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Analysis of Waiting Times in M/M/1 Queue Model Using Fuzzy Control Chart

Mohammed Shapique. A
Assistant Professor (SG), Department of Mathematics,
IFET College of Engineering, Villupuram, Tamil Nadu, India

Abstract:
This paper proposes a technique to construct fuzzy control chart (graphical technique) for the waiting time in the queue model FM/FM /1 where the inter-arrival and service time are Fuzzified. Fuzzy queue is more realistic than the conventional crisp queue models. The basic idea of this paper is based upon Zadeh extension principle .The objective is transform fuzzy control chart of FM/FM /1 model to a family of crisp model by α-cut approach. Parametric (NLP) Non- linear program approach is followed to derive the membership function of upper and lower bounds for control chart parameters Trapezoidal fuzzy numbers are used to determine the validity of the proposed method, numerical example is illustrated.

Keywords: *α-cuts, Membership function, Parametric non –linear program, Fuzzy control chart, Waiting time.*

1. Introduction

Queuing theory deals with one the most unpleasant experience of life, waiting. Queueing is quiet common in many fields, for example, telecommunication, traffic engineering, computing, etc. To characterize a queuing system, we have to identify the probabilistic properties of performance measure of the queueing system All probability queueing models have assumed Poisson input and exponential service times but the fuzzy queues are much realistic than the normally used crisp queues. In real time the arrival rate, service rate is normally defined by linguistic terms such as fast, slow or moderate which can be best described by the Fuzzy sets. Queuing systems in Fuzzy have been described many researchers[1-6,] they have analyzed fuzzy queues using zadeh's extension principle. Parametric non-linear programming approach to fuzzy queues with bulk service and non -linear programming approach to derive the membership function of steady state performance measure in bulk arrival system were developed recently[7-8] .The control chart technique applied for M/M/1 crisp queueing model using weighted variance [9].Standard control chart applied for(M/E_k/1) and(E_k/M/1) models[10] .In this paper the M/M/1 queuing model is analyzed for E(Wq) and E(Ws) considering that the arrival and service rate are fuzzified (i.e)(FM/FM/1),based on zadeh's extension principle the membership function is constructed for the fuzzfied arrivals and service.

2. Model Description

Consider M/M/1 queueing system with infinite capacity. In this queuing system customers arrive according to Poisson distribution with arrival rate λ and inter-arrival arrival time follows exponential distribution with one server. The performance measures of this queueing system (crisp) are as follows

$$L_s = \frac{\lambda}{\mu-\lambda} \quad L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} \quad E(W_s) = \frac{1}{(\mu-\lambda)} \text{ and } E(W_q) = \frac{\lambda}{\mu(\mu-\lambda)}$$

The variance of W_s & W_q are given by

$$\text{Var}(W_s) = \frac{1}{(\mu-\lambda)^2} \quad \text{Var}(W_q) = \frac{2\lambda\mu-\lambda}{\mu^2(\mu-\lambda)^2}$$

3. The Parameters of the Control Chart for the Expected Waiting Time in the Queue

UCL= $E(W_q)+3\sqrt{\text{Var}(W_q)}$

CL= $E(W_q)$

LCL= $E(W_q)-3\sqrt{\text{Var}(W_q)}$

(ii) The parameters of the control chart of the expected waiting time in the system are

UCL= $E(W_s)+3\sqrt{\mathrm{Var}(W_s)}$
CL= $E(W_s)$
LCL= $E(W_s)-3\sqrt{\mathrm{Var}(W_s)}$

4. Model with Fuzzy Parameter

Consider a queueing system in which the customer arrive at a single-server facility with arrival rate $\tilde{\lambda}$ and service rate $\tilde{\mu}$, where $\tilde{\lambda}$ (Poisson rate) is fuzzy in nature and $\tilde{\mu}$ (service rate). Let $\mu_{\tilde{\lambda}}(x)$ and $\mu_{\tilde{\mu}}(x)$ are membership function of arrival rate and service rate respectively.

