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Stability Evaluation of Lattice Boltzmann Method for High Viscosity Fluid Model Applied in Pore Scale Porous Media

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Abstract:
This study examines the applicability of the Lattice Boltzmann Method (LBM) in the term of numerical simulation stability. One of the issues in LBM application is the numerical simulation stability when modeling fluid flow with high viscosity fluid in pore-scale porous media domain. The problem that encountered in the LBM is the range of the time relaxation that represents the fluid viscosity that can be related to the macro scale viscosity in Navier-Stokes equation with the multi-scale method of the Chapman - Enskog. The intention to simulate the high viscosity fluid is to simulate the fluid flow of heavy oil model in porous media. However, during the first attempt, numerical instability occurred when using the lowest relaxation parameter that represents the highest fluid viscosity. Therefore, several evaluations were conducted by manually fine-tuning the relaxation parameter to verify the simulation stability in the pore scale of the deduced porous media domain. The results show, there is a certain boundary limit of the relaxation parameter could be applied to model high viscosity fluid to represent the heavy oil in acceptable stability during the simulation, especially in the complex geometry such as porous media. By doing this verification prior to conducting fluid flow simulation of high viscosity in porous media with LBM, will ensure the simulation will run smoothly and modeling the heavy oil fluid flow in pore-scale porous media could be conducted.

Keyword: *Complex geometric domain, high viscosity fluid, heavy oil, fluid flow simulation, lattice boltzmann method, pore scale porous media*

1. Introduction

The applicability of the Lattice Boltzmann Method (LBM) due to its nature has the limitation in the term of the stability during the fluid dynamic simulation. The instability during modeling such a fluid flow will prevent to conclude the fluid flow model being simulated. In this work, the specific application is intended to simulate the high viscosity fluid such as heavy oil in the pore scale porous media.

The fluid viscosity in the Lattice Boltzmann Method is controlled by relaxation parameter, which could be derived using Chapman-Enskog multi-scale. The mathematical representation of the fluid viscosity determines the low and high value of the relaxation parameter that represents the high and low viscosity respectively. However, the low number of relaxation parameter to represent the high viscosity fluid could not apply straightforward due to stability issues during the fluid flow simulation. Therefore, during the evaluation, the relaxation parameter is fine tuned to the value that the simulation still could run smoothly.

2. Theory

The Lattice Boltzmann Method as alternative numerical fluid flow simulation method is developed based on the Boltzmann equation with the kinetic theory as a foundation. Essentially, the objective is to describe the collective behaviour of particles and represented statistically. In the form of statistic that formulated in particles distribution function equation, solving the transport equation in Equation 1, with collision process on the right-hand side as collision operator.

$$\frac{\partial f}{\partial r}\mathbf{v}+\frac{\mathbf{F}}{\mathbf{m}}\frac{\partial f}{\partial \mathbf{v}}+\frac{\partial f}{\partial t}=\left(\frac{\partial f}{\partial t}\right)_{coll} \quad (1)$$

f is distribution function, the particle velocity **v**, the particle mass **m**, the external force **F** and the time **t**. The particles distribution function that propagate to neighbor site with assigned direction, interacting and colliding. These evolution processes occurred explicitly, and the propagation and collision of the fluid particles movement occurred locally.

Higuera and Jimenez in 1989 introduced the linearized of collision operator in Boltzmann equation by assuming the distribution function close to its equilibrium. Then, the next development for the collision operator is single relaxation time approximation by BGK (Bhatnagar, Gross, Krook), in this model, collisions are defined implicitly.

The collision terms in the Boltzmann equation in Equation (1), could be further simplified by single relaxation time approximation such that:

$$\frac{\partial f}{\partial r}\mathbf{v}+\frac{\mathbf{F}}{\mathbf{m}}\frac{\partial f}{\partial \mathbf{v}}+\frac{\partial f}{\partial t}=-\frac{1}{\tau'}\left(f-f^{eq}\right) \quad (2)$$

τ' is the characteristic relaxation time of collision processes and

$$\frac{\partial f}{\partial r}\mathbf{v}+\frac{\mathbf{F}}{\mathbf{m}}\frac{\partial f}{\partial \mathbf{v}}+\frac{\partial f}{\partial t}=-\omega\left(f-f^{eq}\right) \quad (3)$$

that $\omega=\frac{1}{\tau'}$ is the characteristic frequency or relaxation parameter that represent the fluid viscosity. If there is no external force, the equation will become Boltzmann equation with BGK (P.L. Bhatnagar, E.F. Gross and M. Krook) approximation:

$$\frac{\partial f}{\partial t}+\mathbf{v}\cdot\nabla f=-\omega\left(f-f^{eq}\right) \quad (4)$$

where f^{eq} is the particles Boltzmann distribution function at equilibrium state.

