THE INTERNATIONAL JOURNAL OF SCIENCE & TECHNOLEDGE

Efficiency Measurement and Estimation of Return to Scale in Hospitals – DEA Approach

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Abstract:
This paper discusses the Return to Scale (RTS) in Data Envelopment Analysis (DEA). DEA is a non-parametric method used to evaluate the relative performance of a set of Decision Making Units (DMU's). In this paper we attempt to evaluate the relative performance of District Hospitals in the State of Tamil Nadu. Here each district is considered as DMU. Input and Output variables are chosen based on the judicial selection and prior knowledge about the operation of the Hospitals. The basic models namely Charnes Cooper and Rhodes (CCR) and Banker Charnes and Cooper (BCC) with output orientation have applied. Besides our discussion here we confined to qualitative characterizations, such as, whether RTS is identified as increasing, decreasing or constant. Empirical investigation was carried out and the results for the above model were listed.

Keywords: *Return to Scale, Data Envelopment Analysis, Decision Making Unit.*

1. Introduction

The government provides public services through various departments like transportation, education, health care and other community services. Policy makers are interested in knowing the efficiency (that is the ratio of the outputs produced to the inputs used) of their relevant service.
Primary health care is primary care applied on a population level. As a population strategy, it requires the commitment of governments to develop a population-oriented set of primary care services in the context of other levels and types of services. Primary care is the provision of first contact, person-focused, ongoing care over time that meets the health-related needs of people, referring only those too uncommon to maintain competence, and coordinates care when people receive services at other levels of care. A primary care oriented system is important for improving health (improving effectiveness)
It is very much essential to study the efficient functioning of public Hospitals. So this study have been taken up to identify the factors which causes inefficiency and find the suitable ways to improve their efficiency.
To serve the above purpose we used data envelopment analysis (DEA) to measure technical efficiency and use the benchmarking process and Return To Scale is identified whether increasing, decreasing or constant for a particular category of health care facilities of the state of Tamil Nadu - the district headquarters hospitals.

1.1. Review of Literature

Linear programming is the underlying methodology that makes DEA particularly powerful compared with alternative productivity management tools. DEA has been widely studied, used and analysed by academics that understand linear programming.
The theoretical foundations of efficiency measurement are based on the seminal work of Farrell (1957) and include the measurement of technical and allocative efficiency using radial measures of distance to the production or cost frontier. Technical efficiency refers to the use of the least resources to produce a given level of output. Alternatively, technical efficiency may be defined in terms of maximizing output for a given level of input. Charnes, Cooper and Rhodes (1978) developed DEA to evaluate non-profit and public sector organizations. Modifications on DEA to handle Variable Return to Scale (VRS) categories were first described by Banker et all (1984). This model deals the Constant Return to Scale (CRS) situation.
The empirical measurement of economic efficiency is based on the underlying idea of defining an efficient frontier against which to measure the performance of an economic organization. Jacobs (2001) classifies these methods as statistical or non-statistical, where statistical methods are based on assumptions about the stochastic nature of the data. Non-statistical methods tend to be non-parametric

and deterministic (no statistical noise). Alternatively, Worthington (2004) distinguish between parametric and non-parametric methods that have been used to estimate the efficient frontier. While in the parametric methods a functional form of the efficient frontier is predefined or imposed a priori, the non-parametric methods assume no functional form. Being nonparametric, DEA has the advantage of requiring neither the assumption of a particular functional form for technology nor assumptions regarding how inefficiency error is distributed. The drawback is that DEA assumes that any deviation of a firm from the efficient frontier is attributed to inefficiency. Therefore, DEA makes no allowance for external shocks, statistical noise, measurement error, or omitted variables in the model Greene (2008).

The majority of health care researchers have analyzed the effect of regulatory changes on the efficiency of health care facilities using a Data Envelopment Analysis . Hollingsworth and Peacock (2008), Valdmanis et al (2008) states that DEA is by far the most common method for analyzing efficiency in health care.

Banker (1984) shows that the CCR model can be employed to test for DMU's RTS using the concept of most productive scale size (MPSS), i.e., the sum of the CCR optimal lambda values can determine the RTS classification. Banker et al (1984) report that a new free BCC dual variable estimates RTS by allowing variable return to scale (VRS) for the CCR model, i.e., the sign of dual variable determines the RTS. Finally, Fare et al (1985) provide the scale efficiency index method for the determination of RTS using DEA.

