Method procedure

Figure-5 through 13 are sandstone(x) samples and Figure 14 and 15 are limestone(l) samples with wide range of permeability. Those rock samples are voxelised in 300x300x300 lattice (left pictures) with it 1st slice sample that converted to binary (right pictures). The converted images are cannot directly inputted into LBM simulation, so using some simple algorithm the white and black image color was changed to binary number 1 and 0. In the binary image, white color is representing the pore space and the black color for non-pore space section.

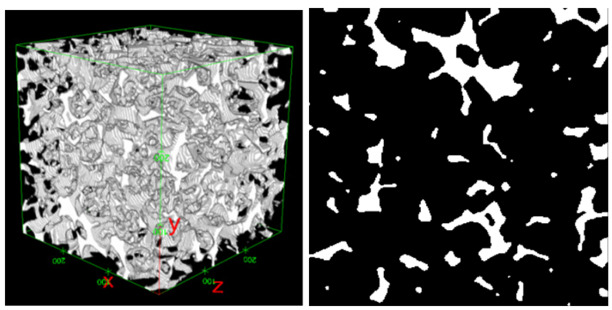

Figure 5: Left is x1D

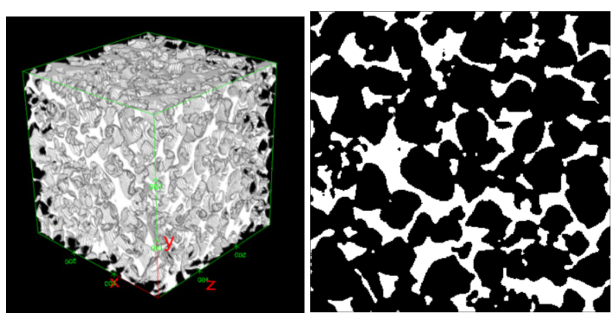

Figure 6: Left is x2D

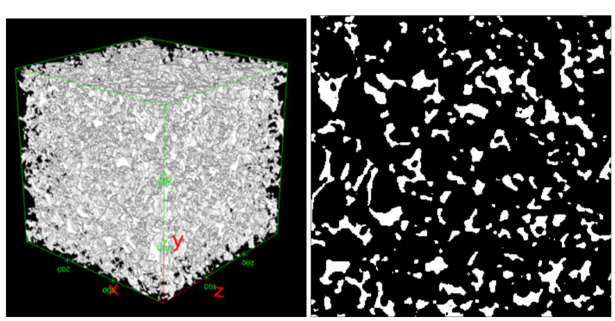

Figure 7: Left is x3D

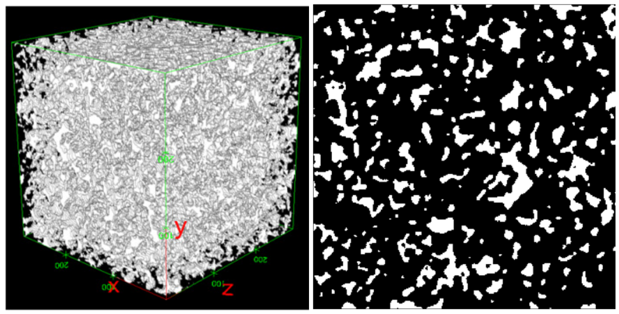

Figure 8: Left is x4D

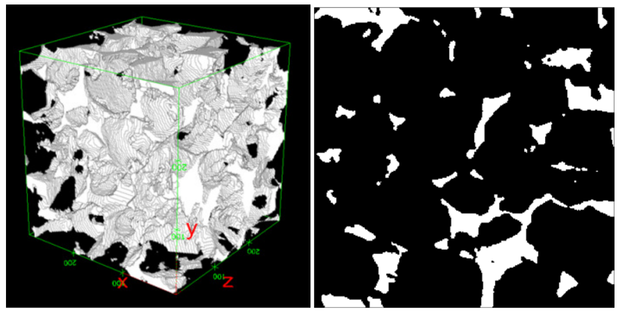