$\tilde{\lambda}=\left\{x,\mu_{\tilde{\lambda}}(x)/\,\mathrm{x}\in S(\tilde{\lambda})\right\}$ and $\tilde{\mu}=\left\{\mathrm{y},\mu_{\tilde{\mu}}(\mathrm{y})/\,\mathrm{y}\in S(\tilde{\mu})\right\}$ Where $S(\tilde{\lambda})$ and $S(\tilde{\mu})$ are the supports of $\tilde{\lambda}$ and $\tilde{\mu}$ which denote the universal set of the arrival rate and service rate respectively.

Let P(x,y) and $\tilde{P}\left(\tilde{\lambda},\tilde{\mu}\right)$ denotes the parameters of the control chart relating to average waiting time of customers in the crisp and fuzzy environment respectively, where p stands for the control chart parameters CL,UCL and LCL. Since $\tilde{\lambda}\,\&\,\tilde{\mu}$ are fuzzy eventually $\tilde{P}\left(\tilde{\lambda},\tilde{\mu}\right)$ will also be fuzzy in nature. Using Zadeh's extension principle the membership function of the control chart $\tilde{P}\left(\tilde{\lambda},\tilde{\mu}\right)$ is given below

$$\mu_{\tilde{P}\left(\tilde{\lambda},\tilde{\mu}\right)}(z)=\sup_{\mathrm{x}\in S(\tilde{\lambda}),\mathrm{y}\in S(\tilde{\mu})}\min\left\{\mu_{\tilde{\lambda}}(x),\mu_{\tilde{\mu}}(x)/\,\mathrm{z}=P(x,y)\right\} \qquad (1)$$

5. Control Chart Parameter for Mean Waiting Time in the Queue

The fuzzy control chart parameters of for the $E(W_q)$ are

$$\mathrm{CL(x,y)}=\frac{x}{y\left(y-x\right)}$$

UCL= $\mathrm{CL(x,y)}+3\sqrt{\mathrm{Var(x,\,y)}}$
LCL= $\mathrm{CL(x,y)}-3\sqrt{\mathrm{Var(x,\,y)}}$

Where $\mathrm{Var(x,\,y)}=\dfrac{2xy-x}{y^2\left(y-x\right)^2}$

Now we intend a mathematical programming for deriving membership function for the control chart parameters CL,UCL and LCL by using α -cuts .

6. The α -Cut Approach Based on the Extension Principle

Denote α cuts of $\tilde{\lambda}$ (fuzzified arrival) and $\tilde{\mu}$ (fuzzified service) as

$$\lambda_\alpha=\left\{x\in X\,/\,\mu_{\tilde{\lambda}}(x)\geq\alpha\right\} \qquad (2)$$

$$\mu_\alpha=\left\{y\in Y\,/\,\mu_{\tilde{\mu}}(\mathrm{y})\geq\alpha\right\} \qquad (3)$$

The above crisp sets may be expressed as

$$\lambda_\alpha=\left[x_\alpha^L,x_\alpha^U\right]=\left[\min_{x\in X}\left\{x\in X\,/\,\mu_{\tilde{\lambda}}(x)\geq\alpha\right\},\max_{y\in Y}\left\{x\in X\,/\,\mu_{\tilde{\lambda}}(x)\geq\alpha\right\}\right] (4)$$

$$\mu_\alpha=\left[y_\alpha^L,\mathrm{y}_\alpha^U\right]=\left[\min_{y\in Y}\left\{y\in Y\,/\,\mu_{\tilde{\lambda}}(\mathrm{y})\geq\alpha\right\},\max_{y\in Y}\left\{y\in Y\,/\,\mu_{\tilde{\lambda}}(\mathrm{y})\geq\alpha\right\}\right] (5)$$

The above provides information on the arrival rate and service rate with possibility α
As a result, the bound of the above intervals can be described as functions of α and can be obtained as

$$x_\alpha^L=\min\mu_{\tilde{\lambda}}^{-1}(\alpha),\; x_\alpha^U=\max\mu_{\tilde{\lambda}}^{-1}(\alpha) \qquad (6)$$

$$y_\alpha^L=\min\mu_{\tilde{\mu}}^{-1}(\alpha),\; y_\alpha^U=\max\mu_{\tilde{\mu}}^{-1}(\alpha) \qquad (7)$$

Therefore, the α -cuts used to form the membership function.