1.2. Objective of the Study

In this study we have the following Objectives

1. To study the Relative Efficiency of 31 District hospitals.
2. Identification of Efficient and Inefficient District Hospitals based on the Efficiency scores.
3. To construct the Peer group for the inefficient hospitals.
4. To carry out the projection analysis.
5. Ranking of DMUs.
6. Identification of DMU's RTS nature.

2. DEA Methodology

DEA is a multi-factor productivity analysis model for measuring the relative efficiencies of a homogeneous set of decision making units. The efficiency score in the presence of multiple input and output factors is defined as:

Efficiency = Weighted sum of outputs / weighted sum of inputs

Assuming that there are n DMUs, each with m inputs and s outputs, the relative efficiency score of a test DMU p is obtained by solving the following model proposed by Charnes et al. (1978):

Mathematical Formulation

$$\text{Max} \quad \frac{\sum_{k=1}^{s} v_k y_{kp}}{\sum_{j=1}^{m} u_j x_{jp}}$$

This maximize the efficiency ratio *for* DMU p. This is subject to the constraint that when the same set of u and v coefficients is applied to all other service units being compared, no DMU will be more than 100% efficient as follows

$$st \qquad 0 \leq \frac{\sum_{k=1}^{s} v_k y_{ki}}{\sum_{j=1}^{m} u_j x_{ji}} \leq 1 \qquad \forall\, i$$

$$v_k, u_j \geq 0 \qquad \forall\, k, j$$

Where

$k = 1$ to s,

$j = 1$ to m,

$i = 1$ to n,

y_{ki} = amount of output k produced by DMU i,

x_{ji} = amount of input j utilized by DMU i,

v_k = weight given to output k,

u_j = weight given to input j.

The above Mathematical programs are fractional. The above fractional program can be converted to a linear program by normalizing the denominator as shown below (Charnes et al. (1978)).

$$\text{Max} \quad \sum_{k=1}^{s} v_k y_{kp}$$

s.t $\sum_{j=1}^{m} u_j x_{jp} = 1$

$$\sum_{k=1}^{s} v_k y_{ki} - \sum_{j=1}^{m} u_j x_{ji} \le 0 \quad \forall \text{ i}$$

$$v_k,\, u_j \ge 0 \quad \forall\ k, j$$

When the above problem is run *n* times it gives efficiency scores, weights of inputs against outputs in identifying the relative efficiency scores of all the DMUs. Each DMU selects input and output weights that maximize its efficiency score. In general, a DMU is considered to be efficient if it obtains a score of 1 and a score of less than 1 implies that it is inefficient.

2.1. RTS Classification

In Economics the laws of returns to scale are a set of three interrelated and sequential laws: Law of Increasing Returns to Scale, Law of Constant Returns to Scale, and Law of Diminishing returns to Scale. If output increases by that same proportional change as all inputs change then there are **constant returns to scale** (CRS). If output increases by less than that proportional change in inputs, there are **decreasing returns to scale** (DRS). If output increases by more than that proportional change in inputs, there are **increasing returns to scale** (IRS). A firm's production function could exhibit different types of returns to scale in different ranges of output. Typically, there could be increasing returns at relatively low output levels, decreasing returns at relatively high output levels, and constant returns at one output level between those ranges.

In the DEA literature there are several approaches for estimating of returns to scale (RTS). Seiford et al (1999) demonstrated that there are at least three equivalent RTS methods. The first CCR RTS method is introduced by Banker(1984). The second BCC RTS method is developed by banker et al (1984) as an alternative approach using the free variable in the BCC dual model. The third RTS method based on the scale efficiency index is suggested by Fare et al (1994).

The CCR model assume the constant the constant returns to scale production possibility set, i.e., it is postulated that the radial expansion and reduction of all observed DMU's and their nonnegative combinations are possible and hence CCR score is called overall technical efficiency. If we add $\sum \lambda_i = 1$ we obtain BCC models assume that convex combinations of observed DMU's form the production possibility set and the BCC score is called local pure technical efficiency. It is important to investigate the source of inefficiency of a DMU. Are they caused by the inefficient operations of the DMU itself or by the disadvantageous conditions under which the DMU is operating? For this purpose the Scale efficiency Score (SS) is defined by the ratio, SS = CCR efficiency/BCC efficiency. This approach depicts the source of inefficiency, i.e., whether it is caused by inefficient operations (the BCC efficiency) or by disadvantageous conditions displayed by the scale efficiency score (SS) or by both.

