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Damping Improvement of a Longitudinal F-16 Aircraft Model through LMI

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Abstract:
This work presents an application of Linear Matrix Inequalities (LMI) for the robust control of an F-16 aircraft through an algorithm that ensuring the damping factor to the closed loop system. The results of some tests show that the zero and gain settings are sufficient to ensure robust performance and stability with respect to various operating points. The technique used is the pole placement, which aims to put the system in closed loop poles in a specific region of the complex plane. Test results using a dynamic model of the F-16 aircraft are presented and discussed.

Keywords:*F-16 Aircraft, linear matrix inequalities, pole placement, robust control*

1. Introduction

The dynamic response characteristics of an aircraft are highly non-linear. Generally, flight control systems have been designed using mathematical models of an aircraft, linearized around several operation points, the controller parameters are programmed in accordance with the flight conditions.
To the F-16 aircraft control, several techniques have been applied. In (Fravolini, Yucelen, Muse and Valigi, 2015), a linear strategy control and adaptive control which parameters are calculated by a convex multi objective optimization, in order to ensure at the same time, the evolution of the error within a minimum invariant set while the linear gain minimized, is performed and applied to the longitudinal dynamic model of the F-16 aircraft. The longitudinal model of a hypersonic flight vehicle was also used for evaluation of the implementation of a robust adaptive controller (Xu, Huang, Wang and Sun, 2013), the methodology of this study addresses the issue of controller design and stability analysis in relation to parametric model uncertainties and saturations entrance to the oriented model control.
In (Holhjem, 2012), the adaptive control technique L1 is applied in closed loop longitudinal F-16 aircraft model linearized around an operating point. In order to guarantee stability and performance of the resulting gain-scheduled controllers, analytical frameworks of gain scheduling have been developed including the technique of linear-parameter-varying (LPV) control (He, Zhao and Dimirovski, 2011; He, Dimirovski and Zhao, 2010). An application of a conditional integrator based sliding mode control design for robust regulation of minimum-phase nonlinear systems to the control of the longitudinal flight dynamics of an F-16 aircraft is made by (Seshagiri and Promtun, 2008). In (Liao, Wang and Yang, 2002), a reliable robust tracking controller design method is developed based on the mixed linear quadratic $(LQ)/H_\infty$ tracking performance index and multi objective optimization in terms of linear matrix inequalities.
Among the techniques presented for control of an F-16 aircraft, the linear matrix inequality become a possible tool in finding solutions for various optimization problems, control systems and recently identification systems. One of the great advantages of this approach is to allow the simultaneous treatment of various performance and robustness requirements. This is because of the emergence of interior point algorithms for the solution of convex optimization problems, which made it possible to numerically solve the linear matrix inequalities faster and more efficiently.
This paper presents the application of linear matrix inequalities for robust control of an F-16 aircraft. Based on the algorithm presented and developed by (Campos, Cruz and Zanetta, 2012), there is the guarantee of the damping factor for the closed-loop system for various operating points by allocating system poles using a predefined controller.

2. Longitudinal Dynamic Model of an F-16 Aircraft

The flat-earth, body-axis 6-Degrees of Freedom (6-DOF) nonlinear control-oriented model for the F-16 fighter aircraft presented in (Stevens, Lewis and Johnson, 2015) and (Russell, 2003) has been employed in this paper. The nonlinear model is linearized around the operating points (altitude = 4,57 *km*; total velocity = 549 *km/h*), and decoupled, to obtain separate longitudinal and lateral-directional linear models. The properties of F-16 aircraft considered in this work are the same in (Stevens, Lewis and Johnson, 2015), with the mass and geometric properties as listed in Table I and only the longitudinal-directional, low fidelity (Russell, 2003) state-space model given by (1) is investigated further under the influence of thrust and elevator control inputs.

Parameter	**Symbol**	**Value**
Weight	*W (kg)*	9298.64
Moment of inertia	J_y (kg/m^2)	75673.62
Wing area	S (m^2)	27.87
Mean Aerodinamic chord	*cbar (m)*	3.45
Reference CG location	x_{cg}	0.35*cbar*

Table 1: Mass and Geometric Properties

$$\begin{bmatrix} \dot{h} \\ \dot{\theta} \\ \dot{V} \\ \dot{\alpha} \\ \dot{q} \\ \dot{\delta}_t \\ \dot{\delta}_e \end{bmatrix} = A.\begin{bmatrix} h \\ \theta \\ V \\ \alpha \\ q \\ \delta_t \\ \delta_e \end{bmatrix} + B.\begin{bmatrix} \delta_t \\ \delta_e \end{bmatrix}, \quad \begin{bmatrix} h \\ \theta \\ V \\ \alpha \end{bmatrix} = C.\begin{bmatrix} h \\ \theta \\ V \\ \alpha \end{bmatrix} + D.\begin{bmatrix} \delta_t \\ \delta_e \end{bmatrix} \quad (1)$$

Where h, θ, V, α, q, δ_t and δ_e are the aircraft's altitude (*km*), pitch angle (degrees), total velocity (*km/h*), angle of attack (degrees), pitch rate (*rad/s*), thrust (*kg*)and elevator deflection (degrees) respectively. The matrix A, B, C and D can be found using the Simulink program based on (Russell, 2003). The eigen values, the damping ζ, the natural frequency *w* (*rad/s*) and the overshoot (%) of the longitudinal model dynamic (1) are shown in Table 2.

