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Unsteady Free Convective MHD Flow through Porous Media in a Rotating System with Fluctuating Temperature and Concentration

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Abstract :
An attempt has been made to study the heat and mass transfer effect of a viscus fluid past a vertical plate. The novelty of the present study is to see the combined effect of 1^{st} order chemical reaction in the presence of heat source under oscillatory suction velocity of the flow past a vertical plate. The method of solution suggested here is the generalization of steady part solution when the amplitude of oscillation becomes zero. Many intresting results exhibiting the effect of chemical reaction, heat source parameter fluctuating temperature and concentration have been discussed carefully.

Keywords : *MHD flow, heat source, heat transfer, mass transfer, chemical reaction, porous medium*

1. Introduction

The study of free convection MHD flow with heat and mass transfer through porous medium with fluctuating temperature and concentration in rotating environment has significant role in the application of astrophysical, geophysical sciences, petrochemical engineering and oceanography. The stimulus for scientific research on rotating fluid systems is basically originated from geophysical and fluid engineering applications. Rotating flow theory is utilized on determining the viscosity of the fluid in the construction of the turbine and other centrifugal machines.

Several researchers have analyzed a bewildering variety of flows connected to MHD free convection flow through porous media of a rotating/non-rotating fluid with heat and mass transfer. Sharma and Mathur (1995) studied the steady laminar free convection flow of an electrically conducting fluid along a porous hot vertical plate in the presence of heat source/sink. Unsteady two-dimensional laminar flow of a viscous incompressible electrically conducting fluid in the vicinity of a semi-infinite vertical porous moving plate was discussed by Kim (2000). Dash and Rath (1997) have studied the problem of laminar flow and heat transfer of an electrically conducting fluid between parallel porous plates by applying explicit finite difference scheme. The hydromagnetic Raleigh problem in a rotating fluid has been studied by Rapties and Singh (1986). Flow and heat transfer of an electrically conducting visco-elastic fluid between two horizontal squeezing/stretching plates has been studied by Rath *et al*.(2001).

Ece (2005) has studied the free convection flow about a cone under mixed thermal boundary conditions and a magnetic field. Non-parallel vortex instability of natural convection flow over a non-isothermal inclined flat plate with simultaneous thermal and mass diffusion has been studied by Acharya *et al*.(2006). Parvazinia and Nassehi (2006) have investigated the study of shear thinning fluid flow through highly permeable porous media. MHD flow through a porous medium past a stretched vertical permeable surface in the presence of heat source/sink and a chemical reaction has been studied by Dash *et al*. (2008). An analytical study of boundary layer flows on a continuous stretching surface was reported by Bararnia *et al*. (2009).

The main objective of the present problem is to study the effect of unsteady free convective viscous flow through porous media in a rotating system with fluctuating temperature and concentration.

2. Mathematical Formulation

We consider the unsteady free convective flow of a viscous incompressible fluid bounded by a vertical infinite porous surface in a rotating system. The temperature on the surface varies with the time about a non-zero constant mean while the temperature of the free stream is taken to be constant. We assume that the fluid properties are not affected by the fluctuating temperature and concentration differences except that of the density ρ in the body force term and the influence of the density variations in the momentum and energy equations is negligible. A vertical infinite porous plate rotating with constant angular velocity Ω about an axis which is perpendicular to the vertical plane surface is considered. The cartesian co-ordinate system is chosen such that x, y-axis, respectively, are in the vertical upward and perpendicular directions on the plane of the vertical porous surface z = 0, while z – axis is normal to it (Fig. 1).

The interaction of Coriolis force with the free convection sets up a secondary flow in addition to primary flow and hence the flow becomes three dimensional. With the above frame of reference and assumption (the plate is infinite in extent and the flow is unsteady), the physical variables are functions of z and time t only. Consequently the equations expressing the conservation of mass, momentum, energy and mass transfer, under usual Boussinesq approximation, are given by

$$\frac{\partial w}{\partial z} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + w\frac{\partial u}{\partial z} - 2\Omega v = g\beta\left(T - T_\infty\right) + g\beta^*(C - C_\infty) + \nu\frac{\partial^2 u}{\partial z^2} - \frac{\sigma B_0^2 u}{\rho} - \frac{\nu u}{k^*} \tag{2}$$

$$\frac{\partial v}{\partial t} + w\frac{\partial v}{\partial z} + 2\Omega u = \nu\frac{\partial^2 v}{\partial z^2} - \frac{\sigma B_0^2 v}{\rho} - \frac{\nu v}{k^*} \tag{3}$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial z} - \frac{\nu w}{k^*} \tag{4}$$

$$\frac{\partial T}{\partial t} + w\frac{\partial T}{\partial z} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial z^2} \tag{5}$$

$$\frac{\partial C}{\partial t} + w\frac{\partial C}{\partial z} = D\frac{\partial^2 C}{\partial z^2} \tag{6}$$

In this equation ρ is the density of the fluid, σ is the electrical conductivity of the fluid, B_0 is the uniform magnetic field strength, ν is the coefficient of the kinematic viscosity, k is the thermal conductivity of the fluid, C_p is the specific heat of the fluid, β is the volumetric coefficient of thermal expansion, β^* is the volumetric coefficient of thermal expansion with concentration, g is the acceleration due to gravity, D is the chemical diffusivity of the fluid and k* is the permeability of the medium.

