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Predator - Prey Model of Security Forces Versus Criminals in a Contemporary Ghanaian Community

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Abstract:
A Predator prey model is developed from the population dynamics of security forces versus criminals in a contemporary community.Using five major crime data covering the period of 2000 to 2010 from Ghana Statistical Service, two models were developed and a local stability analysis of the model after determination of the equilibrium points was investigated. With some assumptions, the parameters of the model are estimated and the simulation of the model for various scenarios using MATLAB is done.It was observed that, by analysing the same point in sequential phases and finding the time in between them the approximate periods of the phase plane cycle is found to be 12 months. The results obtained suggest that criminal activities persist if not the introduction of voluntary reinforcement.

Keywords: *Predator - Prey, Stability, Asymptotically stable, Unstable*

1. Introduction

The word crime, from the root of latin $cer\bar{n}o$ means 'I decide, I give judgement'.

According to Fafa (2010), crime is the act of breaking rule(s) or regulation(s) for which a governing authority (via mechanisms such as legal system) can ultimately prescribe a conviction.

Crime has also been defined in social or non-legal terms. The social definition of crime is that it is a behaviour or an activity that offends the social code of a particular community. Mower (1959) has defined it as 'an anti-social act'.

All crimes either major or minor have negative effects both on individuals and societies or the nation at large. These negative effects range from destruction and lost of property, lost of innocent lives, fear and panic, security threat, budget constraints and many more.

The security forces include the Police service, the Bureau of National Investigations (BNI) and the Armed Forces. The GPS (Ghana Police Service) under the ministry of interior is responsible for maintaining law and order.

The military continued to participate in law enforcement activities. The Bureau of national investigations handled cases considered critical to the state security and answered directly to the ministry of national security. The police maintained specialized units in Accra for homicide, forensics, domestic violence, trafficking in persons, visa fraud, narcotics, and cybercrimes. Jane's, Sentinel Country Risk Assessment - Ghana, Security and Foreign Forces, updated 7 December 2011, further observed that the, Police-associated departments of the interior ministry include ... the Criminal Investigations Department (CID); ... Narcotics Control Board (NCB); ... Immigration Service; and Customs and Excise Service, in addition Jane explained that Ghana's Customs and Excise Service operates as part of the Police Service, and that border checkpoints are manned by the Immigration Service (under which there is a Border Patrol Unit) and the Customs and Excise Service. The army also conducts limited border security patrol. Several interventions have been put in place in preventing crime in the country. Some were time-based or discrete whilst others were continuous or permanent interventions. Some specific interventions put in place in Ghana in recent times to combat crime includes the following; Police partner transport owners in crime combat, Xmas crime combat, Government ordering assemblies to name streets, Ghana and Togo police join forces to combat crime, police launched 'operation calm life', crime education and the establishment of community policing service. According to statistical service, the level of criminal activities in the past 3 years has increased to 75 percent. Although all these attempts are good, criminal activities are still in the increase, there is therefore the need to come out with a model which will help explain, predict and further curtail the propagation of criminals by deploying security forces at post accordingly.

2. Methodology

A predator - prey model of population dynamics of security forces versus criminals was formulated as a system of differential equations and the equilibrium points determined, the stability of the equilibrium points was also determined. Simulation using MATLAB was done. Graphs were plotted to show the trend of the incidence of security forces versus criminals.

3. Model Derivation

3.1. Model A

We let

$H(t)$ be the number of criminals at time t

$P(t)$ be the number of security forces at time t

N be the population at time t

Then;

$$H(t) + P(t) = N \qquad (1.0)$$

equation (1.0) represent the security forces and criminals cases at time t

3.1.1. Linear Term

In the absence of security forces, criminals tend to increase exponentially to their current population

$$\frac{dH}{dt} = \mu H\,,\ \mu > 0\text{, when P = 0} \qquad (1.2)$$

In the absence of criminals the security forces population tends to decrease exponentially

$$\frac{dP}{dt} = -\beta P\,,\ \beta > 0\text{, when H = 0} \qquad (1.3)$$

3.1.2. Non - Linear Term

Are obtained through the mass action law, that is, as the security forces grows in number the interaction rate between the security forces and criminals also increases.

$$\frac{dP}{dt} = \gamma HP \text{ where } \gamma > 0 \qquad (1.4)$$

Increase in the security forces in a criminal prone community will reduce the activity of the criminals.

