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On Extended Generalised ϕ-Reccurent Para Sasakian Manifold

N. V. C. Shukla
Assistant Professor, Departmant of Mathematics and Astronomy, Lucknow University, Lucknow, India
Anurag Dixit
Research Scholar, Departmant of Mathematics and Astronomy, Lucknow University, Lucknow, India

Abstract:
The purpose of the present paper is to study the notion of extended generalized ϕ-recurrency to para Sasakian manifold. Some geometric properties with the existence have been studied. Among the result established here. It is shown that an extended generalized ϕ-recurrent para Sasakian manifold is an Einstein manifold. Further we study extended generalized ϕ-recurrent Para Sasakian manifold and obtain some results which reveal the nature of its associated 1-forms.

1. Introduction

The notion of locally symmetry of a Riemannian mani fold has been weakened by many authors in several directions such as recurrent manifolds by Walker [1], semi-symmetric manifold by Soapy[2], pseudo symmetric manifold by Chaki [3], pseudo-symmetric manifold by Deszcz [4], weakly symmetric manifold by Tamassy and Binh [5], weakly symmetric manifold by Selberg [6]. As a weaker version of locally symmetry, in 1977 Takahashi [7] introduced the notion of local ϕ -symmetry on a Sasakian manifold.By extending this notion, De et al. [8] introduced and studied the notion of ϕ-recurrent Sasakian manifolds. The notion of generalized recurrent manifolds was introduced by Dubey[9] and then studied by De and Guha[10]. A Riemannian manifold $(M^n, g), n > 2$, is called generalized ϕ- recurrent if its curvature tensor R satisfies the condition

$$\nabla R = A \otimes R + B \otimes G \qquad (1.1)$$

Where A and B are two non-vanishing 1-forms defined by $A(\circ) = g(\circ, \rho_1)$, $B(\circ) = g(\circ, \rho_2)$ and the tensor G is defined by

$$G(X,Y)Z = g(Y,Z)X - g(X,Z)Y \qquad (1.2)$$

for all $X, Y, Z \in T(M), T(M)$ being the Lie algebra of smooth vector fields and $\tilde{\nabla}$ denotes the covariant differentiation with respect to the metric g. Here ρ_1, ρ_2 are vector fields associated with 1-forms A and B respectively. Especially, if the 1 -form B vanishes, then $(1,1)$ turns into the notion of recurrent manifold introduced by Walker [1].

[2010] 53C15, 53A25 Generalized ϕ-recurrent para Sasakian manifold, Generalized recurrent para Sasakian manifold, extended generalized ϕ-recurrent para Sasakian manifold, Einstein manifold, T -curvature tensor and Extended T-ϕ-recurrent para Sasakian manifold.

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A Riemannian manifold (M^n, g) is called a generalized Ricci-recurrent [11] if its Ricci tensor S of type $(0,2)$ satisfies the condition

$$\nabla S = A \otimes S + B \otimes G \qquad (1.3)$$

Where A and B are defined in (1.1). In particular, if $B = 0$, then (1.3) reduces to the notion of Ricci-recurrent manifolds introduced by Patterson [12] .

In 2007, Ozgur [13] studied generalized recurrent Kenmotsu manifold. Generalizing this notion recently, Basari and Murathan [15] introduced the notion of generalized ϕ-recurrency to Kenmotsu manifolds. Also, the notion of generalized ϕ-recurrency to para Sasakian manifolds and Lorentzian α -Sasakian manifolds are respectively studied in [15,16]. By extending the notion of generalized ϕ -recurrency, Shaikh and Hui [17] introduced the notion of extended generalized ϕ -recurrency to β-Kenmotsu manifolds.