Figure 9: Left is x5D

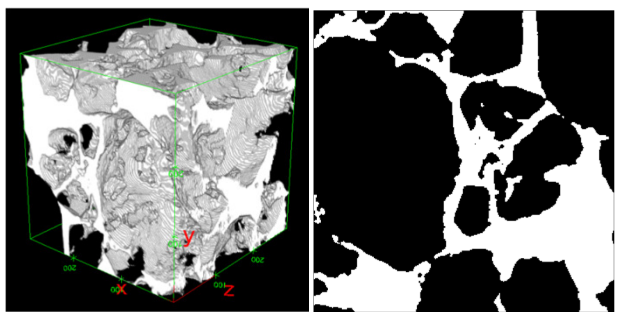

Figure 10: Left is x6D

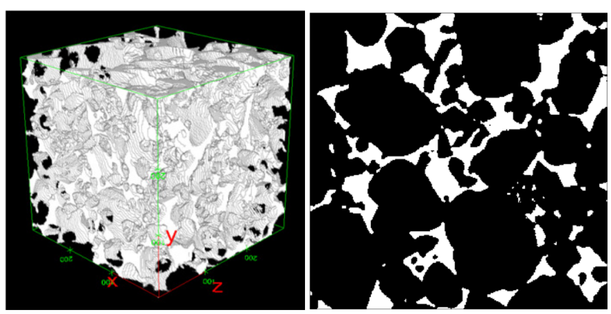

Figure 11: Left is x7D

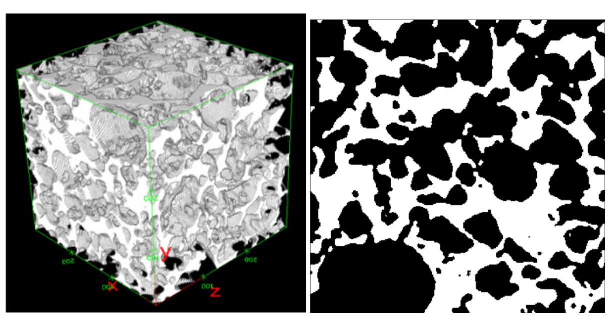

Figure 12: Left is x8D

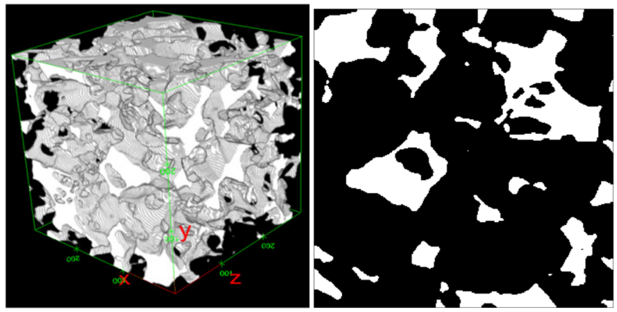

Figure 13: Left is x9D

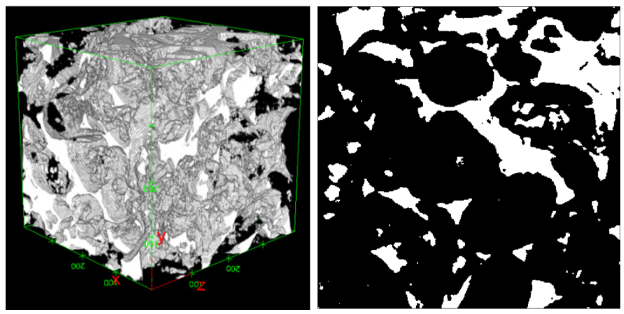

Figure 14: Left is l1D

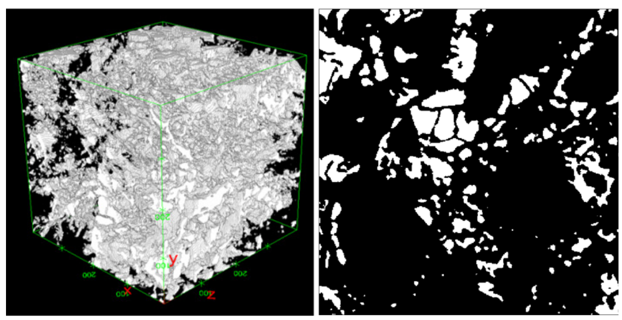

Figure 15: Left is l2D

The second one is the Lattice Boltzmann Method numerical simulation to simulate single phase that simulate every lattice, especially that occupied by the fluid, therefore the density, velocity and pressure could be calculated then finally summarized with the macroscopic parameters such as average velocity, lattice viscosity, and final pressure gradient. The simulation is done with the condition that the Darcy's law is valid.