$$\frac{dH}{dt} = -\alpha HP \qquad (1.5)$$

Combining the linear and non-linear parts of the model, we have

$$\frac{dH}{dt} = \mu H - \alpha HP$$
$$\frac{dP}{dt} = -\beta P + \gamma HP \qquad (1.6)$$

From our equation (1.0) we have,

$$H(t) + P(t) = N$$
$$P(t) = N - H$$
$$H(t) = N - P$$

substituting P(t) and H(t) into equation (1.6)

$$\frac{d(N-P)}{dt} = \mu(N-P) - \alpha(N-P)(N-H)$$
$$\frac{d(N-H)}{dt} = -\beta(N-H) + \gamma(N-P)(N-H) \quad (1.7)$$

The constants μ, α, β and γ are all positive where μ and γ are the growth rate constants and α and β are measures of effect of their interactions.

3.2. Model B

Model B we consider here is a slight modification of model A, where we introduce two more parameters (i)The recruitment rate of volunteer guards and (ii) The rate at which the volunteer guards come into contact with the criminals.

$$\frac{dH}{dt} = \mu H - \alpha HP - \tau u_1 H \quad (3.1)$$
$$\frac{dP}{dt} = -\beta P + \gamma HP + \tau u_1 H \quad (3.2)$$

4. Analysis of the Model

Parameter	Parameter values
μ	0.4
β	0.9
α	0.001
γ	0.001
τ	1×10^{-4}
u_1	0.03
H_0	600
P_0	400
N_0	1000

Table 1: Parameters and their definitions

Taking the first linear term equation (1.2)

$$\frac{dH}{dt} = \mu H$$

Equation (1.2) models the criminals that are not affected by the security forces, hence growing exponentially according to a rule of the form

$$H(t) = Ce^{\mu t}$$

Where C is a positive constant representing the criminals population when $t = 0$, $H = C$

For the second linear term, equation (1.3)

$$\frac{dP}{dt} = -\beta P$$

Equation (1.3) represents a declining population of security forces in the community exponentially due to the absence of criminals given by

$$P(t) = De^{-\beta t}$$

where D is a positive constant representing initial security forces population.

However for interaction populations where encounters are unavoidable we assume that the number of encounters between security forces and criminals is proportional both to the population of P security forces and the population of H criminals in the community.

The growth rate of criminals decreases by a factor proportional to the number of encounters between security forces and the criminals that is by a factor HP.

We revise our first model to include this extra term

$$\frac{dH}{dt} = \mu H - \alpha HP \tag{3.3}$$

for some positive constant α

From equation (3.8) we let $\alpha = \frac{\mu}{K}$, where $K > 0$, we now have

$$\frac{dH}{dt} = \dot{H} = \mu H(1 - \frac{P}{K}) \tag{3.4}$$

which is a non-linear term.
Similarly for a population of security forces in a criminal free environment we have from equation (1.3)

$$\frac{dP}{dt} = -\beta P$$

where β is a positive constant. This represents exponential decay. However if there is a population H criminals for the security forces to overpower, we expect the growth rate $\frac{dP}{dt}$ of the security forces to increase and a simple assumption is that the growth rate $\frac{dP}{dt}$ of security forces increases by a factor proportional to the number of encounters between the criminals and the security forces. Our revised model for the security forces is given by

$$\frac{dP}{dt} = \dot{P} = -\beta P + \gamma HP \tag{3.5}$$

for some positive constant γ.

Again it is convenient to let $\gamma = \frac{\beta}{Q}$, where $Q > 0$, thus ,

$$\frac{dP}{dt} = \dot{P} = -\beta P(1 - \frac{H}{Q}) \tag{3.6}$$

which is also a non - linear term
We now have our two equations as

$$\frac{dH}{dt} = \dot{H} = \mu H(1 - \frac{P}{K}) \tag{3.7}$$

$$\frac{dP}{dt} = \dot{P} = -\beta P(1 - \frac{H}{Q}) \tag{3.8}$$

Which is in the form of Lotka - Volterra equations where $P \geq 0$ and $H \geq 0$

Rearranging equation (3.7), we have $\frac{\dot{H}}{H} = \mu - \frac{\mu}{K}P$.