The link between the above distribution function equation with the macroscopic quantity such density and velocity can be obtained from the microscopic velocity moments of the density distribution f :

$$\rho=\int f d\mathbf{v}=\int f^{eq} d\mathbf{v} \quad (5)$$

$$\rho u=\int \mathbf{v} f d\mathbf{v}=\int \mathbf{v} f^{eq} d\mathbf{v} \quad (6)$$

Fluid density and velocity can be obtained from discrete distribution function,

$$\rho=\sum f_i \quad (7)$$

$$\rho u=\sum \mathbf{c}_i f_i \quad (8)$$

where discrete distribution function during time interval, could obtain

$$f_i\left(x+\mathbf{c}_i\delta t, t+\delta t\right)-f_i(x,t)=-\omega\left[f_i(x,t)-f_i^{eq}(x,t)\right] \quad (9)$$

That describe the evolution of particle distribution function, where f_i (**x**, t) represents the probability of finding a particle with velocity $\mathbf{c}_i$ at position **x** and time t.
With the multi-scale approach, such as Chapman-Enskog method, the Navier-Stoke equations could be recovered as described in the Figure 1.

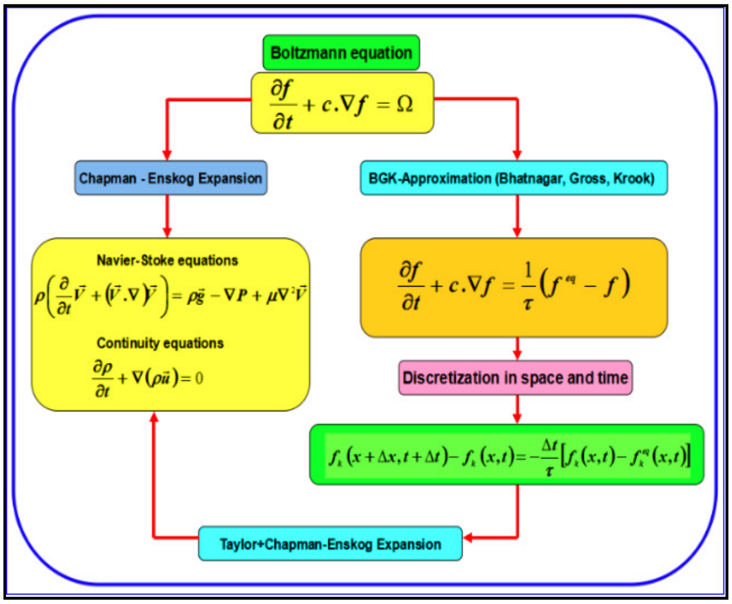

Figure 1: The Boltzmann transport equation linked to the Navier-Stokes equations that bridged directly through multi-scale technique such as Chapman-Enskog method as well as through the discretization of linearized Lattice Boltzmann BGK approximation

The relation of the macroscopic viscosity with the relaxation parameter is stated as for two-dimensional and 9 particles lattice model such as D2Q9:

$$\nu \equiv \frac{1}{3}\left(\frac{1}{\omega} - \frac{1}{2}\right) \quad (10)$$

where ν is kinematic viscosity. It can be seen in the Equation 10, the relaxation parameter ωhas certain range to ensure stability that ω: $0<\omega<2$,the term inside the parentheses need to be greater than zero to ensure positive fluid viscosity. The lower the ωnumber, the higher the fluid viscosity. The lower boundary of the relaxation parameter could approach zero value, however it can be seen that to simulate the high fluid viscosity with lower relaxation parameter number has a limit in the term of stability.

Previous known evaluation of relaxation parameter was conducted by Qian, Humieres and Lallemand (1992), that conducted in three-dimensional and 15 particles D3Q15 on a 128 x 1 x 1 lattice domain, described that the relaxation parameter (ω) is stable at the range of $0.2 \leq \omega < 2.0$ as shown in Figure 2. Therefore, the stability evaluation is likewise conducted for the D2Q9 lattice model which will be used along this work.

