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Stability Analysis of Axially Compressed Rectangular SSSS & CSCS Plates Using MATLAB Programming

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Abstract:
Two thin rectangular plates, one simply supported on all sides (SSSS), and the other plate simply supported and fixed on alternate opposite sides(CSCS), were analyzed for buckling or stability. Polynomial series were used to formulate shape functions for plates loaded axially along the x- axis. The shape functions were used in Ritz energy equation to obtain the critical buckling loads 'Nx' for the plates studied. Computer programs based on the formulated critical buckling load equation were developed in MATLAB language for easier and faster stability analysis of aforementioned plates. In order to validate the results obtained from the computer program, comparison was made with other results from literature. For simply supported plate with aspect ratios of 1.0, 1.2, 1.5, 1.6 and 2.0, the maximum percentage difference is 0.02%. And for the plate with alternate opposite simply supported and fixed edge, the maximum percentage difference from the computed values of 'n' and those in literature is -0.19%. In both cases, the percentage differences are less than 1% which is consider very insignificant. Therefore, the developed MATLAB programs can be used to analyze accurately the critical buckling load of SSSS and CSCS plates.

Keywords: *Critical buckling load, polynomial shape function, MATLAB program, Ritz energy equation, rectangular plates.*

1. Introduction

Nearly all structural materials have the tendency to buckle under load in the direction and proportional to the magnitude of the applied load. Mostprevious studies were based on the classical thin plate theory. Kang and Leissa (2001) presented the buckling factors for SS-F-SS-F plate subjected unidirectional in-plane moment. Thereafter, Leissa and Kang (2002) extended their solution to SS-C-SS-C plate under the same type of loading. For these cases, the stability equation for the thin plate theory can be treated in the two directions as a product of two one-variable functions, exact solution.

Others who work on plate buckling using the trigonometric shape functions are Szilard(2004), Ventsel and Krauthamer (2001), Timoshenko & Woinowsky-Kreiger(1959), Iyenger (1988), Ugural and Fenster(2003).Of note is that all these works were based on trigonometric functions.

However, some researchers who recently took a different approach used polynomial shape functions to simplify the buckling analysis using conventional method. Some of them are Ibearugbulem et al. (2012), Onwuka et al. (2013), Ezeh et al. (2014a),Eziefula et al.(2014), and Ezeh et al. (2014).

Of all the research works available in literatures, there is no evidence of any computer program based on polynomial shape functions developed for analyzing buckling of SSSS and CSCS rectangular plates.

2. Methodology

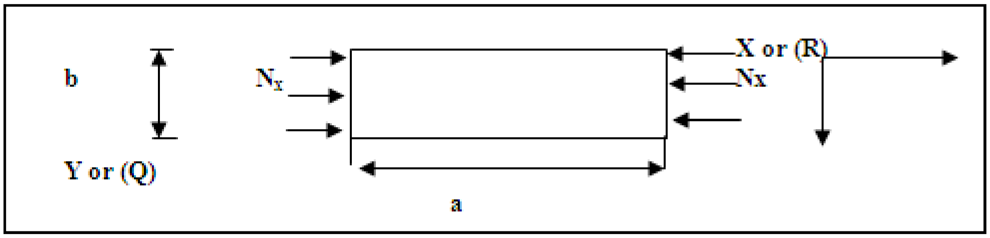

Figure 1: Schematic diagram of axially loaded SSSS Rectangular Plate.