2. Preliminaries

A $(2n+1)$-dimensional smooth manifold M is said to be an almost contact metric manifold [14] if it admits an $(1,1)$-tensor field ϕ, a vector field ξ, a 1-form η and a Riemannian metric g, which satisfy

$$(a)\ \phi.\xi = 0 \quad (b)\ \eta(\phi X) = 0 \quad (c)\ \phi^2 = X - \eta \otimes \xi, \tag{2.1}$$

$$(a)\ g(\phi X, Y) = g(X, \phi Y)\ (b) \eta(X) = g(X, \xi)\ \ (c)\ \eta(\xi) = 1, \tag{2.2}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y). \tag{2.3}$$

$\forall\ X, Y \in \chi(M)$

An almost contact metric manifold is said to be para Sasakian manifold if the following conditions hold [15]

$$(\nabla_X \phi)Y = g(X, Y)\xi - \eta(Y)X, \tag{2.4}$$

$$\nabla_X \xi = \phi X. \tag{2.5}$$

In a Para Sasakian manifold $M^{2n+1}(\phi, \xi, \eta, g)$ the following properties hold

$$(\nabla_X \eta)Y = g(X, \phi Y), \tag{2.6}$$

$$R(X, Y)\xi = \eta(Y)X - \eta(X)Y, \tag{2.7}$$

$$R(\xi, X)Y = (\nabla_X \phi)Y, \tag{2.8}$$

$$S(X, \xi) = -2n\eta(X), \tag{2.9}$$

$$S(\phi X, \phi Y) = S(X, Y) - 2n\eta(X)\eta(Y), \tag{2.10}$$

$$(\nabla_W R)(X, Y)\xi = g(\phi X, W)Y - g(\phi Y, W)X + R(X, Y)\phi W. \tag{2.11}$$

For any vector field $X, Y, Z \in \chi(M)$

3. Extended generalized $\phi-$ recurrent para Sasakian manifolds

- Definition 1 .A para Sasakian manifold $M^{2n+1}(\phi, \xi, \eta, g), n \geq 1$ is said to be an extended generalized $\phi-$recurrent para Sasakian manifold if its curvature tensor R satisfies the relation

$$\phi^2((\nabla_W R)X, Y)Z = A(W)\phi^2(R(X, Y)Z) + B(W)\phi^2(G(X, Y)Z) \tag{3.1}$$

$\forall\ X, Y, Z, W \in \chi(M)$ where A and B are two non-vanishing 1 -forms such that and $A(X) = g(X, \rho_1), B(X) = g(X, \rho_2)$ .Here ρ_1, ρ_2 are vector fields associated with 1-forms A and B respectively.

Now we begin with the following

- Theorem 1. An extended generalized $\phi-$ recurrent para Sasakian manifold

$M^{2n+1}(\phi, \xi, \eta, g), n \geq 1$ *is generalized Ricci recurrent if and only if the associated 1-form* A *and* B *are identically equal.*

- Proof.Let us consider an extended generalized $\phi-$recurrent Para Sasakian manifold. Then by virtue of (2.1), we have form (3.1) that

$$\begin{aligned}(\nabla_W R)(X, Y)Z - \eta(\nabla_W R)(X, Y)Z)\xi &= A(W)[R(X, Y)Z - \eta(R(X, Y)Z)\xi] \\ &+ B(W)[G(X, Y)Z - \eta(G(X, Y)Z)\xi\end{aligned} \tag{3.2}$$

$$\begin{aligned}g((\nabla_W R)(X, Y)Z, U) - \eta((\nabla_W R(X, Y)Z\eta(U) &= A(W)[R(X, Y)Z, U) - \eta((R(X, Y)Z\eta(U)] \\ &+ B(W)[g(G(X, Y)Z, U) - \eta(G(X, Y)Z)\eta(U)]\end{aligned} \tag{3.3}$$

Let $\{e_i : i = 1, 2, 3....2n+1\}$ be an orthonormal basis of the tangent space at any point of the manifold. Setting $X = U = e_i$ in (3.3) and taking summation over i, $1 \leq i \leq 2n+1$ and using (1.2) ,we get

$$\begin{aligned}(\nabla_W S)(Y, Z) - g((\nabla_W R)(\xi, Y)Z, \xi) &= A(W)[S(Y, Z) - \eta(R(\xi, Y)Z] \\ &+ B(W)[(2n-1)g(Y, Z) + \eta(Y)\eta(Z)\end{aligned} \tag{3.4}$$