7. Building Membership Function

Consider the membership function relating to CL for $E(W_q)$ as $\mu_{\tilde{CL}}(z)$ where $\mu_{\tilde{CL}}(z) = \min\left\{\mu_{\tilde{\lambda}}(x), \mu_{\tilde{\mu}}(x) / z = CL(x, y)\right\}$

To deal with the above membership function, we need atleast one of the following two cases to hold such that $z = CL(x, y)$ and $\mu_{\tilde{CL}}(z) = \alpha$

Case (i) $\mu_{\tilde{\lambda}}(x) = \alpha,\ \mu_{\tilde{\mu}}(y) \geq \alpha$ (8)

Case (ii) $\mu_{\tilde{\lambda}}(x) \geq \alpha,\ \mu_{\tilde{\mu}}(y) \square\ \alpha$ (9)

Using (2), (3), (4) & (5) we can write $x \in \left[x_\alpha^L, x_\alpha^U\right], y \in \left[y_\alpha^L, y_\alpha^U\right]$

8. Non -Linear Program for Lower and Upper Bounds

Using non-linear program (NLP) we find the lower and upper bounds of the α -cut of $\mu_{\tilde{CL}}(z)$ corresponding to the two cases mentioned in (8) & (9)

Case (i)

$$(CL)_\alpha^{L_1} = \min \frac{x}{y(y-x)}, \text{subject to } x_\alpha^L \leq x \leq x_\alpha^U, y \in \mu_\alpha$$

$$(CL)_\alpha^{U_1} = \max \frac{x}{y(y-x)}, \text{subject to } x_\alpha^L \leq x \leq x_\alpha^U, y \in \mu_\alpha$$

Case (ii)

$$(CL)_\alpha^{L_2} = \min \frac{x}{y(y-x)}, \text{subject to } y_\alpha^L \leq y \leq y_\alpha^U, x \in \lambda_\alpha$$

$$(CL)_\alpha^{U_2} = \max \frac{x}{y(y-x)}, \text{subject to } y_\alpha^L \leq y \leq y_\alpha^U, x \in \lambda_\alpha$$

Using the above mentioned case(i) and (ii) we find the left shape function L(z) and the right shape function R(z) of $\mu_{\tilde{CL}}(z)$ using which we can find the $(CL)_\alpha^L$ and $(CL)_\alpha^U$.

$$(CL)_\alpha^L = \min_{x \in X, y \in Y} \frac{x}{y(y-x)}, \text{subject to } x_\alpha^L \leq x \leq x_\alpha^U, y_\alpha^L \leq y \leq y_\alpha^U \quad (10)$$

$$(CL)_\alpha^U = \max_{x \in X, y \in Y} \frac{x}{y(y-x)}, \text{subject to } x_\alpha^L \leq x \leq x_\alpha^U, y_\alpha^L \leq y \leq y_\alpha^U \quad (11)$$

To satisfy the condition $\mu_{\tilde{CL}}(z) = \alpha$ atleast one x and y should falls on the boundary of the equation (10) and (11) which falls under the category of parametric NLP. Based on the zadeh's extension principle and by convexity principle of fuzzy we obtain $\left[(CL)_{\alpha_1}^L, (CL)_{\alpha_1}^U\right] \subseteq \left[(CL)_{\alpha_2}^L, (CL)_{\alpha_2}^U\right] \textit{for } 0 < \alpha_2 < \alpha_1 < 1$

At α =0, the range for the support of the $E(W_s)$ and $E(W_s)$ is calculated and at α =1 the greatest possible range of the $E(W_s)$ and $E(W_s)$ is calculated.