In this method three different models are solved. First model is the CCR DEA model, which does not have any convexity constraint involving $\sum \lambda_i$ let the optimal objective function value for reference DMU be denoted as *p*. The second model is the BCC DEA model. This model has the additional constraint $\sum \lambda_i = 1$, the optimal objective function value be denoted as *q*. If the ratio *p*/*q* is the scale efficiency of the reference DMU. If the scale efficiency is 1 i.e., $p = q$, then the reference DMU exhibits CRS. If $p \neq q$, a third model, the NIRS DEA models needs to be solved. The output oriented envelopment NIRS DEA model has the additional constraint, $\sum \lambda_i \ge 1$, as compared to the basic CCR DEA model, let the optimal objective function value be denoted as *r*. If $q = r$, the reference DMU exhibits DRS and if $q < r$, the reference DMU exhibits IRS.

2.2. Model Description

In our study we use output oriented model. i.e., the variables considered as output is under our control . So output oriented model is suggested.

Output oriented model

CRS	VRS
Max φ	Max φ
s.t $Y\lambda \ge \varphi Y_0$	s.t $Y\lambda \ge \varphi Y_0$
$X\lambda \le X_0$	$X\lambda \le X_0$
$\sum \lambda_i = 1$	
$\lambda \ge 0$, $\varphi \ge 1$	$\lambda \ge 0$, $\varphi \ge 1$

3. Data Structure

The data Analyzed here is obtained from the Directorate of Medical Sciences, DMS complex, Chennai-6, for the year 2013-2014. Here 31District General Hospitals of Tamil Nadu state numbered from 1 to 31 is considered as DMU's. For each DMU four inputs are considered, they are,

1. Number of Hospitals
2. Number of Beds available
3. Number of Staff Nurses.
4. Number of Surgeons

For each DMU the following four outputs are considered

1. Number of Outpatients treated.

2. Number of Inpatients treated.
3. Total Major Surgeries conducted.
4. Total Deliveries performed.

3.1. Descriptive Statistics

	Mean	Std. deviation	Minimum	Maximum	N
OP1	2217337	914354.3	749148	4984101	31
OP2	61802.13	28335.97	23162	129192	31
OP3	7631.7097	3757.374	1393	17755	31
OP4	6241.9677	3260.317	1254	16517	31
IP1	9.8065	3.55358	3	18	31
IP2	780.6774	316.81429	207	1375	31
IP3	126.1935	45.67306	24	198	31
IP4	38.9355	17.03709	14	80	31

Table 1: Descriptive Statistics for District Hospitals
Source: DEA solver

Descriptive statistics for the district hospitals are presented in the above table. In 2013-14 the hospitals, on an average, employed 39 doctors and 126 staff nurses, have a mean capacity of 780 beds. The variability in utilisation of resources, and more evidently in their output. The average number of outpatients treated was 2217 thousands, ranging from 749 thousands to 4984 thousands providing a first-hand indication of increased overall inefficiency in the operation of these facilities.

S.NO	DMU	CRS TE (p)	VRS TE (q)	SCALE (r)	EFF.STA -TUS
1.	ARIYALUR	1.00	1.00	1.00	
2.	COIMBATORE	1.06	1.00	1.00	Drs (q=r)
3.	CUDDALORE	1.00	1.00	1.00	
4.	DHARMAPURI	1.00	1.00	1.00	
5.	DINDIGUL	1.28	1.22	1.28	Irs (q<r)
6.	ERODE	1.54	1.49	1.54	Irs (q<r)
7.	KANCHEEPURAM	1.19	1.18	1.19	Irs (q<r)
8.	KANYAKUMARI	1.00	1.00	1.00	
9.	KARUR	1.25	1.22	1.22	Drs (q=r)
10.	KRISHNAGIRI	1.00	1.00	1.00	
11.	MADURAI	1.00	1.00	1.00	
12.	NAGAPATTINAM	1.05	1.01	1.05	Irs (q<r)
13.	NAMAKKAL	1.45	1.43	1.43	Drs (q=r)
14.	PERAMBALUR	1.00	1.00	1.00	
15.	PUDUKOTTAI	1.30	1.29	1.30	Irs (q<r)
16.	RAMANATHAPURAM	1.60	1.56	1.60	Irs (q<r)
17.	SALEM	1.33	1.31	1.33	Irs (q<r)
18.	SIVAGANGAI	1.85	1.78	1.78	Drs (q=r)
19.	THANJAVUR	1.36	1.30	1.36	Irs (q<r)
20.	THE NILGIRIS	1.89	1.47	1.47	Drs (q=r)
21.	THENI	1.13	1.05	1.05	Drs (q=r)
22.	THOOTHUKUDI	1.29	1.28	1.28	Drs (q=r)
23.	TIRUCHIRAPALLI	1.40	1.39	1.39	Drs (q=r)
24.	TIRUNELVELI	1.00	1.00	1.00	
25.	TIRUPPUR	1.25	1.23	1.23	Drs (q=r)
26.	TIRUVALLUR	1.00	1.00	1.00	
27.	TIRUVANNAMALAI	1.00	1.00	1.00	
28.	TIRUVARUR	1.30	1.25	1.25	Drs (q=r)
29.	VELLORE	1.00	1.00	1.00	
30.	VILLUPURAM	1.00	1.00	1.00	
31.	VIRUDHUNAGAR	1.35	1.35	1.35	Drs (q=r)