The corresponding initial boundary conditions are

$$\left.\begin{array}{l} u = 0,\ v = 0,\ T = T_w + \varepsilon\,(T_w - T_\infty)\,e^{i\omega t},\ C = C_w + \varepsilon\,(C_w - C_\infty)e^{i\omega t} \text{ at } z = 0, \\ u, v \to 0,\ T \to T_\infty,\ C \to C_\infty \text{ as } z \to \infty \end{array}\right\} \tag{7}$$

where $\varepsilon << 1$ and ω is the frequency of oscillation.

There will be always some fluctuations in the temperature. The plate temperature is assumed to vary harmonically with time. It varies from $T_w \pm \varepsilon\,(T_w - T_\infty)$ as t varies from 0 to $\pi/2w$. Moreover, the species concentration is subjected to fluctuate harmonically and varies as $C_w \pm \varepsilon\,(C_w - C_\infty)$. Now there may also occur some variation in suction at the plate due to the variation of the temperature. Here we assume that the frequency of suction and temperature variation are same. Integrating equation (1) we get

$$w\,(t) = -\,w_0\,(1 + \varepsilon\,Ae^{i\omega t}) \tag{8}$$

Where A is the suction parameter, w_0 is the constant suction velocity and ε is the small positive number such that $\varepsilon A \le 1$. Since ε is small, the plate temperature and the suction velocity at the plate vary slightly from the mean value T_w and w_0 respectively. Equation (4) determines the pressure distribution along the axis of rotation and the absence of $\frac{\partial p}{\partial y}$ in equation (3) implies that there is a net cross flow in y-direction.

Considering U = u + iv and equation (8), the momentum equation (2) and (3) can be written as

$$\frac{\partial U}{\partial t} - w_0\,(1 + \varepsilon\,Ae^{i\omega t})\,\frac{\partial U}{\partial z} + 2\Omega iU = g\beta(T - T_\infty) + g\beta^*(C - C_\infty) + \nu\frac{\partial^2 U}{\partial z^2} - \sigma\frac{B_0^2}{\rho}U - \frac{\nu U}{k^*} \tag{9}$$

Now, we introduce the following non-dimensional quantities.

$$z' = \frac{w_0 z}{\nu},\ U' = \frac{U}{w_0},\ t' = \frac{t w_0^2}{\nu}, \omega' = \frac{\nu\omega}{w_0^2}$$

$$T' = \frac{T - T_\infty}{T_w - T_\infty},\ C' = \frac{C - C_\infty}{C_w - C_\infty}, S_c = \frac{\nu}{D}, P_r = \frac{\rho\nu C_p}{k}$$

$$G_r = \frac{g\beta\nu(T_w - T_\infty)}{w_0^3},\ G_c = \frac{g\beta^*\nu(C - C_\infty)}{w_0^3}$$

$$R_0 = \frac{\Omega\nu}{w_0^2}, M = \sqrt{\frac{\sigma B_0^2 \nu}{\rho w_0^2}},\ k_p = \frac{w_0^2 k^*}{\nu^2}$$

With the above non-dimensional quantities (dropping the dashes) equation (9), (5) and (6) can be written as

$$\frac{\partial U}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial U}{\partial z} + 2iR_0 U = G_r T + G_c C + \frac{\partial^2 U}{\partial z^2} - \left(M^2 + \frac{1}{k_p}\right)U \tag{10}$$

$$\frac{\partial T}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial T}{\partial z} = \frac{1}{P_r}\frac{\partial^2 T}{\partial z^2} \tag{11}$$

$$\frac{\partial C}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial C}{\partial z} = \frac{1}{S_c}\frac{\partial^2 C}{\partial z^2} \tag{12}$$

and the boundary conditions (7) become

$$\left.\begin{array}{l} \text{at } z = 0:\ U = 0,\ T = 1 + \varepsilon e^{i\omega t},\ C = 1 + \varepsilon e^{i\omega t} \\ \text{as } z \to \infty:\ U \to 0,\ T \to 0,\ C \to 0 \end{array}\right\} \tag{13}$$

In view of equations (8) and oscillating plate temperature T, we look for the solutions of the forms

$$\left.\begin{array}{l} U(z, t) = U_0(z) + \varepsilon e^{i\omega t} U_1(z), \\ T(z, t) = T_0(z) + \varepsilon e^{i\omega t} T_1(z), \\ C(z, t) = C_0(z) + \varepsilon e^{i\omega t} C_1(z), \end{array}\right\} \tag{14}$$

which are valid for small amplitude of oscillation.

Substituting (14) into the system (10) – (12) and equating the harmonic and non-harmonic terms, we get

$$\frac{d^2U_0}{dz^2} + \frac{dU_0}{dz} - \left(2iR_0 + M^2 + \frac{1}{k_p}\right)U_0 = -G_r T_0 - G_c C_0 \tag{15}$$

$$\frac{d^2U_1}{dz^2} + \frac{dU_1}{dz} - \left[i(2R_0 + \omega) + M^2 + \frac{1}{k_p}\right]U_1 = -G_r T_1 - G_c C_1 - A\frac{dU_0}{dz} \tag{16}$$

$$\frac{d^2T_0}{dz^2} + P_r \frac{dT_0}{dz} = 0 \tag{17}$$

$$\frac{d^2T_1}{dz^2} + P_r \frac{dT_1}{dz} - i\omega P_r T_1 = -AP_r \frac{dT_0}{dz} \tag{18}$$

$$\frac{d^2C_0}{dz^2} + S_c \frac{dC_0}{dz} = 0 \tag{19}$$

$$\frac{d^2C_1}{dz^2} + S_c \frac{dC_1}{dz} - i\omega S_c C_1 = -AS_c \frac{dC_0}{dz} \tag{20}$$

and the boundary conditions (13) become

$$\left\{\begin{array}{ll} \text{at } z = 0: & \begin{array}{l} U_0 = 0,\ T_0 = 1, C_0 = 1 \\ U_1 = 0,\ T_1 = 1, C_1 = 1 \end{array} \\ \text{as } z \to \infty: & \begin{array}{l} U_0 \to 0,\ T_0 \to 0, C_0 \to 0 \\ U_1 \to 0,\ T_1 \to 0, C_1 \to 0 \end{array} \end{array}\right. \tag{21}$$