The graph of the proportionate growth rate $\frac{\dot{H}}{H}$ of criminals as a function of the population P of security forces is given by

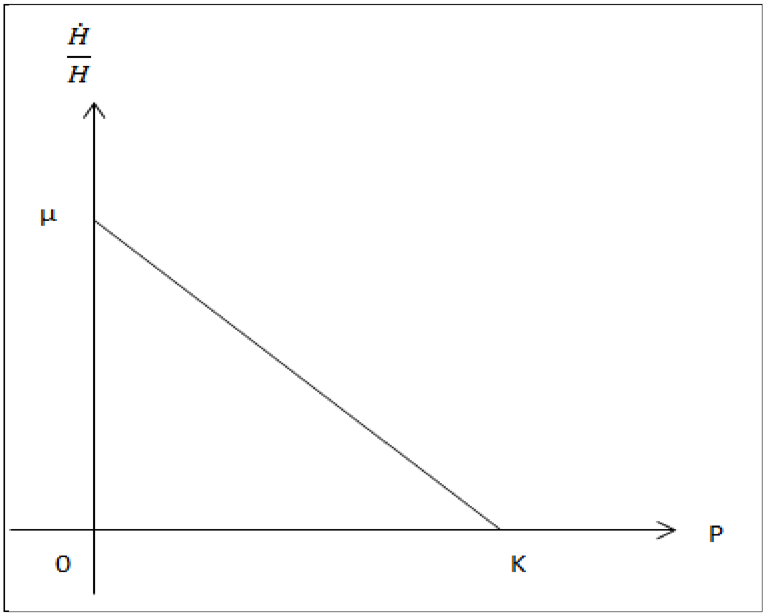

Figure 1: Proportionate growth rate of criminals as a function of the population of security forces

The proportionate growth rate $\frac{\dot{H}}{H}$ of the criminals decreases as the population P of security forces increases, becomes zero when $P = K$. The population H of criminals will increase if the population P of the security forces is less than K but will decline if $P > K$

Similarly, from equation (3.10) we rearrange to have

$$\frac{\dot{P}}{P} = -\beta + \frac{\beta H}{Q}$$

and the graph $\frac{\dot{P}}{P}$ of the security forces as a function of the population H of criminals is given by figure below

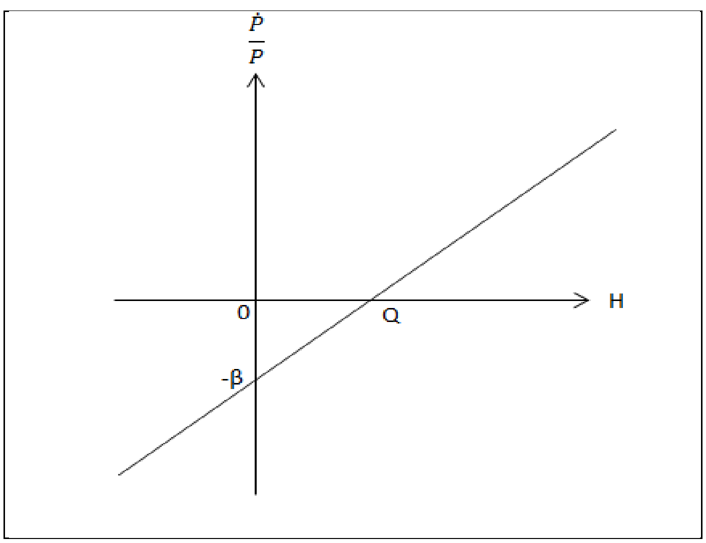

Figure 2: Proportionate growth rate of the security forces as a function of the population of criminals

The proportionate growth rate $\frac{\dot{P}}{P}$ of security forces increases linearly as the population H of criminal's increases. The security forces population will decrease if the population of the criminals is less than Q, but will increase if H is greater than Q

4.1. The Equilibrium Point

We equate equation 1.6 to zero to have,

$$\mu H(1-\frac{P}{K})=0 \quad (4.1)$$

$$-\beta P(1-\frac{H}{Q})=0 \quad (4.2)$$

Hence (0,0) and (Q,K) are the two equilibrium points.

4.1.1. Stability of Equilibrium Points

We find the Jacobian matrix on equation 1.6 to have

$$J(H,P)=\begin{pmatrix} \mu(1-\frac{P}{K}) & -\frac{\mu H}{K} \\ \frac{\beta P}{Q} & -\beta(1-\frac{H}{Q}) \end{pmatrix} \quad (4.3)$$

At the critical point (0,0)

$\lambda_1=\mu$ and $\lambda_2=-\beta$

$\lambda_1=0.4$ and $\lambda_2=-0.9$.