Eigenvalues	**Damping ζ**	***w* (*rad/s*)**	**Overshoot (%)**
$1.03x10^{-13}$	-1.00	$1.03x10^{-13}$	0
-0.00523+0.0634.i	0.0822	0.0636	77.2
-0.00523-0.0634.i	0.0822	0.0636	77.2
-1.00	1.00	1	0
-1.06+1.69.i	0.53	1.99	14
-1.06-1.69.i	0.53	1.99	14
-20.2	1.00	20.2	0

Table 2: properties of longitudinal f-16 aircraft dynamic model in open loop

As we can see, the longitudinal model has a pole on the right side of the complex plane, moreover, has poles -0.00523 ± 0.0634.i, which shows that the system has insufficient damping. Our goal will be to improve the system damping, whether it be only in an operating point or not. The theory for this will be presented in the next section.

3. The Predefined Controller and System Closed Loop Structure

The structure of the predefined controller to be used to control the F-16 aircraft is given by the following transfer function (Campos, Cruz and Zanetta, 2012):

$$K_{y \to u} = \frac{a_{y \to u}.s^2 + b_{y \to u}.s + c_{y \to u}}{s^2 + (p_1 + p_2).s + p_1.p_2} \quad (2)$$

Where the notation y→u indicates the controller of the output *y*of the longitudinal F16 modelto input *u*. The poles p_1 and p_2 are pre-determined. In this scheme, we work with pre-defined poles and we have to obtain the gain and the zeros, given by the values $a_{y \to u}$, $b_{y \to u}$ and $c_{y \to u}$of the controller, constrained to feasible values. Our control method comprises applying an output feedback for the F-16 system. The closed-loop system is given in Figure1.

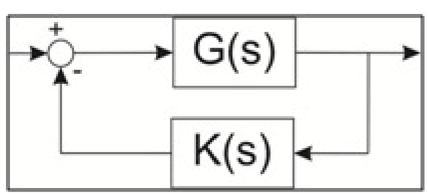


Figure 1: Structure of the closed loop system

Based on the application of the controller to the closed-loop system, evaluating the controller in the state space, we note that the matrix A_c and $C_c$controller are also predefined because they depend only on the values of the poles p_1 and p_2(Campos, Cruz and Zanetta, 2012). While B_c and $D_c$matrices are the variables that need to determine. The determination of these matrices will be considering the location in the complex plane that will be used for allocation of system poles. We will see in the next section.

4. Pole Placement through Linear Matrix Inequalities

Linear matrix inequalities are mathematical tools that have various applications in control theory, especially in the robust control area. For purposes of pole placement, it is important to define regions in a linear matrix inequality. Two regions of a linear matrix inequality interest in control applications for pole placement are the conical sector with vertex at the origin and interior angle $2.\theta$ and the half-plane Re(z) < α. The intersection of this regions are also a region of a linear matrix inequality. This region can see in Fig. 2. In this work, the goal is to allocate the system poles of F-16 aircraft in closed loop at this region, ensuring a minimum damping coefficient $\zeta = \cos\theta$ and a transient response with minimum decay rate equal to α for the closed-loop system. Thereby allocating the poles closed loop in this region guarantee adequate performance. To do this, we use the linear matrix inequality.

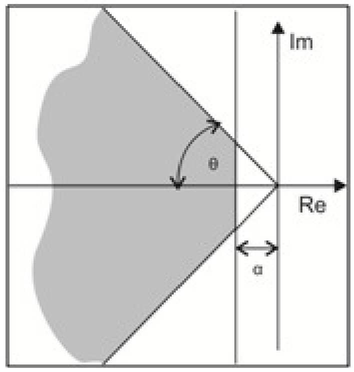


Figure 2: Complex region plan for pole placement closed loop.

The system of the aircraft is described by a set of nonlinear equations are linearized around some operating points. However, these operating points only represent the system behavior in a specific condition, and changes in operating points often occur. Thus, it is necessary to ensure that the F-16 will present good performance of the system in case of changes in operating points. To overcome this problem, we will make use of polytopic models.

