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Generalization of Convolution Based Hankel Transform

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Abstract
The object of the present paper is to generalize the Fourier solution of the Convolution Hankel based transform. There are some novel functions that form special cases of the generalized function. In the present paper, we shall give the solution of convolution equations involving two of such novel functions as kernels.

Keywords and Phrases: *Convolution integral equation, Generalized Hankel Transform, Laplace transform*

1. Introduction

Generalized convolution of f and g under three operator K,K_1 ,K_2 , and with some weight function γ is a function denoted by the symbol f * g, such that the following factorization propert holds:

$K(f * g) = \gamma$ (x) (k_1f) (x)(k_2g)(x)…………………………..(.1.1)

If k= K_1 = K_2 =F_H (the Fourier Hankel transform)

F_H(x) = $\sqrt{\frac{2}{\pi}} \int_0^\infty f(y) \cos xy \, dy$ …….(1.2)

The convolution has the form

$(f * g)(x) = \sqrt{\frac{1}{2\pi}} \int_0^\infty f(y)$ [g (!x-y!)+ g(x+y)]dy ……(1.3)

And the property (1.1) hold

F_H $(f * g)$ (x) = F_Hf(x) F_Hg(x) ……..(1.4)

Otherwise, then it there appear convolution.

An example of generalized convolution was first introduced by Churchill

$(f * g)(x) = \sqrt{\frac{1}{2\pi}} \int_0^\infty f(y)$ [g(!x-y!)- g(x+y)]dy …..(1.5)

And the respective factorization property (1.1) and (1.5) has the form

$F_S(f * g)(x) = (F_S f)(x) F_H$ g(x) …(1.6)

Where F_S is the Fourier sine transform?

$(F_S f)(x) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(y) \sin xy \, dy$ …….(1.7)

Now we shall proof the main theorems, which are completely based on above results:

1.1. Theorem1

Let f, g$\in L(R_+)$, then the convolution $f * g \in L(R_+)$,

And

$F_s(f * g)(x) = (F_s f)(x) F_H \quad g(x) \quad , x\in L(R_+)$, …… (2.2)

Proof. We have

$\int_0^\infty |(f * g)(x)|\, dx \le \sqrt{\frac{1}{2\pi}} \int_0^\infty \int_0^\infty |f(y)|(|g(|x-y|)| + |g(x+y)|)dxdy$

$\le \sqrt{\frac{1}{2\pi}} \int_0^\infty |f(y)|\, dy \left[\int_{-y}^\infty |g(|x|)|dx + \int_y^\infty |g(|x|)|dx\right]$

$= \sqrt{\frac{1}{2\pi}} \int_0^\infty |f(y)|dy \int_0^\infty |g(x)|\, dx < \infty$.

Hence, the convolution (2.2) $\in L(R_+)$.

Furthermore

$(F_s f)(x) F_H \quad g(x)$

$= \frac{2}{\pi} \int_0^\infty \int_0^\infty \text{sinxusinxuf(u)g(v)dudv}$

$= \frac{1}{\pi} \int_0^\infty \int_0^\infty \cos(u - v) f(u) g(v) dudv - \frac{1}{\pi} \int_0^\infty \int_0^\infty \cos(u + v) f(u) g(v) dudv$

$= \frac{1}{\pi} \int_0^\infty \int_0^\infty cosxt\, [f(y)g(t+y) + f(y+t)g(y)]dydt$

$- \frac{1}{\pi} \int_0^\infty \int_0^\infty cosxt\, f(y)g(t-y)dydt$

$= \frac{1}{\pi} \int_0^\infty cosxt\, [\int_0^\infty f(y)g(y+t)dy \int_t^\infty f(y)g(y-t)dy \int_0^t f(y)g(t-y)dy]dt$

$= \frac{1}{\pi} \int_0^\infty cosxt\, [\int_0^\infty f(y)g(y+t)dy + \int_0^\infty \sin(y-t) f(y) g(|y-t|)dy]dt$

$= (f * g)(x)$. Proved.

1.2. Theorem 2

Let the function f, g, h belong to $L(R_+)$,

Then the following formulas hold

(f * g) * h = (f * h) * g = f * (g * h) …..(2.5)

f * (g * h) = g * (h * f) = h * (g * f) …..(2.6)

f * (g * h) = g * (f * h) = h * (f * g) …..(2.7)

f*(g * h) = g * (f * h) = h * (f * g) ……(2.7)

the proof follows easily from the formulas (1.4) ,(1.6) and (1.2).

we have

$F_s[(f * g) * h] = F_s[(f * g) F_H \quad (h) = F_H\, (f) \quad (g) F_H \quad (h)$

$= [F_s(f)\; F_c(h)\; F_c(g)] = F_s[(f * g) F_H \quad (g)$

$= F_s[(f * g) * h]$.

Hence,

(f * g)* h = (f * h)* g.

On other hand, $F_s[(f * g) * h] = F_H\, (f) \quad F_H \quad (g) F_H \quad (h)$

$= F_H \quad (g * h) = F_s$ [(f * g) * h] ,therefore

(f * g)* h = (f * h)* g , and the formula (2.5) is proved. By the same way, we can verify the other.

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