For each time step, the average velocity was calculated and as well as other macro parameters prior to get single phase permeability in lattice unit square. Then, based on the physical unit, the lattice unit is converted to Darcy's unit.

4.2. Single Phase Permeability Calculation on Three-Dimensional Pore Scale Porous Media with Lattice Boltzmann Method

The simulation running time is depend on the properties of the samples. The higher permeability has a shorter running time between 2 to 3 hours in portable computer with i7 3630 MQ processor, while lower permeability could run 4 to 5 hours.

The simulation results processed for data analysis and visualization. The visualization results are also presented in following figures, which show the velocity streamlines. It could also show the iso-surface of the velocity; therefore, we can see the contour of the pore network.

All of the results, visually shown the interconnected of the "flow path" which is very complicated and different if we compare with the common model of capillary tubes bundle. Generally, the constrictions area for all samples will be shown by brighter color that indicating high velocity. It showed very random interconnection at any direction, which actually quite challenging to model the pore network.

It also shown that the "flow path" have many different types of forms. In the relatively low permeability samples such as x3 and x4 (Figure 16 and 17) and l1 and l2 (Figure 18 and 19), many dense tinny "flow path" randomly distributed and connected to the larger "flow path". Significant different form shown in the limestone samples, that shows only at some part of the area have the flow paths and the calculated permeability are significantly low (see Table 2).

In samples with higher permeability; x-1, 2, 5, 6, 7, 8 and 9, the flow path has bigger form and dispersedly distributed, sometime with tunnel form or just like wide ribbon. It also shown similarly with lower permeability sample, that the interconnection of the flow path occurred randomly which can be seen from the streamlines inside the flow paths.

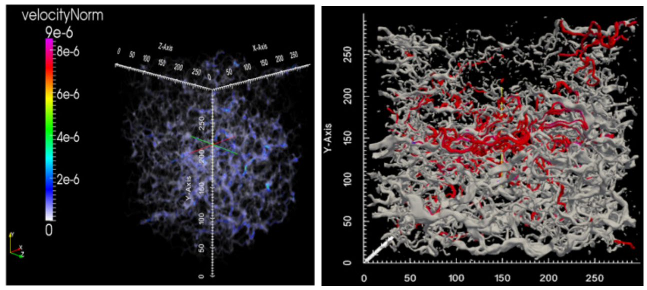

Figure 16: x3D velocity field(left), right picture is iso-surface velocity and streamlines on X -axis direction

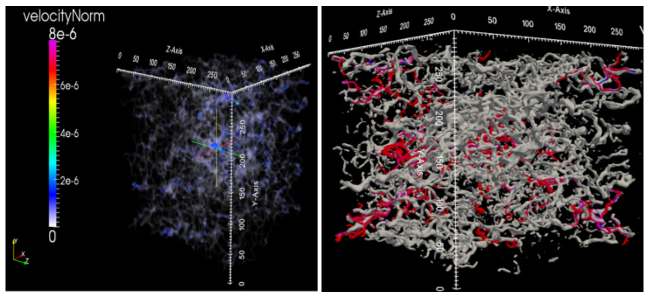

Figure 17: Left is x4D velocity, right is iso-surface velocity and streamlines on X -axis direction

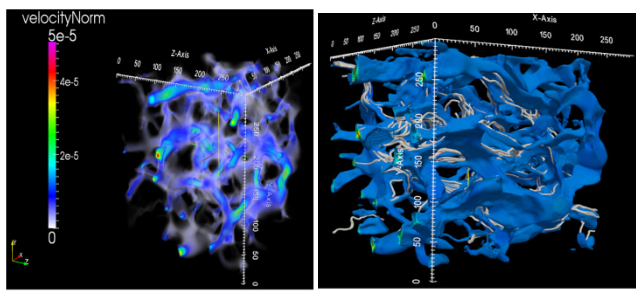

Figure 18: Left is x5D velocity, right is iso-surface velocity and streamlines on X -axis direction

