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Nonnegative Matrix Factorization with Nonlinear Constraints

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Abstract:
Processing of data with large dimensions has been a hot topic in recent decades. Various techniques have been proposed to execute the desired information or structure. Non- Negative Matrix Factorization-based on non-negatives data has become one of the favorite methods for shrinking dimensions. The main strength of this approach is the non-negative object, the object modeled by a combination of some basic non-negative parts so as to provide a physical interpretation of the object construction. NMF methods include the use of text mining, pattern recognition, and bioinformatics. The mathematical formulation for NMF did not appear as a convex optimization problem, and various types of model mathematics have been proposed to solve the problem Framework for Alternative Nonnegative Least Square. (ANLS) Are coordinates of the block formulation approaches that have been proven reliable theoretically and empirically efficient. This dissertation proposes a new algorithm to solve NMF based on the framework ANLS. This algorithm is put forward primary pivot methods to least squares problem with not – negative constraints which can overcome the limitations of the active set method. The proposed algorithm explores the reduced gradient method is a method that efficiently blocks the central pivot in the context of NMF. These algorithms also own ANLS convergence properties of the framework can be developed for the formulations and NMF with other constraints.

Keywords: *Matrix factorization, nonnegative matrix, sparse matrix, nonlinear constraints.*

1. Introduction

Every second in the modern era, a lot of data set. Imagine the number of people online write their blogs, their homepage design and disseminate their experiences through digital support, videos, images and others - others. Imagine also the data formed from living organisms and gene research data obtained from space or our earth.

Data of transactions through e-banking and others. Be helpful if the data has processed associated with the boom of data so rapidly, there are several approaches to the processing of data, the application of classical methods, designing more robust computational structures such as distributed computing, multi-core processors, supercomputers, and others - others. However, the amount and complexity of data growth exceed the trend increase in computing capability (1). One very popular approach called model of reduction that attempts to reduce the complexity while maintaining the assumption of the problem (preliminary data).

The use of the right model can save time. Also, different types of data require different models to capture the understanding of the data. Of course, a proper model considered so that the model can survive better met. An ideal model no, for example, the use of sub-dominant space with Singular Value Decomposition (SVD) has been proposed as the best model to reduce the complexity of data and the complexity of the system (10-11)

2. Literature Review

2.1. Matrix Factorization Nonnegative

This chapter is a presentation of Nonnegative matrix factorization problem. It consists of a description of the settlement of the problem formulation and some observations. This review gives the basics for part of this dissertation. Some comments more carefully examined in other chapters. (10,18).

It model could suggest that there is a non-negative matrix factorization name may be confusing in some cases and that the Matrix Approximation Nonnegative which should use instead. The term "factorization" may understand as an exact decomposition such as Cholesky decomposition, LU decomposition, and others (9-13). Where factored as an inexact input matrix as a matrix multiplication of the other. However, "non-negative matrix factorization" has been so popular that it works for non-negative matrix approximation problems with the multiplication of two matrices nonnegative. Determined using this term and nonnegative matrix factorization launched inexact to an exact case.

Non-Negative Matrix Factorization approach can formulate as follows:

A particular cover image C with size mm, can be factorization C into two non-negative matrices B and H where. Nonnegative matrix B contained the vector base, FMN and weighted non-negative matrix H, has a coefficient of association (nonnegative weights). To measure the quality of the factorization C = BH approach, the cost function between C and BH need to optimize subject to nonnegative constraints on B and H. This is done by minimizing Z conflicting information provided by:

$$Z(C\|BH)=\sum_{ij}(C_{ij}Log\frac{C_{ij}}{(BH)_{ij}}-C_{ij}+(BH)_{ij})$$

That will result in legal *multiplicative update*.

$$H_{kj}\leftarrow H_{kj}\frac{\sum_i B_{ik}C_{ik}\big/(BH)_{ij}}{\sum_i B_{ik}}$$

$$B_{jk}\leftarrow B_{jk}\frac{\sum_j B_{kj}C_{ij}\big/(BH)_{ij}}{\sum_j B_{kj}}$$

Even the model matrix B and H the start of a random non-negative matrix and update alternative did so once in the update of the H line, need to update the column corresponding to B. In other words, no need to refresh the matrix H is accompanied by the first update of the model B. non-negative matrix factorization algorithm is an iterative optimization algorithm. Where modifications at each iteration nonnegative basis functions and encodings to converge (3-15).