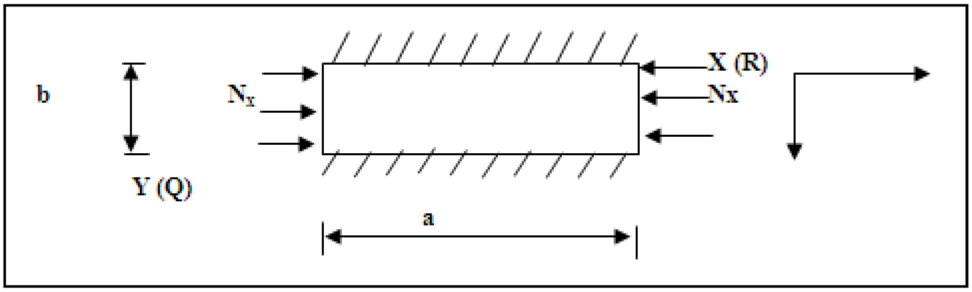

Figure 2: Schematic diagram of axially loaded CSCS Rectangular Plate.

Consider axially- or in-plane loaded SSSS and CSCS plates shown in Fig.1 and Fig.2 respectively. Ibearugbulem (2012) gave the total potential energy functional, Π_x for a rectangular thin isotropic plate subject to in-plane load in x-direction as Equation (1)

$$\Pi_x = \frac{D}{2b^2}\iint \left[\frac{b^3}{a^3}(w''^R)^2 + \frac{2b}{a}(w''^{RQ})^2 + \frac{a}{b}(w''^Q)^2\right]\partial R\partial Q - \frac{bN_x}{2a}\iint (w'^R)^2 \partial R\partial Q \quad (1)$$

Where $w''^R = \frac{\partial^2 w}{\partial R^2}$; $w''^Q = \frac{\partial^2 w}{\partial Q^2}$; $w''^{RQ} = \frac{\partial^2 w}{\partial R\,\partial Q}$; $w'^R = \frac{\partial w}{\partial R}$; D = flexural rigidity of the plate, and R & Q are dimensionless quantities in x & y axis taken between $0 \le R \le 1$; $0 \le Q \le 1$; Mathematically, $R = \frac{x}{a}$; $Q = \frac{y}{b}$

Let aspect ratio $s = b/a$, (2)

Substituting Equation(2) in Equation(1), yields Equation (3)

$$\Pi_x = \frac{D}{2a^2}\iint \left[s(w''^R)^2 + \frac{2}{s}(w''^{RQ})^2 + \frac{1}{s^3}(w''^Q)^2\right]\partial R\partial Q - \frac{N_x}{2}\iint s(w'^R)^2 \partial R\partial Q \quad (3)$$

But, the formulated polynomial based deflectedshape functions for SSSS and CSCS plates were derived by Ibearugbulem (2012) as $W = A(R-2R^3+R^4)(Q-2Q^3+Q^4)$ and $W = A(R-2R^3+R^4)(Q^2-2Q^3+Q^4)$ respectively.

Hence, for SSSS plate,

$$W = A(R-2R^3+R^4)(Q-2Q^3+Q^4) \quad (4)$$

For CSCS plate

$$W = A(R-2R^3+R^4)(Q^2-2Q^3+Q^4) \quad (5)$$

Fromequations (4) and (5), Let $W = Ak$ (6)

And $k = UV$ (7)

Where U =R terms and V = Q terms in Equations (4) and (5).

Substituting Equation (6) into Equation(3), yields the total potential energy functional as Equation (8),

$$\Pi_x = \frac{DA^2}{2a^2}\iint \left[s(k''^R)^2 + \frac{2}{s}(k''^{RQ})^2 + \frac{1}{s^3}(k''^Q)^2\right]\partial R\partial Q - \frac{N_x A^2}{2}\iint s(k'^R)^2 \partial R\partial Q \quad (8)$$

Where $k''^R = \frac{\partial^2 k}{\partial R^2}$; $k''^Q = \frac{\partial^2 k}{\partial Q^2}$; $k''^{RQ} = \frac{\partial^2 k}{\partial R\,\partial Q}$; $k'^R = \frac{\partial k}{\partial R}$;