Using (2.7) and (2.11) and the relation $g(\tilde{\nabla}_W R)(X,Y)Z,U) = -g((\tilde{\nabla}_W R(X,Y)U,Z)$ We have

$$g((\nabla_W R)(\xi,Y)Z,\xi) = 0 \tag{3.5}$$

By virtue of (2.8) and (3.5) ,it follows from (3.4) that

$$(\nabla_W S)(Y,Z) = A(W)S(Y,Z) + [(2n-1)B(W) + A(W)]g(Y,Z) + [B(W) - A(W)]\eta(Y)\eta(Z) \tag{3.6}$$

If $A(W) = B(W)$ that is,associated forms are identically equal then (3.6) reduces to

$$\nabla S = A \otimes S + \psi \otimes g \tag{3.7}$$

Where $\psi(W) = 2n\beta$ for all $W \in \chi(M)$

This completes the proof.

- Theorem 2.An extended generalized $\phi-$ recurrent para Sasakian manifold $M^{2n+1}(\phi,\xi,\eta,g), n \geq 1$ is an Einstein manifold and moreover the associated 1-form A and B are related by $A(W) = B(W)$.
- Proof.Setting $Z = \xi$ in (3.6) and using $(2.2(b))$ and (2.9) ,we obtain

$$(\nabla_W S)(Y,\xi) = 2n\{B(W) - A(W)\}\eta(Y) \tag{3.8}$$

Also we have

$$(\nabla_W S)(Y,\xi) = (\nabla_W S)(Y,\xi) - S(\nabla_W Y,\xi) - S(Y,\nabla_W \xi) \tag{3.9}$$

Using (2.6) and (2.9) in (3.9) , it follows that

$$(\nabla_W S)(Y,\xi) = -2ng(Y,\phi W) - S(Y,\phi W) \tag{3.10}$$

By (3.8) and (3.10) we have

$$-2ng(Y,\phi W) - S(Y,\phi W) = 2n\eta(Y)[B(W) - A(W)] \tag{3.11}$$

Again Setting $Y = \xi$ in (3.11) we get

$$A(W) = B(W) \qquad \text{forall} W \tag{3.12}$$

Taking account of (3.12) in (3.11)

$$S(\phi W,Y) = -2ng(\phi W,Y) \tag{3.13}$$

Substituting Y by ϕY in (3.13) and using (2.3) and (2.10) we have

$$S(W,Y) = -2ng(W,Y) \tag{3.14}$$

From (3.12) and (3.14) the theorem follows.

- Theorem3 .In extended generalized $\phi-$ recurrent para Sasakian ,manifold

$M^{2n+1}(\phi,\xi,\eta,g)$, $\dfrac{r+2n(2n-1)}{2}$ *is an eigen value of the Ricci tensor* S *corresponding to the eigen vector* ρ_1 .

- Proof. Changing W, X, Y cyclically in (3.3) and adding them , we get by virtue of Bianchi identity an (3.12) that

$$A(W)[\{g(R(X,Y)Z,U) + g(G(X,Y)Z,U)\} - \{\eta(R(X,Y)Z) + \eta(G(X,Y)Z)\}\eta(U)] \tag{3.15}$$
$$-A(X)[\{g(R(Y,W)Z,U) + g(G(Y,W)Z,U)\} - \{\eta(R(Y,W)Z) + \eta(G(Y,W)Z)\}\eta(U)]$$
$$-A(Y)[\{g(R(W,X)Z,U) + g(G(W,X)Z,U)\} - \{\eta(R(W,X)Z) + \eta(G(W,X)Z)\}\eta(U)]$$

Replacing $Y = Z = e_i$ in (3.15) and taking summation over i ,$1 \leq i \leq 2n+1$, we get

$$A(W)[S(X,U) - 2ng(X,U)] - A(X)[S(U,W) - 2ng(U,W)] - A(R(W,X)U) \tag{3.16}$$
$$-A(R(W,X)\xi)\eta(U) - A(X)g(W,U) + A(W)g(X,U) - \{A(X)\eta(W) - A(W)\eta(X)\} = 0$$

Again putting $X = U = e_i$ in above relation and taking summation over i $1 \leq i \leq 2n+1$,we have

$$S(W,\rho_1) = \frac{r+2n(2n-1)}{2}A(W)$$

This proves the theorem.