To set a polytopic model, consider that only the matrix A of the system varies due to changes in operating points of the F-16 aircraft. Therefore, a polytope Ω is set (Boyd, Ghaoui, Feron and Balakrishnan, 1994):

$$\Omega = \{A / A \in R^{nxn}, A = \sum_{i=1}^{m} \lambda_i . A_i, \lambda_i \geq 0, \sum_{i=1}^{m} \lambda_i = 1\} \quad (3)$$

where n is the dimension of the matrices A_i and m the number of operating points. The matrices $A_i$are called polytope vertices. To ensure that the poles of any closed loop system associated with the matrix A ϵΩ are in the region of the complex plane defined in the previous section, it is necessary to resolve m linear matrix inequalities according (Campos, Cruz and Zanetta, 2012).

5.Results and Discussion

The method presented in previous section was applied to the dynamic longitudinal-directional model of F-16 aircraft to the system considering three operation points, the only parameter that changed was the total velocity V to obtain the new linearized system around these points operation, the altitude navigation was kept constant at 4.57 *km*. The velocities considered are 549*km/h* (nominal point), 658 *km/h* (operation point 2) and 768 *km/h* (operation point 3).

For each point of operation, new matrices were obtained for the system in the state space, through simulations using the program based on (Russell, 2003), however, as described in robust method, consider only the variations in matrix A of the system, the matrices B, C and D were considered to be the nominal system.

According to equation fixed controller (2), the poles of the controller were chosen p_1= -5000 and p_2 = -5000, with a value of $\alpha = 0$ and the following performance index (minimum damping coefficient of eigen values of the closed loop system): $\zeta = \cos\theta = 0.9$.

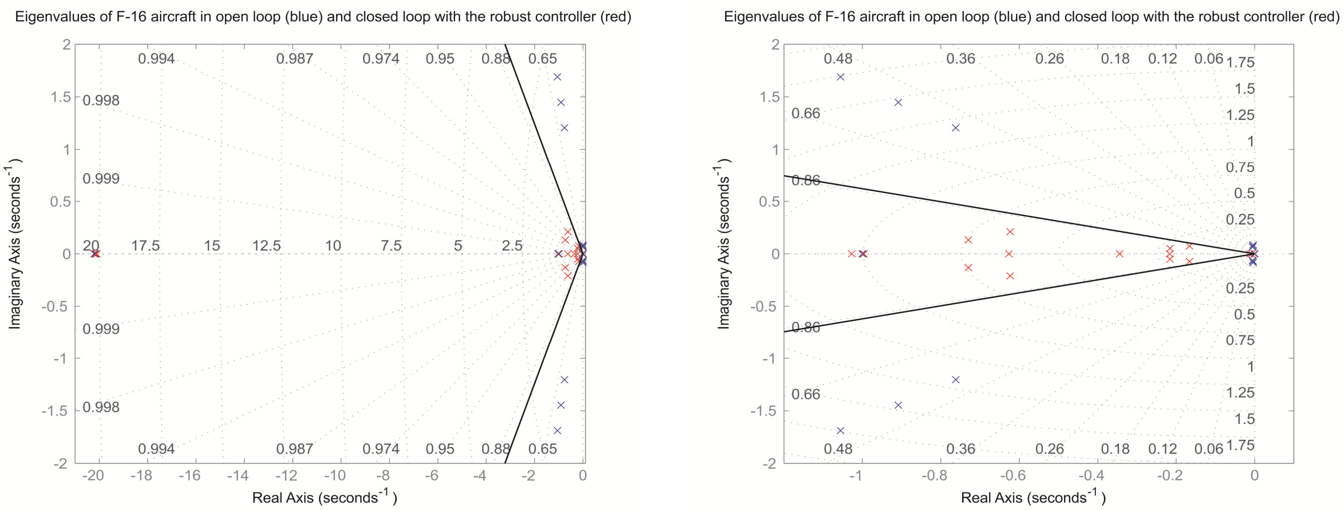
His two zeros were left free, as well as its static gain. The simulation was performed using the software Matlabversion R2013b with his toolbox for calculation of linear matrix inequalities. For this case the simulation lasted 3.48 *s* using an Intel Core i5 computer 2.20 GHz, 4GB of RAM, 64-bit. The parameters obtained for the robust controller are shown in Table III. The controller parameters are referenced to the (2).

y	u	$a_{y\rightarrow u}$	$b_{y\rightarrow u}$	$c_{y\rightarrow u}$
h	δ_t	-0.1277	-1277	3.191×10^6
	δ_e	0.0003772	4.191	8437
θ	δ_t	-1.596	-15960	-3.99×10^7
	δ_e	0.04666	484.2	1.06×10^6
V	δ_t	-2.402	-24020	-6.005×10^7
	δ_e	0.0002307	2.011	6039
α	δ_t	1.581	15810	3.953×10^7
	δ_e	-0.232	-2321	-5.857×10^6

Table 3: controller parameters found for the longitudinal f-16 control system

To evaluate system performance in closed loop, we were plotted all the eigenvalues of the matrices A in closed loop for the three operation points, together with the eigenvalues of the original systems, as can be seen in Fig. 3. In this figure, with two expands, we just show the regions of interest. The black line is an approach to the conic section defined by the angle θ.