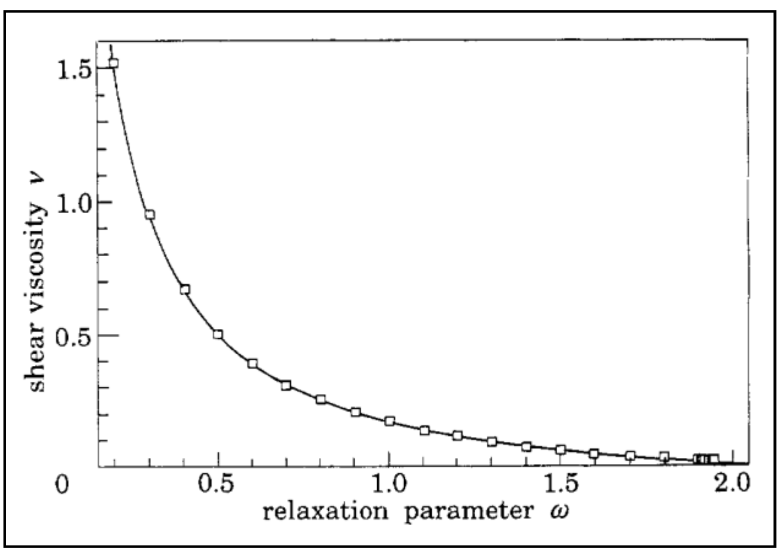

Figure 2: Qian, Humieres and Lallemand(1992), viscosity function of relaxation parameter for the 15-particle model D3Q15, solid line represents Equation 12, square points are numerical result

The above result is not applicable when modeling the high viscosity fluid applied in the pore scale porous media that represented with low relaxation parameter number. Therefore, in this work, the evaluation is conducted to determine the lowest stable relaxation parameter that can be applied in the pore scale porous media.

3. Method

The domain of simulation is prepared with the deduced two-dimensional pore structures from rock samples as described in the flowchart by Figure 4. It started with scanned thin slice of the porous media, that converted into digitally image as shown in Figure 5.

The digitally image is consisting of binary number 1 and 0, where 1 is represented the solid part and 0 is the deduced pore space. The fluid is modeled with simple algorithm to fill the pore space and bounce back scheme is applied during propagation and collision including when the fluid hit the solid part.

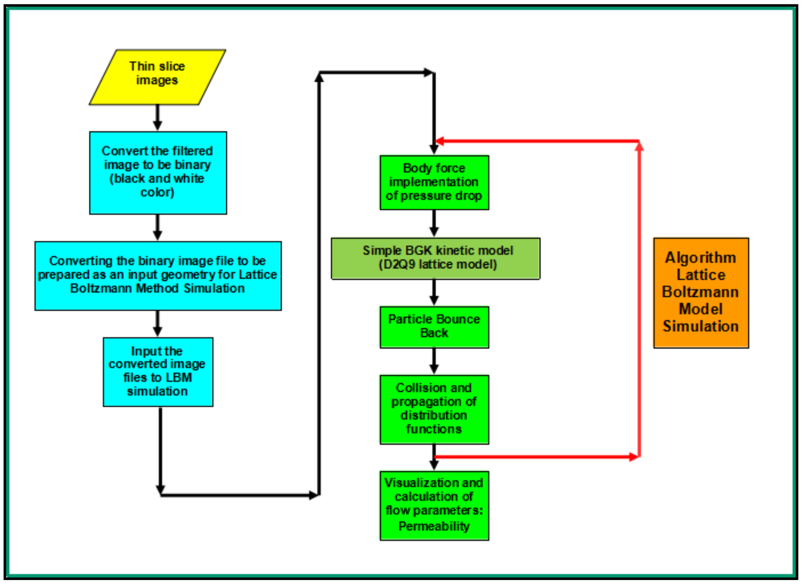

Figure 3: Two-dimensional deduced pore structure to use as complex geometry domain in single phase fluid flow numerical simulation by Lattice Boltzmann Method with D2Q9 lattice model

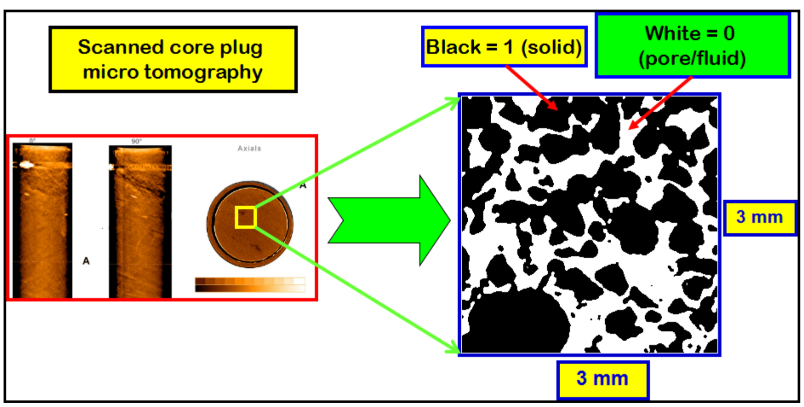

Figure 4: Converting scanned core plug into two-dimensional deduced pore structure as binary image complex geometry domain