Even the non-negative matrix factorization is used to cluster genes and the modified algorithm based on non-negative matrix factorization, which used for the same purposes. Non-negative matrix factorization is used to analyze protein sequences. Only non-negative matrix factorization is used directly to sample cluster expressed genes compared to the group as a hierarchy and mapping organization own, non-negative matrix factorization is used to bio clustering of data expressed genes. Here introduce sharper basic idea of non-negative matrix factorization, by providing a set of information in the form of a matrix consisting of m samples in n-dimensional space, where each entry is non-negative (ie for all i, j), is a non-negative matrix factorization to define an approach, where B is a matrix and H a matrix nod DXM and the second B, H is also nonnegative. Each column of B can view as a base vector, and each column of H can see as new vector images comparable with real data. More details, basically there are two algorithms to enhance the implementation of the update via the multiplicative decomposition as follows: $b_{ik}\leftarrow b_{ik}\frac{(XH^T)_{ik}}{(BHH^T)_{ik}}$

$h_{kj}\frac{(B^TX)_{kj}}{(B^TBH)_{kj}}$ and

$$b_{ik}\leftarrow\frac{\sum_j h_{kj}\frac{x_{ij}}{(BH)_{ij}}}{\sum_j h_{kj}}\quad h_{kj}\leftarrow h_{kj}\frac{\sum_j b_{ik}\frac{x_{ij}}{(BH)_{ij}}}{\sum_i b_{ik}}$$

Legal updates can be followed non-negative multiplicative with initialization of non-negative.

In many types of linear systems that arise in the application, the elements of the coefficient matrix show no negative magnitude. It deals with the knowledge of such matrices and their properties.

The definition of an nxt matrix A with entries called non-negative real numbers if for every i and j, and is called active if for every i and j.

2.2. Program Non-Linier

In the chapter, it was found a certain optimization procedure to maximize or minimize a function of n variables. Worse, as noted earlier, this procedure can't be used to solve problems that are very much exceeds its variable. Here will be presented a few, computationally feasible method to solve certain types of problems, non-linear programming. One problem, nonlinear programming problem which is a must to maximize or minimize a function f ($\bar{x}$), the constraints of a set of equations or inequality constraints. Where f ($\bar{x}$) or at least one of the functions that appear in the set of constraints or both, is a non-linear function (7-13). Problems, non-linear programming, can generally be expressed as follows:

Maximize Z = f ($\bar{x}$)

constraints gi ($\bar{x}$) {≤, =, ≥} bi

Where, one of the three relations {≤, =, ≥} is designed against each of the m constraints the function f ($\bar{x}$) in equation (4.1) is called the objective function. In connection with the non-negativity constraint (xj ≥ 0) on some, or all, of the variables stated, separately or may be deemed to be included in the. constraint. There is no known method of determining the global maximum of the problem, non-linear programming in general. Even if the objective function and constraints in accordance with the properties of certain global

maximum can sometimes be acceptable. For example, proves that the global maximum of a function on a set konfek limited and on, an extreme point of the set konfek.
Here will be lowered "Kuhn-Tucker condition is" a set of necessary conditions for maximizing a non-linear problem. This condition also results in certain cases the global maximum. Kuhn-Tucker conditions are very useful indeed, in lowering method to resolve some, this type of problem, non-linear. It can be seen how, using Kuhn-Tucker conditions to develop an algorithm, using the simple method to solve a quadratic program. In one problem, the objective function is a quadratic program and constrain was linear quadratic.

3. Algoritma

Mathematical formulation of NMF can be expressed in the following way. Known input matrices A ε RM xn, in which each element and a nonnegative integer k <min {m, n}. NMF aims to determine two facts W ε Rm × k and H with the element xn ε Rk nm negative so:

$$A = W H$$

By using Fortenius norms, facts W and H is obtained by solving the following problems sptinuseri:

$$\frac{min}{N\geq 0, H\geq 0}\ f(N, H) = \frac{1}{2} \| A - WH \|_F^2$$

Where the inequality W ≥ 0 and H ≥0 means that all elements of W and H nm negative.
The basic framework from negative alternative least square (ANLS). before the proposed Algorithm to solve. This is necessary because dotted proposed algorithm of ANLS. In the basic framework ANLS, variable