The equilibrium point is a saddle point, therefore unstable, that is if a perturbation results in a catastrophe change with the population of security forces or criminals collapsing to zero or increasing without limit, we say that the equilibrium point is unstable

For the second critical point (Q, K)

$$\lambda^2+\mu\beta=0, \Rightarrow \lambda=\pm i\sqrt{\mu\beta}$$

$$\lambda_{1,2}=\pm i\sqrt{0.36}$$

$$\lambda_{1,2}=\pm 0.6i,$$

hence the steady state is purely a centre and therefore stable. From the analysis of this we arrive at a number of results.

The criminals (prey) depends on parameter associated with the security forces (predators) $P=K$. A similar result holds for steady state levels of security forces (predators) $H=Q$, it is the particular coupling of the variables that leads to this effect. To paraphrase, the presence of security forces (predator) $P\neq 0$, means that the available criminals (prey) has to just suffice to make growth rate due to predation, $\frac{\beta H}{Q}$ equal security forces (predator) rate β for a steady predator population to persists.

similarly, when criminals (prey) are present ($H\neq 0$) security forces (predators) can only keep them under control when prey growth rate μ and predation rate $\frac{\mu}{K}$ are equal.

4.1.2. Model B

4.1.3. Equilibrium Point

we equate equation (14) and (15) to zero to have;

$$\frac{dH}{dt}=0$$

$$\frac{dP}{dt}=0$$

$$\frac{dH}{dt}=\mu H-\alpha HP-\tau u_1 H=0 \quad (4.3)$$

$$\frac{dP}{dt}=-\beta P+\gamma HP+\tau u_1 H=0$$ (4.4) The equilibrium points are

$$(0,0) \text{ and } (\frac{\mu-\tau u_1}{\alpha},\frac{\beta\mu-\beta\tau u_1}{\gamma\mu-\gamma\tau u_1+\tau u_1\alpha})$$

4.1.4. Stability of Equilibrium Points

Taking the Jacobian matrix on (4.3) and (4.4) we have,

$$J(H,P)=\begin{pmatrix}\mu-\alpha P-\tau u_1 & -\alpha H\\ \gamma P+\tau u_1 & -\beta+\gamma H\end{pmatrix}$$

At the first critical point (0,0) we have;

$$((\mu-\tau u_1)-\lambda)(-\beta-\lambda)=0$$

$$\lambda_1=\mu-\tau u_1 \text{ and } \lambda_2=-\beta$$
$$\lambda_1=0.4,\quad \lambda_2=-0.9$$

The critical point is a saddle point, therefore unstable.

For the second equilibrium point $(\frac{\mu H-\tau u_1 H}{\alpha H},\frac{\beta\mu-\beta\tau u_1}{\gamma\mu-\gamma\tau u_1+\tau u_1\alpha})$, we have

$$J(\frac{\mu H-\tau u_1 H}{\alpha H},\frac{\beta\mu-\beta\tau u_1}{\gamma\mu-\gamma\tau u_1+\tau u_1\alpha})=\begin{pmatrix}\frac{\mu(\gamma\mu-\gamma\tau u_1+\alpha)-(\alpha\beta\mu+\alpha\beta\tau u_1)-\tau u_1(\gamma\mu-\gamma\tau u_1\alpha)}{\gamma\mu-\gamma\tau u_1+\tau u_1\alpha} & \frac{-\alpha\mu H+\tau\alpha H u_1}{\alpha H}\\ \frac{\gamma\beta\mu-\gamma\beta\tau u_1+\tau u_1+\tau u_1\gamma\mu-\tau^2u_1^2\gamma+\tau^2u_1^2\alpha}{\gamma\mu-\gamma\tau_1+\tau u_1\alpha} & \frac{-\beta\alpha H+\gamma\mu H-\gamma\tau H u_1}{\alpha H}\end{pmatrix}$$

$$J(\frac{\mu H-\tau u_1 H}{\alpha H},\frac{\beta\mu-\beta\tau u_1}{\gamma\mu-\gamma\tau u_1+\tau u_1\alpha})=\begin{pmatrix}a & b\\ c & d\end{pmatrix}$$

$$det\,|J-\lambda I|=\begin{pmatrix}a-\lambda & b\\ c & d-\lambda\end{pmatrix}$$

$$(a-\lambda)(d-\lambda)-bc=0$$
$$ad-a\lambda-d\lambda+\lambda^2-bc=0$$
$$\lambda^2-\lambda(a+d)+(ad-bc)=0$$