If the lower and upper bound of $(\tilde{CL})$ are invertible with respect to α then the left shape and the right shape function L(z) and R(z) may be obtained as $L(z) = \left[(CL)_\alpha^L\right]^{-1}$ and $R(z) = \left[(CL)_\alpha^U\right]^{-1}$.Then the membership function

$$\mu_{\tilde{CL}}(z) = \begin{cases} L(z), & (CL)_{\alpha=0}^L \leq z \leq (CL)_{\alpha=1}^L \\ 1, & (CL)_{\alpha=1}^L \leq z \leq (CL)_{\alpha=1}^U \\ R(z), & (CL)_{\alpha=1}^U \leq z \leq (CL)_{\alpha=0}^U \end{cases}$$

Yaker ranking method is applied to defuzzify $(\tilde{CL})$ of $E(W_q)$ into crisp one. The yaker ranking index is

$$\tilde{Y}(\tilde{CL}) = 0.5\int_0^1 [(\mathrm{CL})_\alpha^L + (\mathrm{CL})_\alpha^U]d\alpha$$

By applying similar method, the membership functions of lower and upper bounds of $(\tilde{CL})$ (i.e) $\mu_{\tilde{UCL}}(z)$, $\mu_{\tilde{LCL}}(z)$ and yager ranking indices relating control chart parameters can be derived.

9. Control Chart Parameters for the $E(W_s)$

The parameters of the fuzzy control chart for the average waiting time in the system are

$$\mathrm{CL(x,y)} = \frac{1}{(y-x)}$$

$$\mathrm{UCL} = \mathrm{CL(x,y)} + 3\sqrt{\mathrm{Var(x,y)}}$$

$$\mathrm{LCL} = \mathrm{CL(x,y)} - 3\sqrt{\mathrm{Var(x,y)}}$$

Where $\mathrm{Var(x,y)} = \dfrac{1}{(y-x)^2}$

By following the same procedure as we did for previous section we can form the membership function and yaker ranking indices relating to $E(W_s)$

10. Numerical Example

Consider a single server Poisson queue with mean arrival rate $\tilde{\lambda} = [3,4,5,6]$ customers per unit time and service according to exponential with mean service rate $\tilde{\mu} = [14,15,16,17]$ .lets analyses the mean waiting time in the queue and in the system using fuzzy control chart.

$$\left[x_\alpha^L, x_\alpha^U\right] = [\alpha+3, 6-\alpha], \left[y_\alpha^L, y_\alpha^U\right] = [14+\alpha, 17-\alpha]$$

CONTROL CHART PARAMETERS FOR THE $E(W_q)$

The upper and lower bounds of fuzzy control parameters for $E(W_q)$ are derived as follows

$$(CL)_\alpha^L = \frac{3+\alpha}{(17-\alpha)(14-2\alpha)}; (CL)_\alpha^U = \frac{16-\alpha}{(14+\alpha)(8+2\alpha)}$$

$$(\mathrm{Var})_\alpha^L = \frac{-3\alpha^2+22\alpha+93}{(17-\alpha)^2(14-2\alpha)^2}; (\mathrm{Var})_\alpha^U = \frac{-3\alpha^2-4\alpha+132}{(\alpha+14)^2(2\alpha+8)^2}$$

$$(\mathrm{U}CL)_\alpha^L = (CL)_\alpha^L + 3\sqrt{(Var)_\alpha^L}; (\mathrm{U}CL)_\alpha^U = (CL)_\alpha^U + 3\sqrt{(Var)_\alpha^U}$$

$$(\mathrm{L}CL)_\alpha^L = (CL)_\alpha^L - 3\sqrt{(Var)_\alpha^L}; (\mathrm{L}CL)_\alpha^U = (CL)_\alpha^U - 3\sqrt{(Var)_\alpha^U}$$

For different values of $\alpha \in [0,1]$ are the values control chart parameters are calculated in table 1.