Directly divided into two groups. Then both groups updated.
This basic framework can be summarized as follows:
1 Initialization H ε Rm with elements xl nm negative
2 Complete the following problems until termination criteria are met:

$$\frac{min}{w\geq 0} (| H^T W^T - A^T |)_F^2$$

With H fix, and

$$\frac{min}{H\geq 0} \| WH - A \|_F^2$$

And W fix

3. Column W is normalized into the unit L2 - nom and the corresponding rows of H scaled.

Alternatively, W can be just as initial iterations of data later in the Press then Release Note that each sub problems is a matter least - square wills nm negative (NNLS). Although the initial problem convex.
NNLS issues and has certain characteristics. NMF is an algorithm to reduce the dimension. Initial dimensions are very large, for example thousands, of small dimensions reduced, Because the matrix ε Rm W x L in is very long and thin shape (m >> k) and the coefficient matrix HT ε Rm xl in Eq. is also long and thin (n >> k). Because the WT and the H matrix of the variables in Eq. and respectively - each, data and wide.
To design the NMF algorithm which is based on ANLS framework, the need for a method to resolve the sub problem in Eq. and for NNLS algorithm is an active set method by Lawson and Hanson (). Active set methods seek the optimal set of active and passive to maintain between the two sets of variables. If the variable is passive (i.e., positive) solution are known, then the NNLS problem can be solved by the least squares procedure was control against passive variable.
The main limitation of the method is the active set of variables that satisfy the set of retained proceeds vector nm negative completion while ensuring that the objective function decreases in each iteration. Means algorithm is slow if the number of large unknown. The method proposed piust main block can know the limitations of active set methods.

4. Analysis and Result

First - assessed all major blocks pivot method developed for NNLS problems with the right-hand side and then proposed a method that was developed to handle the right side a lot.
.1NNLS with Vector Segment One Right
First - first noticed by a vector NNLS right hand side can be written mathematically as:

$$\frac{min}{x\geq 0} \| Cx - b \|_2^2$$

Rules support, where feasible only variable with the largest index is exchanged, a pivot rule - the lead single. This rule guarantees finite termination. By combining the full exchange rule or rules supporters, the main block pivot method for NNLS problems with one segment is obtained. This determinant algorithm listed in Algorithm 1.
Algorithm1
Input : $C\ \varepsilon\ R^{P \times q}$
and $b\ \varepsilon\ R^{P}$

output : x ($\varepsilon\ R^{q \times 1}$) = arg min x ≥ 0 $\|C_X - b\|_2^2$
Initializes F = ∅ , G = {1, …., q}, x = 0, y = - $C^T b$, $\propto = 3, \beta = q + 1$

calculates X_F and Y_G
While (X_F, Y_G) reject do
Calculate
V
if| V | <, for $\beta = |V|, \propto = 3$
and $\hat{V}$ = V if | V | $\geqq \beta$ and $\propto \geq 1$, for $\propto -1$and $\hat{V}$ = V
if | V | $\geqq \beta$ and $\propto = 0$, for $\hat{V}$
End while

Cases supporting for the rule is slower than the full exchange rule, it is only in use if the full exchange rule does not work. Finite termination of Algorithm 1 is achieved by controlling the number of variables is not feasible. The variable α is used as a buffer for full exchange rules in testing. If the full exchange rule increases the number of variables is not feasible, α minus one.

After α to zero, supporting the rule is used so that he made a number of variables worth less than the smallest value is reached, which is filled in β. It occurs in a finite number of steps because the rules buffer has finite termination properties. As soon as the buffer rule reaches the lowest amount of new variables unfit, returned to full exchange rules. Use as the default value for the three α, which means that the full exchange rule tested up to three times until he has a decent amount of variable. Because the variable is not feasible systematic reduced.

5. Conclusion

This dissertation proposes a new algorithm to solve FMN based on the framework ANLS. Thus, the algorithm is built on top of the block pivot method for nonnegative least squares problems. The method covers the exchange of some variables among the set of the processes with the aim to reach the final partition into active and passive sets quickly. Improved methods of applying power so reduced gradient method to handle the case of multiple right-hand side of nonnegative least squares problems efficiently. This new algorithm has the same convergence properties as ANLS framework can be extended to formulations not FMN FMN way so as sparse matrix power. Further research that can be done is to add the Quasi-Newton method and Variable- Matrix besides reduced gradient method. The addition of this method will be able to further accelerate convergence.

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