Minimizing Equation (8) and making the critical buckling load 'N_x' the subject of the formula gives:

$$N_x = \frac{D\int\int \left[(k''^R)^2 + \frac{2}{s^2}(k''^{RQ})^2 + \frac{1}{s^4}(k''^R)^2\right]\partial R\,\partial Q}{a^2\int\int (k'^R)^2\,\partial R\,\partial Q} \quad (9)$$

Let $$n_x = \frac{\int\int \left[(k''^R)^2 + \frac{2}{s^2}(k''^{RQ})^2 + \frac{1}{s^4}(k''^R)^2\right]\partial R\,\partial Q}{\int\int (k'^R)^2\,\partial R\,\partial Q} \quad (10)$$

Then, Equation (10) becomes:

$$N_x = n_x\frac{D}{a^2} \quad (11)$$

Where n_x is the critical buckling factor or coefficient in x- direction,

The individual 'k' value from Equation (7) can be substituted in equation (10) and 'n'values evaluated. Programs are develop for this analysis as presented in appendices 1& 2 for SSSS and CSCS plate respectively and values of'n' (= n_x)obtained are presented and analyzed in Tables 1&2 for SSSS and CSCS plates respectively.

3. Design of Computer Program

Based on the equations derived, programs were designed to carry out the critical buckling analysis of SSSS and CSCS rectangular thin isotropic plates. The algorithm is as follows:

ALGORITHM

- ✓ Start
- ✓ Input the dimensions of plate, a, and b; Poisson ratio, v; plate thickness, h; and Young's modulus, E. Press 'Enter' for each variable inputted.
- ✓ Process aspect ratio , s = b/a
- ✓ Process flexural rigidity, $D = Eh^3/12(1-v^2)$
- ✓ Process the buckling load factor or coefficient , $n_x = \frac{\int\int [(k''^{r})^2 + \frac{2}{S^2}(k''^{rq})^2 + \frac{1}{S^4}(k''^{r})^2]\, \partial r\, \partial q}{\int\int (k'^{r})^2\, \partial r\, \partial q}$
- ✓ Process n_x in terms of denominator b^2 , then $n_{1x} = n_x/s^2$
- ✓ Process$n_{2x} = n_{1x}/\pi^2$
- ✓ Process the Critical Buckling Load, $N_x = n_x * D/a^2$
- ✓ End

NOTE: MATLAB uses lower case letters most, thus R= r, Q = q, in parameter, k.

The results from the execution of the programs are presented and compared with relevant studies in Table 1 and Table 2 for SSSS and CSCS plate respectively.

4. Results and Discussions

In order to validate these results, a comparative analysis was carried out in Table 1. It indicates that the percentage differences obtained when the results (n-values) were compared with those of Ibearugbulem et al. (2014)are 0.00 except for aspect ratio of 1.2 where it is -0.024, which is very insignificant. This implies that the results from this present study are very close to those of energy methods. Also, comparing the results obtained for aspect ratio 1 with those of Iyeenyer (1988), shows a percentage difference of 0.025%, indicating that the solutions from the present program are closer to those of the exact solution. Hence, the present program is satisfactory.

A similar comparison in Table 2 of the results (n-values) obtained for a CSCS plate with the results obtained by Ibearugbulem et al.(2014) shows that the percentage differences are less than 0.2% with an average value of -0.168% which means that the results are very close. As a matter of fact, this results are very accurate and the program faster to execute.

Aspect Ratio S = b/a	Present Study $N_x=n_1\pi^2\frac{D}{b^2}$ n_1	Ibearugbulem et al.(2014) $N_x=n_1\pi^2\frac{D}{b^2}$ n_2	% difference $100(n_1 - n_2)/n_2$	Iyeenyer(1988) $N_x=n_1\pi^2\frac{D}{b^2}$ n_3	% difference $100(n_1 - n_3)/n_3$
1.0	4.003	4.003	0.00	4.002	0.025
1.2	4.137	4.138	-0.024		
1.5	4.698	4.698	0.00		
1.6	4.955	4.955	0.00		
2.0	6.256	6.256	0.00		
Aver. %diff.			-0.005		