3. Solution

Thus, the solutions are given by

$$U(z,t) = L_1 e^{-P_r Z} + L_2 e^{-S_c Z} + L_3 e^{M_6 Z} + \varepsilon e^{i\omega t}\left[\frac{(AN_1-1)G_r}{(M_2-M_7)(M_2-M_8)}\left(e^{M_2 Z}-e^{M_8 Z}\right) + \frac{A(P_r L_1 - G_r N_1)}{(P_r+M_7)(P_r+M_8)}\left(e^{-P_r Z}-e^{M_8 Z}\right) + \frac{A(S_c L_2 - G_c N_2)}{(S_c+M_7)(S_c+M_8)}\left(e^{-S_c Z}-e^{M_8 Z}\right) + \frac{(AN_2-1)G_c\left(e^{M_4 Z}-e^{M_8 Z}\right)}{(M_4-M_7)(M_4-M_8)} - \frac{AL_3 M_6}{(M_6-M_7)(M_6-M_8)}\left(e^{M_6 Z}-e^{M_8 Z}\right)\right] \quad (22)$$

$$T(z,t) = e^{-P_r Z} + \varepsilon e^{i\omega t}\left[e^{M_2 Z} + \frac{AP_r^2}{(P_r+M_1)(P_r+M_2)}\left(e^{-P_r Z}-e^{M_2 Z}\right)\right] \quad (23)$$

$$C(z,t) = e^{-S_c Z} + \varepsilon e^{i\omega t}\left[e^{M_4 Z} + \frac{AS_c^2}{(S_c+M_3)(S_c+M_4)}\left(e^{-S_c Z}-e^{M_4 Z}\right)\right] \quad (24)$$

where

$$M_1 = \frac{1}{2}\left[-P_r + \left(P_r^2 + 4i\omega P_r\right)^{1/2}\right],\quad M_2 = \frac{1}{2}\left[-P_r - \left(P_r^2 + 4i\omega P_r\right)^{1/2}\right],$$

$$M_3 = \frac{1}{2}\left[-S_c + \left(S_c^2 + 4i\omega S_c\right)^{1/2}\right],\quad M_4 = \frac{1}{2}\left[-S_c - \left(S_c^2 + 4i\omega S_c\right)^{1/2}\right],$$

$$M_5 = \frac{1}{2}\left[-1 + \left\{1 + 4\left(2iR_0 + M^2 + \frac{1}{k_p}\right)\right\}^{1/2}\right],$$

$$M_6 = \frac{1}{2}\left[-1 - \left\{1 + 4\left(2iR_0 + M^2 + \frac{1}{k_p}\right)\right\}^{1/2}\right],$$

$$M_7 = \frac{1}{2}\left[-1 + \left[1 + 4\left\{i(2R_0+\omega) + M^2 + \frac{1}{k_p}\right\}\right]^{1/2}\right],$$

$$M_8 = \frac{1}{2}\left[-1 - \left[1 + 4\left\{i(2R_0+\omega) + M^2 + \frac{1}{k_p}\right\}\right]^{1/2}\right],$$

$$N_1 = \frac{P_r^2}{(P_r+M_1)(P_r+M_2)},\quad N_2 = \frac{S_c^2}{(S_c+M_3)(S_c+M_4)},$$

$$L_1 = \frac{-G_r}{(P_r+M_5)(P_r+M_6)},\quad L_2 = \frac{-G_c}{(S_c+M_5)(S_c+M_6)},\quad L_3 = -(L_1+L_2)$$

Equation (22) reveals that, the steady part of the velocity field has three-layer character, while the oscillating part of the velocity field exhibits a multilayer character.

From equations (23) and (24), we observe that in case of considerably slow motion of the fluid i.e., when viscous dissipation term is neglected, the temperature and the concentration profiles are mainly affected by Prandtl number (P_r) and Schmidt number (S_c) of the fluid respectively.

Considering $U_0 = u_0 + iv_0$ and $U_1 = u_1 + iv_1$, we write the primary and secondary velocity fields in terms of the fluctuating parts as

$$\frac{u}{w_0}(z,t) = u_0(z) + \varepsilon\left(u_1 \cos\omega t - v_1 \sin\omega t\right) \quad (25)$$

and $\frac{v}{w_0}(z,t) = v_0(z) + \varepsilon\,(u_1 \sin\omega t + v_1 \cos\omega t)$ (26)

where

$u_0(z) = L_{11}e^{-P_r Z} + L_{21}e^{-S_c Z} + \left[\left(L_{31}\cos M_{62}z - L_{32}\sin M_{62}z\right)e^{M_{61}Z}\right]$

$v_0(z) = L_{12}e^{-P_r Z} + L_{22}e^{-S_c Z} + \left[\left(L_{32}\cos M_{62}z + L_{31}\sin M_{62}z\right)e^{M_{61}Z}\right]$

$u_1(z) = \left(C_1\cos M_{22}z - D_1\sin M_{22}z\right)e^{M_{21}Z} + C_2 e^{-P_r Z} + C_3 e^{-S_c Z}$
$+ (C_4\cos M_{42}z - D_4\sin M_{42}z)e^{M_{41}Z} - (C_5\cos M_{62}z - D_5\sin M_{62}z)e^{M_{61}Z}$
$-(C_6\cos M_{82}z - D_6\sin M_{82}z)e^{M_{81}Z}$