α	x_α^L	x_α^U	y_α^L	y_α^U	$(CL)_\alpha^L$	$(CL)_\alpha^U$	$(\mathrm{U}CL)_\alpha^L$	$(\mathrm{U}CL)_\alpha^U$	$(\mathrm{L}CL)_\alpha^L$	$(\mathrm{L}CL)_\alpha^U$
0.0000	3.0000	6.0000	14.0000	17.0000	0.0126	0.0536	0.1342	0.3613	-0.1090	-0.2542
0.1000	3.1000	5.9000	14.1000	16.9000	0.0133	0.0510	0.1388	0.3487	-0.1122	-0.2466
0.2000	3.2000	5.8000	14.2000	16.8000	0.0140	0.0486	0.1435	0.3366	-0.1155	-0.2393
0.3000	3.3000	5.7000	14.3000	16.7000	0.0147	0.0463	0.1484	0.3251	-0.1189	-0.2324
0.4000	3.4000	5.6000	14.4000	16.6000	0.0155	0.0442	0.1533	0.3140	-0.1223	-0.2257
0.5000	3.5000	5.5000	14.5000	16.5000	0.0163	0.0421	0.1584	0.3035	-0.1258	-0.2192
0.6000	3.6000	5.4000	14.6000	16.4000	0.0171	0.0402	0.1637	0.2934	-0.1294	-0.2130
0.7000	3.7000	5.3000	14.7000	16.3000	0.0180	0.0384	0.1691	0.2837	-0.1330	-0.2070
0.8000	3.8000	5.2000	14.8000	16.2000	0.0189	0.0366	0.1746	0.2744	-0.1368	-0.2012
0.9000	3.9000	5.1000	14.9000	16.1000	0.0199	0.0349	0.1803	0.2655	-0.1406	-0.1957
1.0000	4.0000	5.0000	15.0000	16.0000	0.0208	0.0333	0.1862	0.2569	-0.1445	-0.1903

Table 1: α cut of arrival and service with fuzzy control chart parameters for $E(W_q)$

We note that at from table 1, at α =0we observe that the value of CL lie in the range [0.0126, 0.0536] which implies that CL of $E(W_q)$ can't exceed 0.0536 or fall before 0.0126 and UCL lies in the range $0.1342 \le \text{UCL} \le 0.3613$.

At α =1 we observe that the value of CL lie in the range [0.0208, 0.0333] which implies that the CL of $E(W_q)$ can't exceed 0.0333 or fall before 0.0208 and UCL lies in the range $0.1862 \le \text{UCL} \le 0.2569$.

By applying Yager ranking index the expected CL and UCL of $E(W_q)$ are

$$\tilde{Y}(\tilde{\text{CL}}) = \frac{1}{2}\int_0^1 [(CL)_\alpha^L + (CL)_\alpha^U]\, d\alpha = 0.03$$

$$\tilde{Y}(\tilde{\text{UCL}}) = \frac{1}{2}\int_0^1 [(\text{UCL})_\alpha^L + (\text{UCL})_\alpha^U]\, d\alpha = 0.23$$

By using MATLAB the inverse function of L(z) and R(z) of $(CL)_\alpha^L, (CL)_\alpha^U, (\text{UCL})_\alpha^L$ & $(\text{UCL})_\alpha^U$ are obtained .the membership function of $\mu_{\tilde{CL}}(z)$ and $\mu_{\tilde{UCL}}(z)$ can be stated as

$$\mu_{\tilde{CL}}(z) = \begin{cases} L(z), & 0.0126 \le z \le 0.0208 \\ 1, & 0.0208 \le z \le 0.0333 \\ R(z), & 0.0333 \le z \le 0.0536 \end{cases} \quad \text{and} \quad \mu_{\tilde{UCL}}(z) = \begin{cases} L(z), & 0.1342 \le z \le 0.1862 \\ 1, & 0.1862 \le z \le 0.2569 \\ R(z), & 0.2569 \le z \le 0.3613 \end{cases}$$

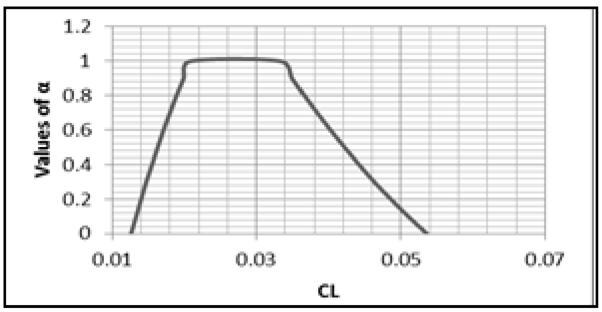

Figure 1 : Membership function of CL for E (Wq)