Trace = $a+d$ Det= $ad-bc$

Trace $=-3.5\times10^{-8}$

det $=3.0\times10^{-7}$

Since $T<0$ and $D>0$ we have a spiral sink, hence asymptotically stable

5. Numerical Analysis

By using the Generalized Euler method (GEM), we obtained the numerical solution of the systems

For the Criminal's we have;

$$t_{i+1}=t_i+h$$

$$H(t_{i+1})=H(t_i)+h^1[\mu H(t_i)(1-\frac{P(t_i)}{K})] \quad (4.5)$$

$$H(t_{i+1})=H(t_i)+h^1[\mu H(t_i)-\alpha H(t_i)P(t_i)-\tau u_1 H(t_i)] \quad (4.6)$$

for $i=0,1,...,z-1$

For the Security forces we have;

$$P(t_{i+1})=P(t_i)+h^1[-\beta P(t_i)(1-\frac{H(t_i)}{Q})] \quad (4.7)$$

$$P(t_{i+1})=P(t_i)+h^1[-\beta P(t_i)+\gamma H(t_i)P(t_i)+\tau u_1 H(t_i)] \quad (4.8)$$

for $i=0,1,...,z-1$

6. Graphs

6.1. Model A

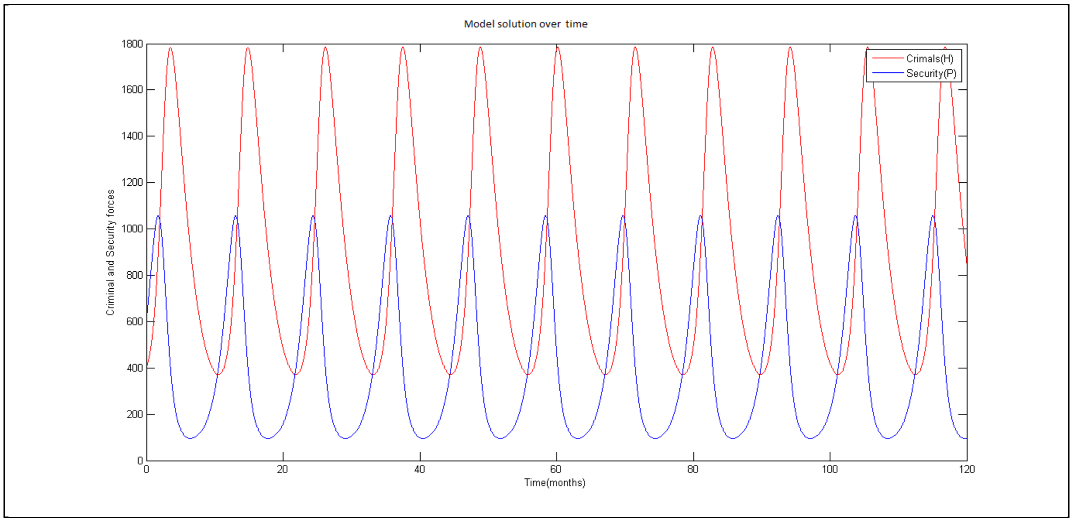

Figure 3: trajectories of model a

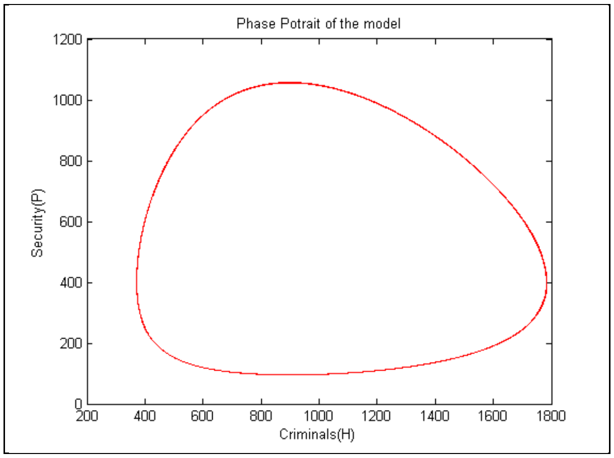

Figure 4: phase portrait of model a

6.2. Model B

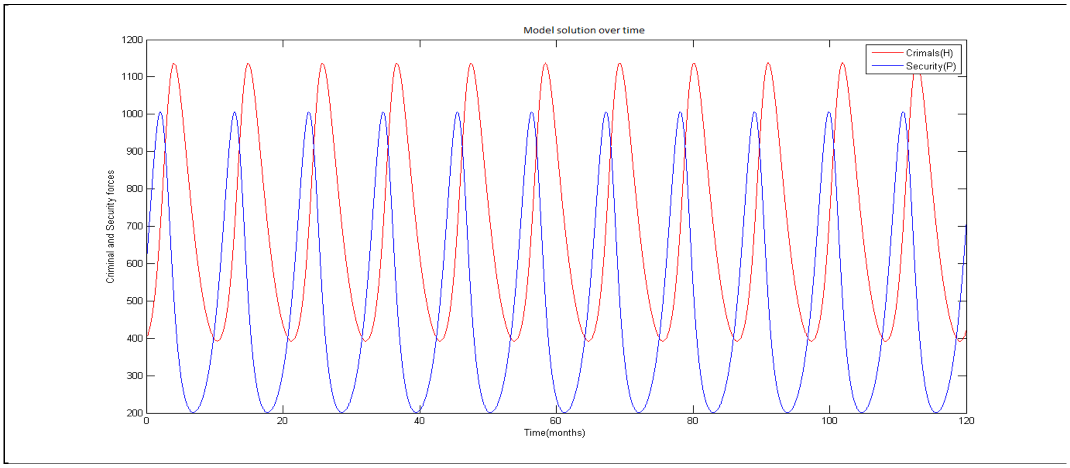

Figure 5: trajectories 0f model b

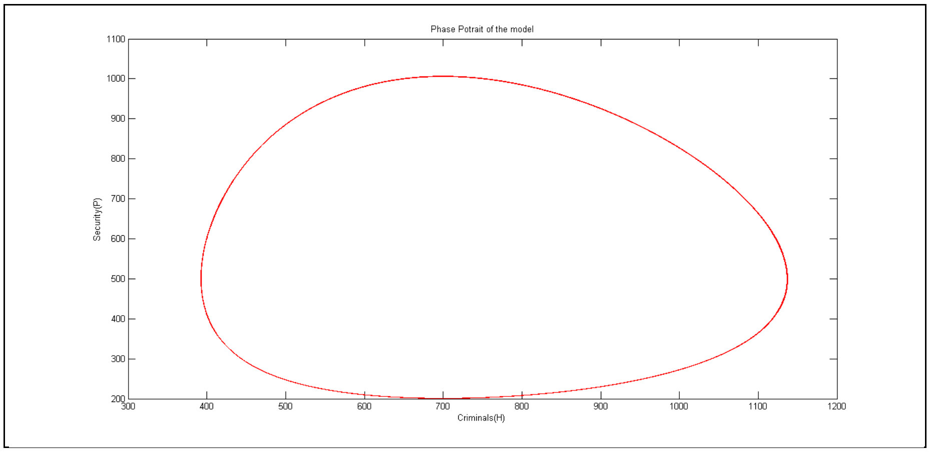

Figure 6: phase portrait of model b

For the model solution over time from model A and model B it was observed that as time progresses in years, security forces population and criminals population clearly fluctuate at cyclic interval. We notice that as the criminal population peaks, security forces population begins to rise rapidly, yet as the security forces population begins to rise the criminals population falls rapidly.
The phase portrait graph is to show the cyclic fluctuations of the security forces versus criminals with respect to each other without showing the change in time.

7. Conclusion

1. The analysis shows that the two models conform to the predator - prey model.
2. The numerical results confirmed that the introduction of volunteer guards helps the security forces in apprehending criminals at a faster rate.
3. Thirdly the predator - prey cycle chart depicts population (in thousands versus time in years) of both criminals and security forces. This is quite useful in order to visualize the population fluctuation of criminals and security forces with respect to time. The average time of the periodic oscillation can be determined graphically in this way and general population variation characteristics can be determined. Notice that by analysing the same point in sequential phases and finding the time in between them the periodic oscillation for the criminals and security forces is 12 months.

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