Every time step, by applying the pressure different to be able the fluid to flow, then the macroscopic parameter such as density and velocity are calculated as mentioned with equations earlier (Equation 7 & 8).

The main concern in this work, when the fluid with high viscosity is modeled by low relaxation parameter number ω, the results showing inappropriate nonphysical phenomena of velocity field visually across the channel of the pore space in porous media domain specifically near the solid boundary that appearing abruptly form of velocity profile. Therefore, verification is required to evaluate the normal velocity profile across the channel with manually simulating the fluid flow with various relaxation parameter.

Initially, the pore scale porous media is saturated with certain fluid viscosity as shown in Figure 5.

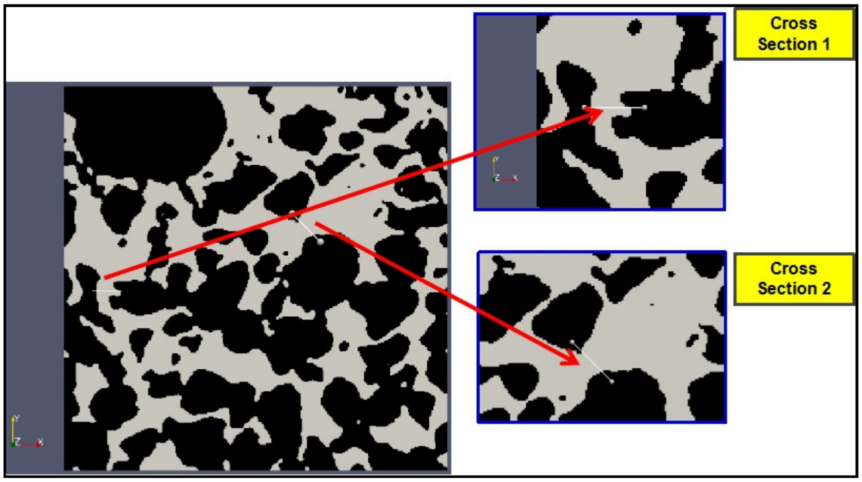

Figure 5: Digitally converted scanned core plug into two-dimensional deduced pore structure as binary image complex geometry domain with 2 cross sections in insert picture

4. Result and Discussion

The fluid flow simulation conducted with various numbers of relaxation parameter. Figure 6 throughout 16 showing the velocity field from lower relaxation parameter number respectively to the higher relaxation parameter number. The image results of velocity field showed on the lower relaxation parameter number showing nonphysical phenomena in the velocity distribution compared to the higher relaxation parameter number. This is the main concern because the high fluid viscosity model represented with low relaxation parameter number giving inappropriate result in the velocity distribution.

It can be seen in the images of velocity field, the velocity profile showing significantly difference between lower and higher relaxation parameter. In the lower relaxation parameter number, see Figure 6 and 7, the simulation resulted rather staggered velocity field and looks overlapping at every channel. While in Figure 8 and 16 for higher relaxation parameter, resulting smoother velocity distribution.

The nonphysical phenomena are emphasized in the velocity plot of channels. In the Figure 3, the velocity profile from the first cross section while Figure 11 is the velocity profile from the second cross section of the channels described in Figure 5, the plot of individually for fluid flow simulation with various relaxation parameter. The velocity profile showing abruptly indication (see green and brown color plot) at the area close to the solid boundary at the cross section mentioned, at either side the velocity value is jump significantly. Those two plots were the fluid flow simulation with $\omega = 0.15$ and 0.25 respectively. While for the higher relaxation parameter number, showing appropriate gradation of the velocity profile.