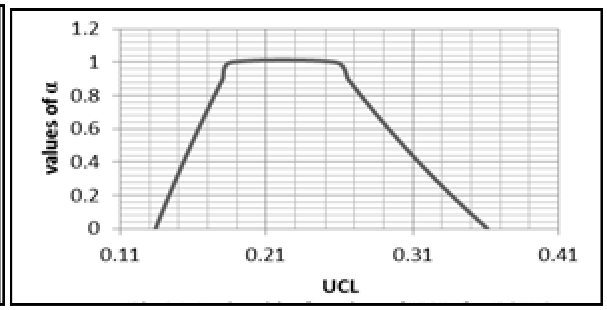

Figure 2: Membership function of UCL for E (Wq)

The graph of the membership function of $\mu_{\tilde{CL}}(z)$, $\mu_{\tilde{UCL}}(z)$ corresponding to α cuts relating to $E(W_q)$ are shown in fig.1 and 2 respectively.

The upper and lower bounds of fuzzy control parameters for $E(W_s)$ are derived as follows

$$(CL)_\alpha^L = \frac{1}{(14-2\alpha)} ; (CL)_\alpha^U = \frac{1}{(8+2\alpha)}$$

$$(\text{Var})_\alpha^L = \frac{1}{(14-2\alpha)^2} ; (\text{Var})_\alpha^U = \frac{1}{(8+2\alpha)^2}$$

$$(\text{U}CL)_\alpha^L = (CL)_\alpha^L + 3\sqrt{(Var)_\alpha^L} \quad (\text{U}CL)_\alpha^U = (CL)_\alpha^U + 3\sqrt{(Var)_\alpha^U}$$

$$(\text{L}CL)_\alpha^L = (CL)_\alpha^L - 3\sqrt{(Var)_\alpha^L} \quad (\text{L}CL)_\alpha^U = (CL)_\alpha^U - 3\sqrt{(Var)_\alpha^U}$$

For different values of $\alpha \in [0,1]$ are the values control chart parameters are calculated in table 2

α	x_α^L	x_α^U	y_α^L	y_α^U	$(CL)_\alpha^L$	$(CL)_\alpha^U$	$(\text{U}CL)_\alpha^L$	$(\text{U}CL)_\alpha^U$	$(\text{L}CL)_\alpha^L$	$(\text{L}CL)_\alpha^U$
0.0000	3.0000	6.0000	14.0000	17.0000	0.0714	0.1250	0.2857	0.5000	-0.1429	-0.2500
0.1000	3.1000	5.9000	14.1000	16.9000	0.0725	0.1220	0.2899	0.4878	-0.1449	-0.2439
0.2000	3.2000	5.8000	14.2000	16.8000	0.0735	0.1190	0.2941	0.4762	-0.1471	-0.2381
0.3000	3.3000	5.7000	14.3000	16.7000	0.0746	0.1163	0.2985	0.4651	-0.1493	-0.2326
0.4000	3.4000	5.6000	14.4000	16.6000	0.0758	0.1136	0.3030	0.4545	-0.1515	-0.2273
0.5000	3.5000	5.5000	14.5000	16.5000	0.0769	0.1111	0.3077	0.4444	-0.1538	-0.2222
0.6000	3.6000	5.4000	14.6000	16.4000	0.0781	0.1087	0.3125	0.4348	-0.1563	-0.2174
0.7000	3.7000	5.3000	14.7000	16.3000	0.0794	0.1064	0.3175	0.4255	-0.1587	-0.2128
0.8000	3.8000	5.2000	14.8000	16.2000	0.0806	0.1042	0.3226	0.4167	-0.1613	-0.2083
0.9000	3.9000	5.1000	14.9000	16.1000	0.0820	0.1020	0.3279	0.4082	-0.1639	-0.2041
1.0000	4.0000	5.0000	15.0000	16.0000	0.0833	0.1000	0.3333	0.4000	-0.1667	-0.2000

Table 2: α cut of arrival and service with fuzzy control chart parameters for $E(W_s)$

We note that at from table 2, at α =0we observe that the value of CL lies in the range [0.0714, 0.1250] which implies that CL of $E(W_q)$ can't exceed 0.1250 or fall before 0.0714 and UCL lies in the range $0.2857 \le \text{UCL} \le 0.5$.