$v_1(z) = \left(D_1\cos M_{22}z + C_1\sin M_{22}z\right)e^{M_{21}Z} + D_2 e^{-P_r Z} + D_3 e^{-S_c Z}$
$- (D_4\cos M_{42}z + C_4\sin M_{42}z)e^{M_{41}Z} - (D_5\cos M_{62}z + C_5\sin M_{62}z)e^{M_{61}Z}$
$-(D_6\cos M_{82}z + C_6\sin M_{82}z)e^{M_{81}Z}$

and

$L_{11} = \frac{-G_r\left(P_r^2 - P_r - M^2 - \frac{1}{k_p}\right)}{\left(P_r^2 - P_r - M^2 - \frac{1}{k_p}\right)^2 + 4R_0^{\,2}}$, $L_{12} = \frac{-2R_0 G_r}{\left(P_r^2 - P_r - M^2 - \frac{1}{k_p}\right)^2 + 4R_0^{\,2}}$

$L_{21} = \frac{-G_c\left(S_c^2 - S_c - M^2 - \frac{1}{k_p}\right)}{\left(S_c^2 - S_c - M^2 - \frac{1}{k_p}\right)^2 + 4R_0^{\,2}}$, $L_{22} = \frac{-2R_0 G_c}{\left(S_c^2 - S_c - M^2 - \frac{1}{k_p}\right)^2 + 4R_0^{\,2}}$

$L_{31} = -(L_{11} + L_{21})$, $L_{32} = -(L_{12} + L_{22})$, $A_1 = P_r^2$, $A_2 = S_c^2$, $A_3 = 1 + 4\left(M^2 + \frac{1}{k_p}\right)$ $A_4 = A_3$, $B_1 = 4\omega P_r$, $B_2 = 4\omega S_c$, $B_3 = 8R_0$, $B_4 = B_3 + 4\omega = 4(2R_0 + \omega)$,

$E_i = \left[\frac{A_i + \left(A_i^2 + B_i^2\right)^{1/2}}{2}\right]^{1/2}, F_i = \left[\frac{-A_i + \left(A_i^2 + B_i^2\right)^{1/2}}{2}\right]^{1/2}$ $i = 1,2,3,4,$

$M_{11} = \frac{-P_r + E_1}{2}, M_{12} = \frac{F_1}{2}, M_{21} = \frac{-P_r - E_1}{2}$, $M_{22} = -M_{12}$,

$M_{31} = \frac{-S_c + E_2}{2}, M_{32} = \frac{F_2}{2}, M_{41} = \frac{-S_c - E_2}{2}$, $M_{42} = -M_{32}$,

$M_{51} = \frac{-1 + E_3}{2}, M_{52} = \frac{F_3}{2}, M_{61} = \frac{-1 - E_3}{2}$, $M_{62} = -M_{52}$,

$M_{71} = \frac{-1 + E_4}{2}, M_{72} = \frac{F_4}{2}, M_{81} = \frac{-1 - E_4}{2}$, $M_{82} = -M_{72}$,

$Y_1 = \left[\frac{1}{2}(P_r - 1)(P_r + E_1) - \left(M^2 + \frac{1}{k_p}\right)\right]$,

$Y_2 = \left[\frac{1}{2}(S_c - 1)(S_c + E_2) - \left(M^2 + \frac{1}{k_p}\right)\right]$

$Z_1 = \left[\frac{1}{2}(P_r - 1)F_1 + \omega P_r - (2R_0 + \omega)\right]$, $Z_2 = \left[\frac{1}{2}(S_c - 1)F_2 + \omega S_c - (2R_0 + \omega)\right]$,

$Y_3 = \left(P_r^2 - P_r - M^2 - \frac{1}{k_p}\right), Y_4 = \left(S_c^2 - S_c - M^2 - \frac{1}{k_p}\right)$,

$Z_3 = -(2R_0 + \omega)$, $Z_4 = Z_3$, $C_1 = \frac{G_r(AP_r z_1 - \omega Y_1)}{\omega(Y_1^2 + Z_1^2)}$,

$$C_2 = \frac{AP_r\{\omega L_{11} Y_3 + (\omega L_{12} - G_r) Z_3\}}{\omega(Y_3^2 + Z_3^2)},$$

$C_3 = \frac{AS_c\{\omega L_{21} Y_4 + (\omega L_{22} - G_c) Z_4\}}{\omega(Y_4^2 + Z_4^2)}, C_4 = \frac{AG_c S_c z_2}{\omega(Y_2^2 + Z_2^2)}$,

$C_5 = \frac{-A\{L_{32} M_{61} + L_{31} M_{62}\}}{\omega}$, $C_6 = C_1 + C_2 + C_3 + C_4 - C_5$,

$D_1 = \frac{G_r(AP_r Y_1 + \omega Z_1)}{\omega(Y_1^2 + Z_1^2)}$, $D_2 = \frac{AP_r[(\omega L_{12} - G_r) Y_3 - \omega L_{11} Z_3]}{\omega(Y_3^2 + Z_3^2)}$,

$D_3 = \frac{AS_c[(\omega L_{22} - G_c) Y_4 - \omega L_{21} z_4]}{\omega(Y_4^2 + Z_4^2)}$, $D_4 = \frac{AG_c S_c Y_2}{\omega(Y_2^2 + Z_2^2)}$,

$D_5 = \frac{A\{L_{31} M_{61} - L_{32} M_{62}\}}{\omega}$, $D_6 = D_1 + D_2 + D_3 + D_4 - D_5$],

$$C_1' = -\frac{G_r Y_1}{(Y_1^2 + Z_1^2)},\ D_1' = \frac{G_r Z_1}{(Y_1^2 + Z_1^2)},$$

(C_1' and D_1' are used when A = 0).