From the fluid flow simulation results using Lattice Boltzmann Method in pore scale porous media, there is a certain condition that during modeling the high fluid viscosity such as heavy oil, there is a limitation in accordance to the applying the relaxation parameter. It cannot use the lowest relaxation parameter number that approach zero. Fine tuning of the relaxation parameter is necessary to be conducted to avoid nonphysical phenomena occurred during the fluid flow simulation in pore scale porous media. Comparing with the previous evaluation conducted by Qian (1992), the lowest relaxation parameter number that could be applied in pore scale porous media using Lattice Boltzmann Method with BGK model is $\omega > 0.25$ instead of 0.2. The simulation results showed the lowest relaxation parameter that could be used start from 0.35 to represent the high viscosity fluid to model heavy oil.

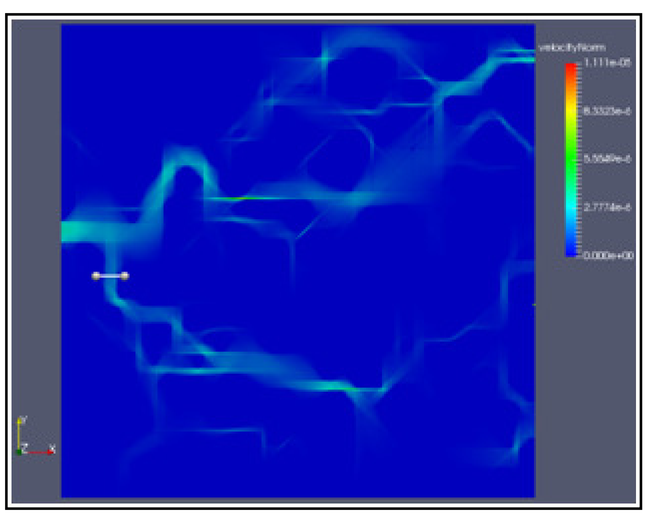

Figure 6: Velocity field in pore space with relaxation parameter $\omega = 0.15$. Fluid flow from left to right side. Bright color indicating higher velocity

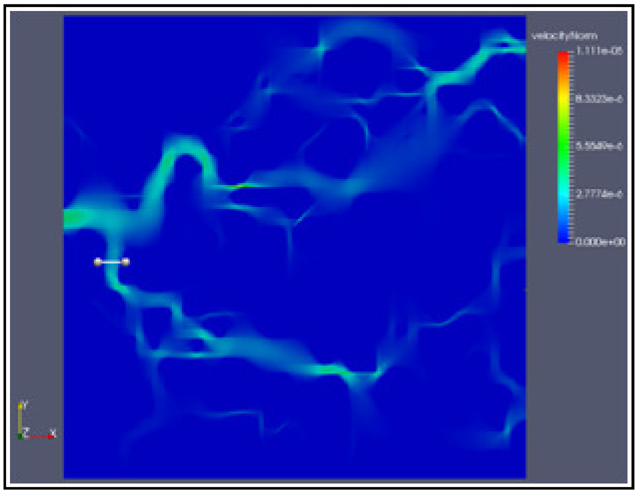

Figure 7: Velocity field in pore space with relaxation parameter $\omega = 0.25$*.*
Fluid flow from left to right side. Bright color indicating higher velocity

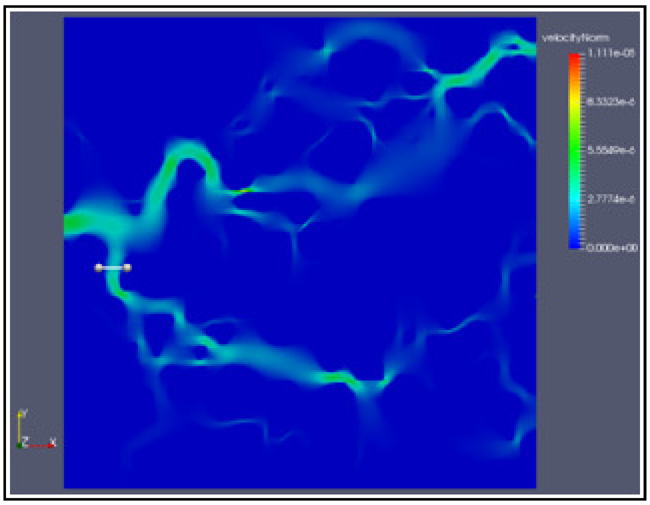

Figure 8: Velocity field in pore space with relaxation parameter $\omega = 0.35$*.*
Fluid flow from left to right side. Bright color indicating higher velocity