At α =1 we observe that the value of CL lies in the range [0.0833, 0.1] which implies that the CL of $E(W_q)$ can't exceed 0.1 or fall before 0.0833 and UCL lies in the range $0.3333 \le \text{UCL} \le 0.4$.

By applying Yager ranking index the expected CL and UCL of $E(W_q)$ are

$$\tilde{Y}(\tilde{\text{CL}}) = \frac{1}{2}\int_0^1 [(CL)_\alpha^L + (CL)_\alpha^U]\, d\alpha = 0.09$$

$$\tilde{Y}(\tilde{\text{UCL}}) = \frac{1}{2}\int_0^1 [(\text{UCL})_\alpha^L + (\text{UCL})_\alpha^U]\, d\alpha = 0.37$$

By using MATLAB the inverse function of L(z) and R(z) of $(CL)_\alpha^L$, $(CL)_\alpha^U$, $(\text{UCL})_\alpha^L$ & $(\text{UCL})_\alpha^U$ are obtained. The membership function of $\mu_{\tilde{CL}}(z)$ and $\mu_{\tilde{UCL}}(z)$ can be stated as

$$\mu_{\tilde{CL}}(z) = \begin{cases} L(z), & 0.0714 \le z \le 0.0833 \\ 1, & 0.0833 \le z \le 0.1 \\ R(z), & 0.1 \le z \le 0.1250 \end{cases} \quad \text{and} \quad \mu_{\tilde{UCL}}(z) = \begin{cases} L(z), & 0.2857 \le z \le 0.3333 \\ 1, & 0.3333 \le z \le 0.4 \\ R(z), & 0.4 \le z \le 0.5 \end{cases}$$

The graph of the membership functions $\mu_{\tilde{CL}}(z)$, $\mu_{\tilde{UCL}}(z)$ corresponding to α cuts relating to $E(W_q)$ are shown in fig.3 and 4 respectively

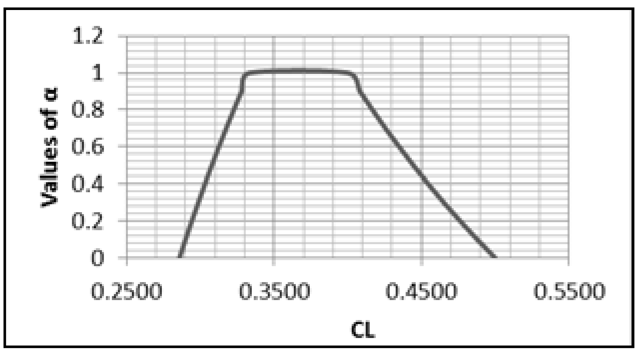

Figure 3 : Membership function of CL for E (Ws)

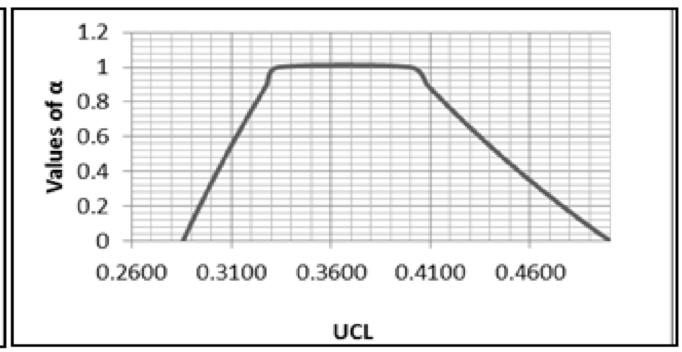

Figure 4: Membership function of UCL for E (Ws)

11. Conclusion

The theory of fuzzy has been applied in queuing systems to provide broader application in many areas. We have analyzed the performance measures of fuzzfied queueing systems in particular waiting times by applying the concept of fuzzy control chart using Parametric non-Linear program through α-cuts of the membership function. This analysis enables system designers in enhancing decision.

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