Table 1: Comparison of the new values of critical buckling load factor 'n' for SSSS Plate with those of Ibearugbulemet al. (2014) and Iyeenger (1988)

Aspect Ratio S = b/a S	Present Study $N_x=n_1\pi^2\frac{D}{b^2}$ n_1	Ibearugbulem et al(2014) $N_x=n_1\pi^2\frac{D}{b^2}$ n_2	% difference $100(n_1 - n_2)/n_2$
1.0	8.606	8.619	-0.15
1.2	7.466	7.478	-0.16
1.5	6.984	6.996	-0.17
1.6	7.016	7.028	-0.17
2.0	7.730	7.745	-0.19
Aver. %diff.			-0.168

Table 2: Comparison of the new values of critical buckling load factor 'n' for CSCS Plate with those of Ibearugbulem et al. (2014)

5. Conclusion

For both SSSS and CSCS plates results, the values discussed above are very close to the existing researches and found satisfactory for analysis of SSSS and CSCS plates. It is also observed that this approach is less time consuming and straight forward. Hence, this present computer program is better and easier approach for stability analysis of SSSS and CSCS thin rectangular plates.

6. References

i. Ezeh, J.C., Ibearugbulem, O.M., Opera, H.E., & Oguagaghamba, O.A. (2014a). Galerkin's Indirect Variational method in Elastic Stability Analysis of all Edges clamped Thin Rectangular Flat Plates. International Journal of Research in Engineering and Technology, 3(4), 674-679.

ii. Ezeh, J.C., Ibearugbulem, O.M., & Echehum, U. T. (2014). Stability Analysis of Orthotropic Reinforce Concrete Shear Wall. International Journal ofResearch in Engineering and Technology, 3(1), 228-231.

iii. Eziefula, U. G., Ibearugbulem, O.M., & Onwuka, D.O. (2014). Plastic Buckling Analysis of an Isotropic C-SS-SS-SS Plate under In-Plane Loading using Taylor's Series Displacement function. International Journal of Engineering and Technology, 4(1), 17-22.

iv. Ibearugbulem, O.M., Osadebe, N.N., Ezeh, J.C., & Onwuka, D.O. (2012). Buckling Analysis of Axially Compressed SSSS Thin Rectangular Plate Using Taylor-Maclaurin Shape Function. International Journal of Civil and Structural Engineering. 2(2).

v. Ibearugbulem, O.M., & Ezeh, J.C. (2012). Instability of axially Compressed CCCC Thin Rectangular Plate Using Taylor's Series Shape Functions on Ritz Method. Academic Research International, 4(1),346-351.

vi. Ibearugbulem, O.M., Ezeh, J.C. & Ettu, L. O.(2014). Energy Methods in Theory of Rectangular Plates: Use of Polynomial Shape Functions. Owerri: Liu House of Excellence Ventures.

vii. Iyenger, N. G.(1988), Structural Stability of Columns and Plate. Chichester: Ellis Horwood.

viii. Kang. J.H. & Leissa, A.W. (2001). Vibration and Buckling of SS-F-SS-F rectangular plates loaded by in-plane moments. International journal of Structural Stability and Dynamics, 1, 527-543.

ix. Leissa, A.W. and Kang, J.H. (2002). Exact solutions for vibration and buckling of SS-C-SS-C rectangular plate loaded by linearly varying in-plane stresses. International journal of mechanical science, 44, 1925-1945.

x. Onwuka, D.O., Ibearugbulem, O.M., Eziefula, U.G. (2013). Plastic Buckling of SSSS Thin Rectangular Plates Subjected to Uniaxial Compression Using Taylor-Maclaurin Shape Function. International Journal of Civil and Structural Engineering Research.2(4), 168-174.

xi. Szilard, R. (2004). Theories and Application of Plate Analysis. NewJersey: John Wiley & sons.

xii. Timoshenko, S.P & Woinowsky-Krieger (1959). Theory of plates and shells. 2nd ed., New York: McGraw Hills.

xiii. Ugural, A.C. & Fenster, S.K. (2003). Advanced Strength & Applied Elasticity. 4th ed., New Jersey: Prentice Hall.

xiv. Ventsel, E. & Krauthamer T. (2001). Thin Plates and Shells: Theory, Analysis and Applications. New York: Marcel Dekker.