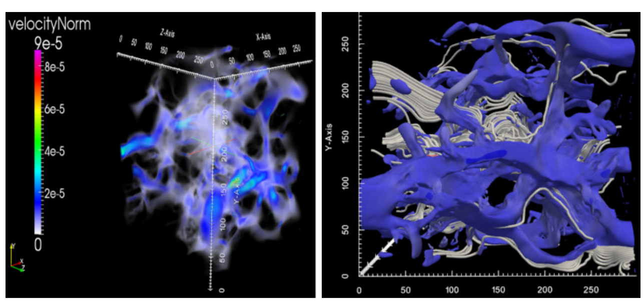

Figure 19: Left is x6D velocity, right is iso-surface velocity and streamlines on X -axis direction

The Lattice Boltzmann Model simulation to calculate permeability from previous samples (9 sandstone samples, 2 limestone sample), have been done and the result shown in Table 2. The simulation results are compared with pore scale network model that developed by Hu-Dong (2007) (see Figure 20). Also compared with the permeability-porosity correlation generated from experiment by Bourbie & Zinszner using Fountaibleau sandstone (see Figure 21) that proposed power law relationship between the porosity and air permeability:

$$\mathbf{k(mD)} = 0.303 * \phi^{3.05} \text{ for } \phi > 8\text{-}9\% \qquad (24)$$

$$\mathbf{k(mD)} = 2.75 * 10^{-5} * \phi^{7.33} \text{ for } \phi < 8\text{-}9\% \qquad (25)$$

The Lattice Boltzmann Method results showing have good agreement with pore network mode, while slightly different with the permeability-porosity correlation by Bourbie&Zinszner due to the sandstone used for the experiment. However, it is shows that, the application of Lattice Boltzmann Method to determine the physical properties of the porous media in pore scale can be applied and reliable, not to mention that the methodology is not hard to be implemented.

Sample	Resolution (µm)	viscosity - lu	Pressure Gradient	Velocity - lattice	Permability in (lu^2)	L(lu)	L(mm)	Perm (in mm^2)	D3Q19 LBM Perm in mD	network K (MB) - mD	ϕ	k by experiment (Bourbie&Zinsner)
Sandstone1D	8.683	0.166667	1.6722E-07	2.915E-08	0.029056	300	2.6049	2.1907E-06	2220	1486	0.141	970
Sandstone2D	4.956	0.166667	1.6722E-07	1.918E-07	0.1911182	300	1.4868	4.6942E-06	4756	3951	0.246	5294
Sandstone3D	9.1	0.166667	1.6722E-07	1.072E-08	0.0106856	300	2.73	8.8487E-07	897	281	0.169	1685
Sandstone4D	8.96	0.166667	1.6722E-07	7.166E-09	0.00714163	300	2.688	5.7334E-07	581	169	0.171	1746
Sandstone5D	3.997	0.166667	1.6722E-07	4.481E-07	0.446602	300	1.1991	7.1349E-06	7229	5369	0.211	3315
Sandstone6D	5.1	0.166667	1.6722E-07	5.500E-07	0.548165	300	1.53	1.4258E-05	14447	11282	0.240	4910
Sandstone7D	4.803	0.166667	1.6722E-07	3.711E-06	0.369868	300	1.4409	8.5324E-06	8645	7926	0.251	5629
Sandstone8D	4.982	0.166667	1.6722E-07	6.684E-06	0.666127	300	1.4946	1.6533E-05	16753	13932	0.340	14205
Sandstone9D	3.398	0.166667	1.6722E-07	2.307E-07	0.229971	300	1.0194	2.6553E-06	2691	3640	0.222	3871
Limestone1D	2.85	0.166667	1.6722E-07	4.571E-08	0.0455623	300	0.855	3.7008E-07	375	556	0.233	4486
Limestone2D	5.345	0.166667	1.6722E-07	1.104E-08	0.011	300	1.6035	3.1426E-07	318	158	0.168	1654