Hence, the expression for the velocity profiles, for $\omega t = \frac{\pi}{2}$, are given by

$$\frac{u}{w_0}\left(z, \frac{\pi}{2\omega}\right) = u_0(z) - \varepsilon v_1(z) \quad (27)$$

$$\frac{v}{w_0}\left(z, \frac{\pi}{2\omega}\right) = v_0(z) + \varepsilon u_1(z) \quad (28)$$

Knowing the velocity field, we now calculate the skin friction at the plate which is given by.

$$\tau^* = \mu\left(\frac{\partial u}{\partial z} + i\frac{\partial v}{\partial z}\right)\Bigg|_{z=0}$$

and in non dimensional form omitting dashes we have

$$\frac{\tau^*}{\rho w_0^2} = \tau_{xz} + i\tau_{yz} = \left(\frac{\partial u}{\partial z} + i\frac{\partial v}{\partial z}\right)_{z=0} \quad (29)$$

From equation (29) the τ_{xz} and τ_{yz} components of the skin friction at the plate are given by

$$\tau_{xz} = \frac{\partial u_0}{\partial z}\Bigg|_{z=0} - \varepsilon\frac{\partial v_1}{\partial z}\Bigg|_{z=0}$$

$= -P_r L_{11} - S_c L_{21} + L_{31} M_{61} - L_{32} M_{62}$
$- \varepsilon[D_1 M_{21} + C_1 M_{22} - D_2 P_r - D_3 S_c + D_4 M_{41} + C_4 M_{42} - D_5 M_{61}$
$- C_5 M_{62} - D_6 M_{81} - C_6 M_{82}]$, (30)

and $\tau_{yz} = \frac{\partial v_0}{\partial z}\Big|_{z=0} + \varepsilon\frac{\partial u_1}{\partial z}\Big|_{z=0}$

$= -P_r L_{12} - S_c L_{22} + L_{32} M_{61} + L_{31} M_{62}$

$+ \varepsilon [C_1 M_{21} - D_1 M_{22} - C_2 P_r - C_3 S_c + C_4 M_{41} - D_4 M_{42}$
$- C_5 M_{61} + D_5 M_{62} - C_6 M_{81} + D_6 M_{82}]$. (31)

Now, from equation (23) and (24) we obtain expressions for the transient temperature and species concentration profiles for $\omega t = \pi/2$, as

$$T\left(z,\frac{\pi}{2\omega}\right) = e^{-P_r Z} - \varepsilon\left[e^{M_{21}Z}\sin M_{22}z + A\frac{P_r}{\omega}\left(e^{-P_r Z} - e^{M_{21}Z}\cos M_{22}z\right)\right], \quad (32)$$

and

$$C\left(z,\frac{\pi}{2\omega}\right) = e^{-S_c Z} - \varepsilon\left[e^{M_{41}Z}M_{42}z + \frac{AS_c}{\omega}\left(e^{-S_c Z} - e^{M_{41}Z}\cos M_{42}z\right)\right] \quad (33)$$

Also, we obtain the expression for the rate of heat transfer from

$$q^* = -k\left.\frac{\partial T}{\partial z}\right|_{z=0}$$

Which reduces to the following non-dimensional form

$$q = -\left.\frac{\partial T}{\partial z}\right|_{z=0} = -P_r + \varepsilon e^{i\omega t}\left[M_2 - \frac{AP_r^{\,2}}{P_r + M_1}\right] \quad (34)$$

The rate of heat transfer q, can also be expressed in terms of the amplitude (Q) and phase angle (γ) as

$q = -P_r + \varepsilon|Q|\cos(\omega t + \Upsilon)$

Where

$Q = Q_r + iQ_i$

$$Q_r = \frac{P_r\left[P_r M_{21} - AP_r^{\,2} + 2M_{11}M_{21} - AP_r M_{11} + \frac{M_{21}}{P_r}\left(M_{11}^2 + M_{12}^2\right)\right]}{\left(P_r + M_{11}\right)^2 + M_{12}^2}$$

$$Q_i = \frac{P_r\left[P_r M_{22} + 2M_{11}M_{22} + AP_r M_{12} + \frac{M_{22}}{P_r}\left(M_{11}^2 + M_{12}^2\right)\right]}{\left(P_r + M_{11}\right)^2 + M_{12}^2}$$

$$\gamma = \tan^{-1}\frac{Q_i}{Q_r}, |Q| = \sqrt{\left(Q_r^2 + Q_i^2\right)}$$

The numerical values of |Q| and tan Υ are entered in Table 2.

4. Results and Discussion

In order to bring out the effects of rotation parameter, suction parameter, magnetic parameter, permeability parameter, mass transfer parameter and oscillation of temperature and concentration, following discussion are carried out. In course of numerical calculation the values of the parameters are so chosen to represent the fluid of common interest such as air, water etc.

Figure 2 shows that modified Grashof number G_c (Curve I, V) and porosity parameter k_p (curve I and IV) have no significant contribution on primary velocity. It is interesting to note that point z = 1 of the flow domain remains unaffected by Grashof number G_r, Modified Grashof number G_c and porosity of the medium as the fine curves I to V pass through a single point

Further it is to remark that high value of thermal bouyancy parameter G_r ($G_r = 8$) enhances the primary velocity sharply near the plate . For high value of rotation parameter and low value of magnetic parameter flow reversal occurs for Z > 0.5 . Another point to note that with low value of rotation, (curve IX) the velocity remains positive throughout. Thus it is inferred that for the flow reversal is prevented due to low speed rotation and high value of magnetic parameter.