Appendix 1

```
clc
%PROGRAM FOR BUCKLING ANALYSIS OF SSSS PLATE
a = input('Enter plate dimensionalong x-axis -length-a(m):');
b = input('Enter plate dimensionalong y-axis -width- b(m):');
v = input('Enter value of poisson ratio v:');
h = input('Enter the thickness h(m):');
E = input('Enter the value of young modulus E:');
p = input('Enter the value of specific density p:');
echo on
s = b/a
echo off
%The flexural Rigidity of plate D is
D = E*h^3/(12*(1-v^2));
syms r q
U = r-2*r^3+r^4;
V = q-2*q^3+q^4;
diff(U,2);
(diff(U,2))^2;
y1 = int((diff(U,2))^2,r,0,1);
z1 = int(V^2,q,0,1);
Y1 = y1*z1;
diff(V,2);
(diff(V,2))^2;
y2 = int(U^2,r,0,1);
z2 = int((diff(V,2))^2,q,0,1);
Y2 = y2*z2;
diff(U,1);
diff(V,1);
y3 = int((diff(U,1))^2,r,0,1);
z3 = int((diff(V,1))^2,q,0,1);
Y3 = y3*z3;
y4 = int(U,r,0,1);
z4 = int(V,q,0,1);
Y4 = y4*z4;
z5 = int(V^2,q,0,1);
Y5 = y3*z5;
%Bulkling Load Factor or Coefficient, nx.
nx = vpa((Y1+(2*Y3/s^2)+(Y2/s^4))/Y5,5)
% For Nx = nx*D/b^2)
n1x = nx*s^2
n2x = n1x/pi^2
% Critical Buckling Load, Nx.
Nx = vpa((nx*D/a^2),7)
```

Appendix 2

```
clc
%PROGRAM FOR BUCKLING ANALYSIS OF CSCS PLATE
a = input('Enter plate dimensionalong x-axis -length- a(m):');
b = input('Enter plate dimensionalong y-axis -width- b(m):');
v = input('Enter value of poission ratio v:');
h = input('Enter the thickness h(m):');
E = input('Enter the value of young modulus E:');
p = input('Enter the value of specific density p:');
echo on
s = b/a
echo off
%The flexural Rigidity of plate D is
D = E*h^3/(12*(1-v^2));
syms r q
U = r-2*r^3+r^4;
V = q^2-2*q^3+q^4;
diff(U,2);
(diff(U,2))^2;
y1 = int((diff(U,2))^2,r,0,1);
z1 = int(V^2,q,0,1);
Y1 = y1*z1;
diff(V,2);
(diff(V,2))^2;
y2 = int(U^2,r,0,1);
z2 = int((diff(V,2))^2,q,0,1);
Y2 = y2*z2;
diff(U,1);
diff(V,1);
y3 = int((diff(U,1))^2,r,0,1);
z3 = int((diff(V,1))^2,q,0,1);
Y3 = y3*z3;
y4 = int(U,r,0,1);
z4 = int(V,q,0,1);
Y4 = y4*z4;
z5 = int(V^2,q,0,1);
Y5 = y3*z5;
%Bulkling Load Factor or Coefficient, nx.
nx = vpa((Y1+(2*Y3/s^2)+(Y2/s^4))/Y5,5)
% For Nx = nx*D/b^2)
n1x = nx*s^2
n2x = n1x/pi^2
% Critical Buckling Load, Nx.
Nx = vpa((nx*D/a^2),7)
```