Table 2: 3D LBM simulation result with D3Q19 Model, single phase permeability

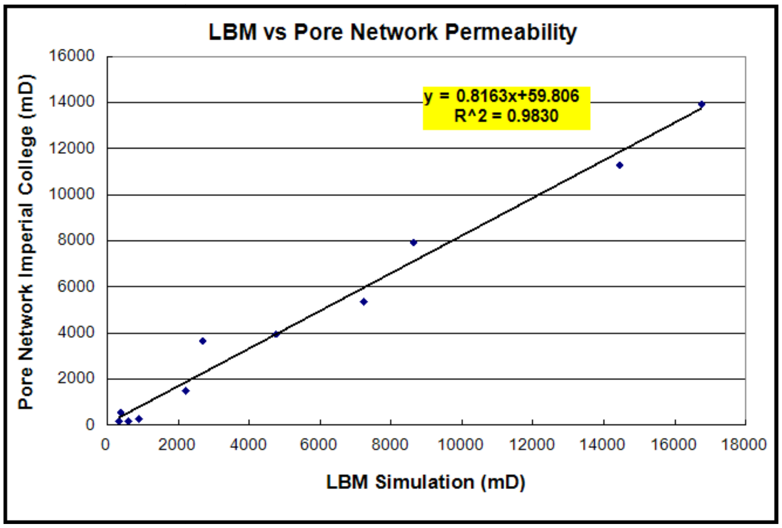

Figure 20: Comparison of one phase permeability calculation between Lattice Boltzmann Method D3Q19 Model and Pore Network

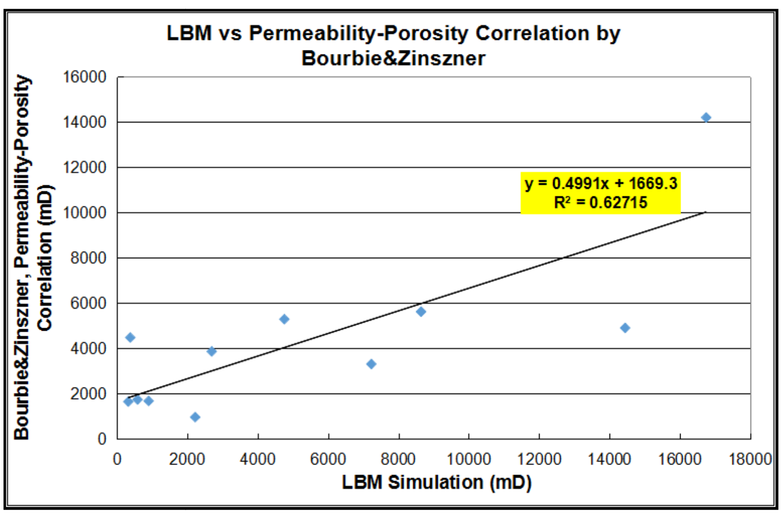

Figure 21: Comparison of one phase permeability calculation between Lattice Boltzmann Method D3Q19 Model and Permeability-Porosity Correlation by Bourbie-Zinszner's experiment on Fontainebleau sandstone

5. Conclusions

1. The Lattice Boltzmann Method is the alternative approach in computerized flow dynamic to evaluate fluid flow in pore scale especially in porous media.
2. With LBM, the evaluation of fluid flow in porous media with complex geometry could be done easily.
3. Estimating single phase permeability from sandstone and limestone samples that obtain from micro-CT and converted to binary images have been done using three-dimensional Lattice Boltzmann Method with D3Q19 model.
4. From visualization results, very complex forms of the flow paths could be investigated which shows the iso-surface for the velocity variable and velocity streamlines.
5. Visually, the flow path (or pore network) in lower permeability of the samples is densely distributed with more thinny size, compare to the higher permeability which dispersedly distributed with bigger path size.
6. Especially for the limestone samples, it shows that the distribution of the flow path in only at certain part area that significantly different with the sandstone samples.
7. Generally, the flow paths are interconnected in any direction, and visually we can see the constrictions of the path from the velocity field which show brighter color.
8. It is clearly shown visually that the flow paths especially in higher permeability have tunnel and ribbon forms.
9. In this work, using Lattice Boltzmann Method, the permeability calculation results are met the Darcy's law, where laminar flow regime could be achieved.
10. This Lattice Boltzmann Method and combining with micro-CT are opening the opportunity to investigate the porous media especially in sandstone and limestone that the samples are not only obtained from coring but also could use from the rock cutting non-destructively and very easy.

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