Figure 3 exhibits the effect of pertinent parameters on secondary velocity. One remarkable observation is that all velocity profiles assume negative values through out the flow field. From Curve III it is seen that for large value of G_r secondary velocity attains the maximum magnitude. This remains also same in case of primary velocity. Larger convection current enhances both the components of the velocity. Further, coincidence of Curves I, IV, V shows that presence of porous matrix and bouyancy effect due to concentration difference have no significant role on secondary velocity which is also true in case of primary velocity. Magnetic parameter and rotation parameter increase the secondary velocity. But opposite effect of rotation parameter is marked on primary velocity.

Figure 4 shows the temperature distribution. High value of prandtl number leads to the decrease the temperature uniformly in all the layers.

Figure 5 shows the concentration variation for different diffusing species such as CO_2, H_2O and NH_3 having Schmidt number 0.30, 0.60 and 0.78 respectively. It is concluded that for heavier species the concentration decreases at all the layers of the flow domain.

Values of skin friction are shown in Table 1. It is to note that all the values of $\square_{xz}$ are positive whereas the value of $\square_{yz}$ are negative. This contributes to the stability of flow. For heavier species $\square_{xz}$ decreases. Further it is to note that and increase in the values of G_r, G_c and k_p contribute to higher values of $\square_{xz}$ whereas other component $\square_{yz}$ decreases.

Table 2 enumerates the values of |Q|, the rate of heat transfer and tan □ the phase of heat transfer. It is seen that magnetic parameter, Grashof number and modified Grashof number increase the rate of heat transfer and no change is marked due to porosity parameter. Presence of bouyancy force and magnetic field accelerate the rate of heat transfer and presence of porous matrix have no effect.

5. Conclusion

- Presence of porous matrix has no significant effect either on velocity or temperature fields.
- Primary velocity at a certain layer remains unaffected by the magnetic field or bouyancy effect .
- Flow reversal is prevented for low rotation and high magnetic field.
- Secondary velocity remains negative throughout the flow domain.
- Secondary velocity is not also affected by porous matrix and modified Grashof number.
- High rotation enhances the secondary velocity but reduces the primary one.
- Fluid with higher prandtl number decreases the temperature at all points.

Greater skin friction $\square_{xz}$ is experienced for high values porosity and bouyancy parameter but other component depicts the opposite effect.