Table 2: Efficiency score of CRS efficiency and Pure Technical Efficiency
Source: DEA solver

Hospitals with score of 1.0 are considered efficient and, hence, lie on the efficient frontier. In this study we have 12 hospitals that are considered to be efficient in CRS model (p), which are also called the scale efficient and 13 are efficient in BCC model (q) which uses the convexity constraint (Table 2). Of all this 11 DMU's exhibits DRS (q=r) and 8 of them exhibits IRS (q<r). DEA helps us to identify a smaller group of best performer specific to the characteristics of an individual hospital (based on the weights given to the inputs and outputs)

4. Projection Analysis

4.1 Peers of Inefficient Firms, Peer Count and It's Rank of Efficient Firms – Bcc Case

In order to identify its group of benchmarking targets or role models, to the inefficient DMU's the reference set and the peer count summary are given below. Based on the peer count Summary, ranks are assigned to each efficient DMU.

S.NO	INEFFICIENT DMU'S		EFFICIENT DMU'S		
	DMU NO.	PEERS	DMU NO.	PEER COUNT	RANK
1.	5	$\lambda_3, \lambda_{10}, \lambda_{26}, \lambda_{29}$	1	6	5
2.	6	λ_3, λ_{10}	2	2	10
3.	7	$\lambda_3, \lambda_4, \lambda_{26}, \lambda_{29}, \lambda_{30},$	3	15	1
4.	9	$\lambda_1, \lambda_3, \lambda_{10}, \lambda_{14}, \lambda_{29}$	4	10	3
5.	12	$\lambda_3, \lambda_{10}, \lambda_{29}$	8	2	10
6.	13	$\lambda_1, \lambda_4, \lambda_{29}$	10	9	4
7.	15	$\lambda_3, \lambda_{10}, \lambda_{26}$	11	1	13
8.	16	$\lambda_3, \lambda_{10}, \lambda_{26}, \lambda_{29}$	14	4	7
9.	17	λ_4, λ_{29}	24	2	10
10.	18	$\lambda_2, \lambda_4, \lambda_8, \lambda_{14}, \lambda_{29}$	26	6	5
11.	19	$\lambda_3, \lambda_4, \lambda_{26}, \lambda_{29}$	27	3	8
12.	20	$\lambda_3, \lambda_4, \lambda_{27}$	29	15	1
13.	21	$\lambda_1, \lambda_3, \lambda_4, \lambda_{29}$	30	3	8
14.	22	$\lambda_3, \lambda_{10}, \lambda_{14}, \lambda_{29}$			
15.	23	$\lambda_1, \lambda_4, \lambda_{29}, \lambda_{30}$			
16.	25	$\lambda_1, \lambda_3, \lambda_{10}, \lambda_{27}, \lambda_{29}$			
17.	28	$\lambda_3, \lambda_4, \lambda_{24}, \lambda_{29}$			
18.	31	λ_3			

Table 3: Inefficient firm's peers and Efficient firm's peer count
Source: DEA solver

The above table describes the peer group for the inefficient DMU's. For e.g., for the inefficient DMU 5 peers are identified as 3, 10, 26 and 29th DMU's. The linear combination of the peers in the reference set will give the input and output target. Then for efficient DMU's peer count is calculated i.e., the reference set for number of inefficient DMU's and its rank is calculated.

5. Conclusion

Hospitals with score of 1.0 are considered efficient and, hence, lie on the efficient frontier. In this study we have 12 hospitals that are considered to be efficient in CRS case and 13 are efficient in VRS case. Out of 19 inefficient DMU's 11 DMU's exhibits DRS (q=r) and 8 of them exhibits IRS (q<r). DEA helps us to identify a smaller group of best performer specific to the characteristics of an individual hospital (based on the weights given to the inputs and outputs)

In this case, DMU 18 is the least efficient unit (efficiency score φ = 1.85, 1.78). In order to identify its group of benchmarking targets or role models, we use DEA to identify a facet or cone in order to create a smaller, more relevant comparison set.

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