On the basis of information obtained by the simulation of the three operating points, the worst obtained damping factor was ζ= 0.914, with an overshoot of 0.086 % and a time of accommodation signal of 3.04 s , as well as all the eigenvalues were contained within the linear matrix inequalities region of the complex plane specified by the intersection between the region of linear inequality matrix formed by the cone sector defined by θangle and the half-plane defined by α.For this longitudinal system of F-16 aircraft, other α values were tested, but for a small increase in its value, the linear matrix inequality became infeasible. Another fact to be noted is that in this case performance specification, the performance obtained by application of the robust controller with this system is that the results obtained were better as the velocity of the F-16 aircraft is increased.



a) Zoom 1 *a) Zoom 2*

Figure 3: Eigen values of F-16 aircraft in open loop (blue) and closed loop with the robust controller (red).

To check system behavior in closed loop with the robust controller, we apply the initial condition x_0= $[1\ 1\ 1\ 1\ 1\ 1\ 1]^T$to the operation nominal point, which showed the worst result among the three operating points, and compared with the open-loop system. Then, we obtain the graph shown in Figure4. As can be seen in this image, the system oscillation and settling time improved significantly compared to the system without the robust controller.

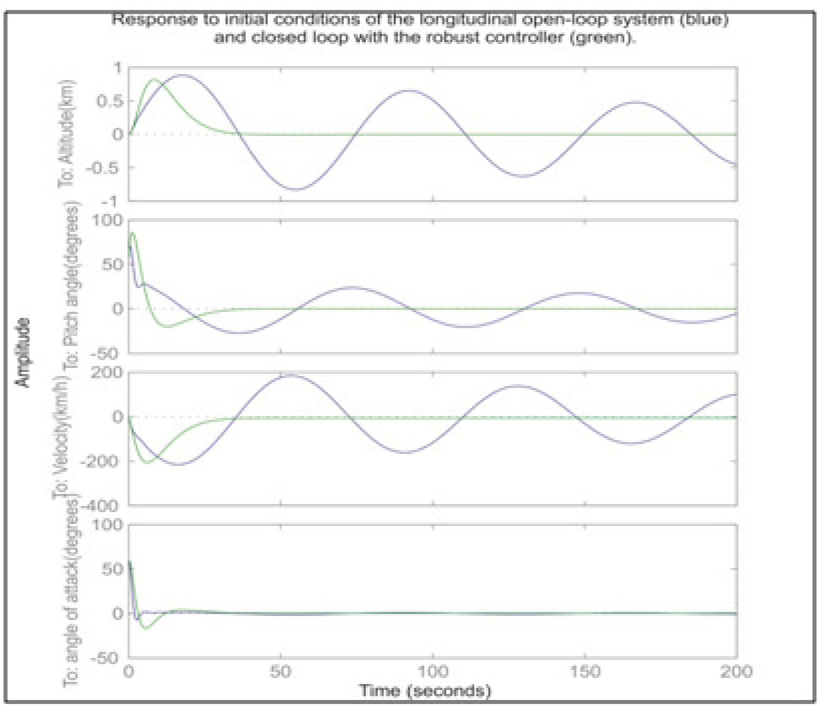


Figure 4: Response to initial conditions of the longitudinal open-loop system (blue) and closed loop with the robust controller (green).

6. Conclusion

In this paper we present a methodology for pole placement of a linearized system around various operating points in a particular region of the complex plane, defined by the intersection of two regions of a linear matrix inequalities. The controller used here has a fixed structure, in which initially define the poles and control through linear matrix inequalities, obtain their gain values and their zeros.

It was applied to pole placement through the linear matrix inequalities longitudinal system F-16 aircraft. For specified performance conditions, there was an improvement in all the properties considered in this work to the closed loop system. For this case, all the eigenvalues of the system were allocated within the complex plane specified region, the value chosen for the damping coefficient was higher than that obtained for the open-loop system, yet the damping obtained for all operating points were better than specified.

As can be seen in the responses to the initial conditions for the system, there is an evident improvement in the response rate and damping output. In addition, the applied controller is robust, which gives it advantages over any other controller, as it considers various system operating points to be controlled. One of the main advantages of the formulation presented in this work is that it generates controllers with a pre-specified structure, which can be applied to control the F-16 aircraft. This makes it easy to practical implementation to test the controllers obtained on stabilization and increased performance.

As a proposal for future work, we suggest testing with various performance specification values, varying for example the value of the angle θ of the linear matrix inequality region and checking the system behavior for all these values.

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