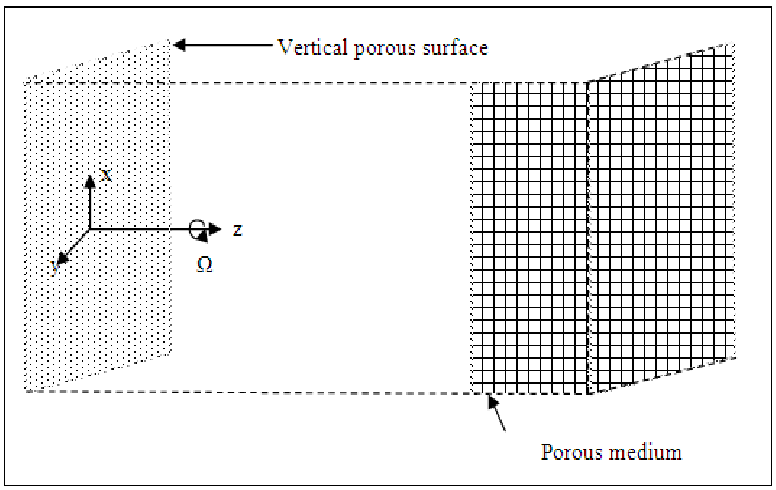

Figure 1: Schematic diagram of the problem

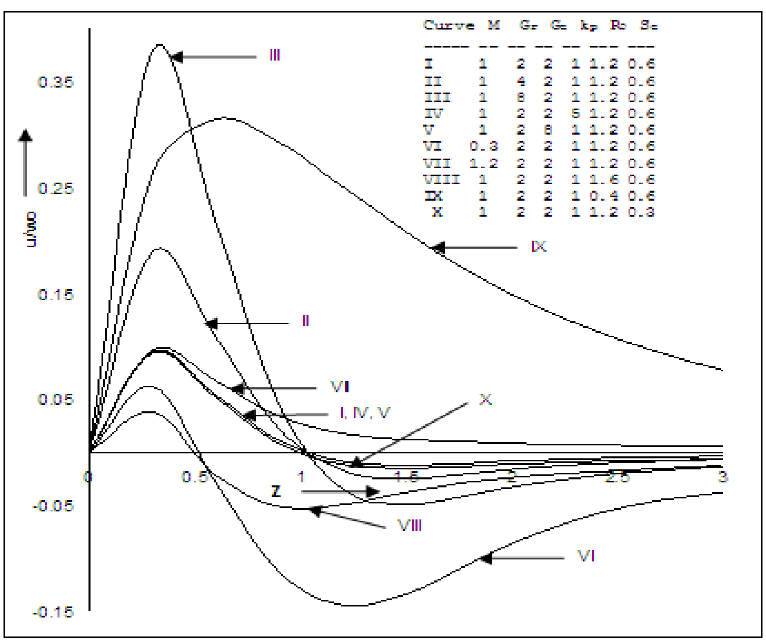

Figure 2: Primary velocity profiles

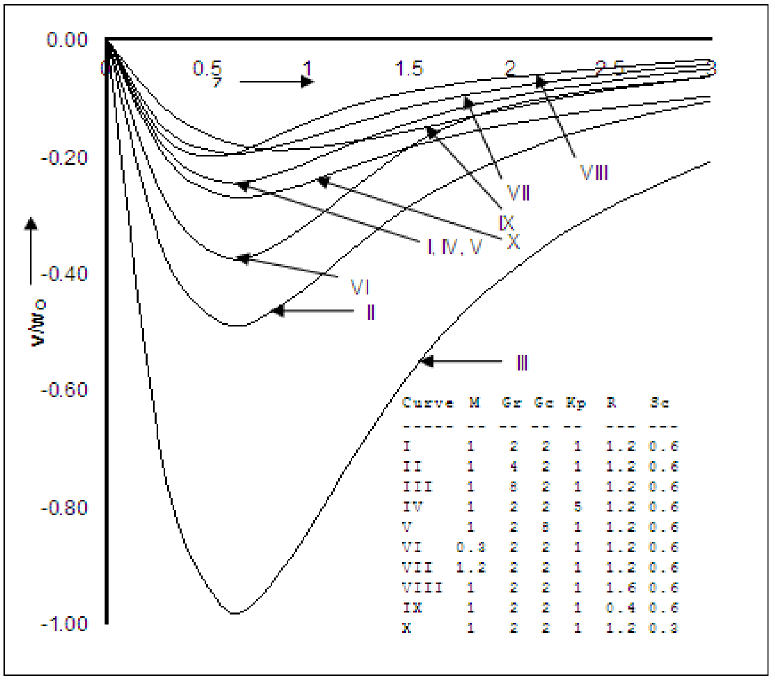

Figure 3:Secondary velocity profiles

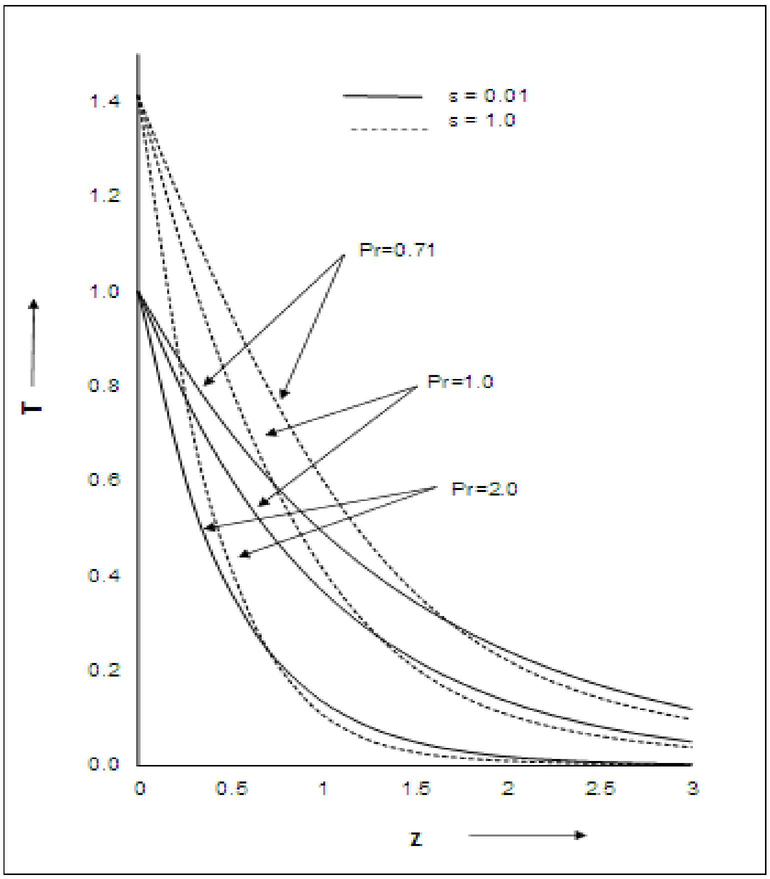

Figure 4: Temperature profiles

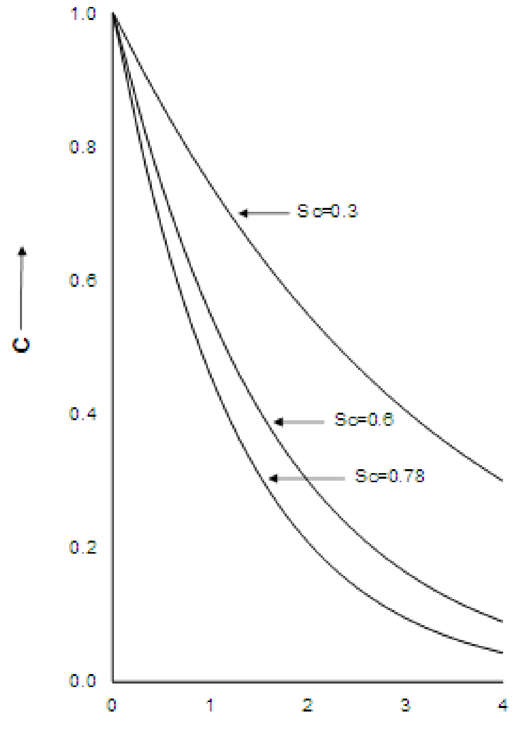

Figure 5: Concentration profiles

P_r	G_r	G_c	R_0	M	ω	k_p	S_c	A=0.0		A=1.0	
								τ_{xz}	τ_{yz}	τ_{xz}	τ_{yz}
0.71	2.0	4.0	0.4	0.3	15	1	0.22	5.84651	-0.00717	5.85191	-0.01717
							0.30	5.48905	-0.0069	5.4939	-0.01626
							0.60	4.54429	-0.00627	4.54773	-0.01392
							0.78	4.16287	-0.00602	4.16574	-0.01297
							1.002	3.80438	-0.00578	3.80672	-0.01207
0.71	2.0	4.0	0.4	0.3	30	1	0.22	5.84434	-0.00499	5.84915	-0.01229
							0.30	5.48699	-0.00481	5.49139	-0.01165
							0.60	4.54251	-0.00437	4.54585	-0.01
							0.78	4.16119	-0.0042	4.1641	-0.00934
							1.002	3.8028	-0.00404	3.80531	-0.00871
0.71	2.0	4.0	0.8	0.3	15	1	0.22	4.92294	-0.00724	4.92703	-0.0155
							0.30	4.6738	-0.00697	4.67751	-0.01478
							0.60	3.98038	-0.00632	3.98305	-0.01288
							0.78	3.68503	-0.00607	3.68727	-0.01209
							1.002	3.3988	-0.00582	3.40062	-0.01131
0.71	2.0	4.0	0.4	1.0	15	1	0.22	5.84651	-0.00717	5.85191	-0.01717
							0.30	5.48905	-0.0069	5.4939	-0.01626
							0.60	4.54429	-0.00627	4.54773	-0.01392
							0.78	4.16287	-0.00602	4.16574	-0.01297
							1.002	3.80438	-0.00578	3.80672	-0.01207
0.71	2.0	8.0	0.4	0.3	15	1	0.22	10.25985	-0.0123	10.26963	-0.0299
							0.30	9.54492	-0.01177	9.55361	-0.02808
							0.60	7.65541	-0.0105	7.66126	-0.02339
							0.78	6.89256	-0.01001	6.89728	-0.0215
							1.002	6.17557	-0.00953	6.17925	-0.0197

P_r	G_r	G_c	R_0	M	ω	k_p	S_c	A=0.0		A=1.0	
								τ_{xz}	τ_{yz}	τ_{xz}	τ_{yz}
0.71	8.0	4.0	0.4	0.3	15	1	0.22	10.14604	-0.01328	10.15451	-0.03048
							0.30	9.78857	-0.01301	9.7965	-0.02957
							0.60	8.84382	-0.01238	8.85033	-0.02723
							0.78	8.46239	-0.01213	8.46834	-0.02628