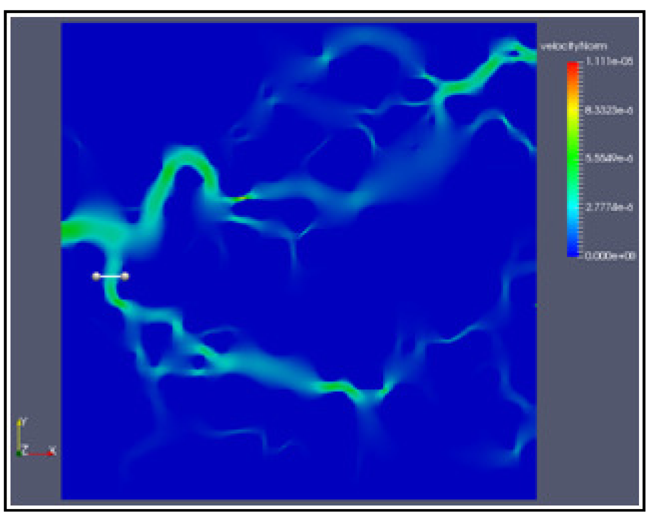

Figure 9: Velocity field in pore space with relaxation parameter $\omega = 0.40$*.*
Fluid flow from left to right side. Bright color indicating higher velocity

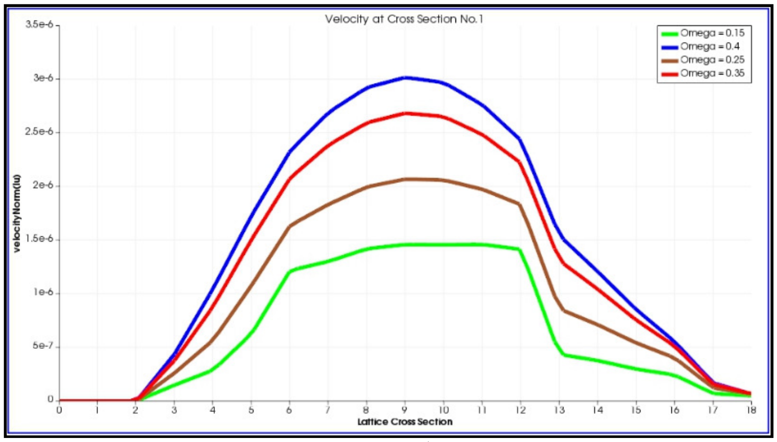

Figure 10:Velocity distribution across the 1st channel cross section from Figure 5, for relaxation parameter number ω = 0.15, 0.25, 0.35 0.40

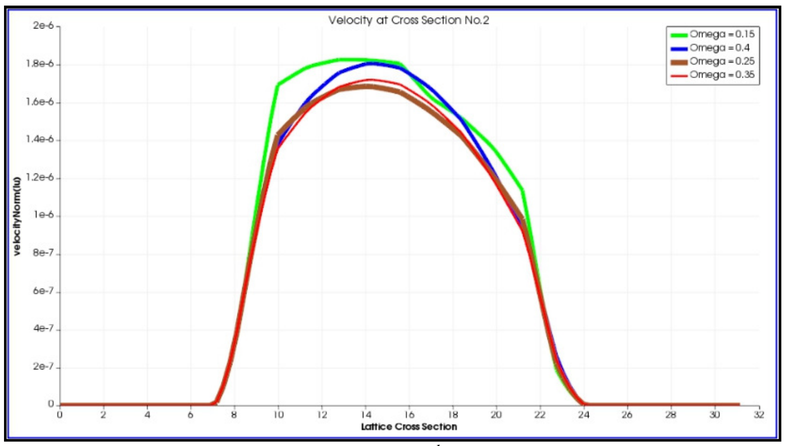

Figure 11: Velocity distribution across the 2nd channel cross section from Figure 5, for relaxation parameter number ω = 0.15, 0.25, 0.35 0.40

5. Conclusions

The application of Lattice Boltzmann Method is not simply could be applied especially when modeling the fluid flow of high viscosity fluid in pore scale porous media. From the simulation results showed, at the lower relaxation parameter number that represent high fluid viscosity indicating nonphysical phenomena of velocity field occurred visually. The phenomena are also confirmed with the velocity profile comparison at channels plotted, the lower relaxation parameter giving sudden jump or abruptly of velocity profile.
Therefore, it can be said that, the application of Lattice Boltzmann Method has certain limit in the term of high fluid viscosity model applied in the porous media to avoid nonphysical phenomena occurred. The evaluation of the lowest relaxation parameter number is necessary to be conducted prior to model the fluid flow of high viscosity fluid such as heavy oil fluid.

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