							1.002	8.1039	-0.01189	8.10932	-0.02538
0.71	8.0	4.0	0.4	0.3	15	5	0.22	16.06953	-0.01307	16.08655	-0.04133
							0.30	14.96994	-0.01281	14.9853	-0.03909
							0.60	12.62489	-0.01221	12.63674	-0.03422
							0.78	11.85434	-0.01197	11.86505	-0.03257
							1.002	11.20172	-0.01174	11.21147	-0.03114
7.0	2.0	4.0	0.4	0.3	15	1	100.0	0.30065	-0.00137	0.30007	-0.00116
							617.0	0.28473	-0.00123	0.28423	-0.00113
7.0	2.0	4.0	0.4	0.3	30	1	100.0	0.30054	-0.00104	0.30018	-0.00098
							617.0	0.28461	-0.00093	0.28434	-0.00093
7.0	2.0	4.0	0.8	0.3	15	1	100.0	0.29438	-0.00137	0.2938	-0.00115
							617.0	0.27851	-0.00123	0.27801	-0.00112
7.0	2.0	4.0	0.8	0.6	15	1	100.0	0.29438	-0.00137	0.2938	-0.00115
							617.0	0.27851	-0.00123	0.27801	-0.00112
7.0	2.0	8.0	0.8	0.6	15	1	100.0	0.33409	-0.00174	0.3334	-0.00118
							617.0	0.30237	-0.00146	0.30181	-0.00113
7.0	4.0	4.0	0.8	0.6	15	1	100.0	0.54904	-0.00236	0.54801	-0.00225
							617.0	0.53318	-0.00222	0.53221	-0.00223

Table 1: Values of skin friction

P_r	G_r	G_c	M	R_0	ω	S_c	A=0.0		A=1.0	
							$\lvert Q\rvert$	tan γ	$\lvert Q\rvert$	tan γ
0.71	2.0	2.0	0.3	0.4	15	0.22	3.5249	0.857151	3.5249	0.79031
						0.60	3.5249	0.857151	3.5249	0.79031
						0.78	3.5249	0.857151	3.5249	0.79031
0.71	2.0	2.0	0.3	0.4	30	0.22	4.87343	0.896831	4.87343	0.859552
0.71	2.0	2.0	0.3	0.4	30	0.60	4.87343	0.896831	4.87343	0.859552
0.71	2.0	2.0	0.6	0.4	30	0.60	4.87343	0.896831	4.87343	0.859552
0.71	2.0	5.0	0.3	0.4	30	0.60	4.87343	0.896831	4.87343	0.859552
0.71	5.0	2.0	0.3	0.4	30	0.60	4.87343	0.896831	4.87343	0.859552
7.00	2.0	2.0	0.3	0.4	15	100.0	13.10426	0.611455	13.10426	0.33997
						617.0	13.10426	0.611455	13.10426	0.33997
7.00	2.0	2.0	0.3	0.4	30	100.0	17.21794	0.708365	17.21794	0.507415
						617.0	17.21794	0.708365	17.21794	0.507415

Table 2: The values of $\lvert Q\rvert$, the rate of heat transfer and tan □, the phase of rate heat transfer

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