4. Extended generalized T - ϕ -recurrent para Sasakianmanifolds

In a $(2n+1)$ -dimensional Riemannian manifold M^{2n+1}, the T-curvature tensor $[18,19]$ is given by

$$T(X,Y)Z = a_0R(X,Y)Z + a_1S(Y,Z)X + a_2S(X,Z)Y + a_3S(X,Y)Z + a_4g(Y,Z)QX \quad (4.1)$$
$$+a_5g(X,Z)QY + a_6g(X,Y)QZ + a_7r(g(Y,Z)X - g(X,Z)Y)$$

Where R, S, Q and r are the curvature tensor,the Ricci tensor,the Ricci opertor and the scalar curvature respectively .In particular, T -curvature tensor is reduced to be quasi-conformal curvature tensor C_*, conformal curvature tensor C, conharmonic curvature tensor L, concircular curvature tensor V, pseudo-projective curvature tensor P_*,projective curvature tensor P , M projective curvature tensor , W_i -curvature tensor $(i = 0,1,2......9)$ and W_j^* -curvature tensors $(j = 0,1)$.

Analogous to the definitions of an extended generalized concircular ϕ- recurrency for $\beta-$ Kenmotsu manifold and an extended generalized projective ϕ - recurrency for LP-Sasakian manifolds, here we define the following:

- Definition 2 A para Sasakian $M^{2n+1}(\phi,\eta,\xi,g), n \geq 1$ is said to be an extended generalized T - ϕ -recurrent if its T - curvature tensor satisfies the relation

$$\phi^2((\nabla_W T)(X.Y)Z) = A(W)\phi^2(T(X,Y)Z)) + B(W)\phi^2(G(X,Y)Z) \quad (4.2)$$

Where A and B are defined as in (1.1)

In particular,an extended generalized T - ϕ -recurrent para Sasakian Manifold $M^{2n+1}(\phi,\xi,\eta,g), n \geq 1$, is reduced to be

(I) an extended generalized $C^* - \phi$ - recurrent if

$$a_1 = -a_2 = a_4 = -a_5 \quad a_3 = a_6 = 0,\ a_7 = -\frac{1}{2n+1}\left(\frac{a_0}{2n} + 2a_1\right)$$

(II) an extended generalized $C - \phi$ - recurrent if

$$a_0 = 1,\ a_1 = -a_2 = a_4 = -a_5 = -\frac{1}{2n-1}, a_3 = a_6 = 0,\ a_7 = -\frac{1}{2n-1}$$

(III) an extended generalized L - ϕ - recurrent if

$$a_0 = 1, a_1 = -a_2 = a_4 = -a_5 = -\frac{1}{2n-1}, a_3 = a_6 = 0, \quad a_7 = 0,$$

(IV) an extended generalized V - ϕ - recurrent if

$$a_0 = 1,\ a_1 = a_2 = a_3 = a_4 = a_5 = a_6 = 0,\ a_7 = -\frac{1}{2n(2n-1)},$$

(V) an extended generalized P_* - ϕ - recurrent if

$$a_0 = 0,\ a_1 = -a_2,\ a_3 = a_4 = a_5 = a_6 = 0, a_7 = -\frac{1}{2n(2n+1)}\left(\frac{a_0}{2n} + a_1\right),$$

(VI) an extended generalized P - ϕ -recurrent if

$$a_0 = 1, a_1 = -a_2 = -\frac{1}{2n}, a_3 = a_4 = a_5 = a_6 = a_7 = 0$$

(VII) an extended generalized M - ϕ - recurrent if

$$a_0 = 1, a_1 = -a_2 = a_4 = a_5 = -\frac{1}{4n}, a_3 = a_6 = a_7 = 0$$

(VIII) an extended generalized W_0 - ϕ - recurrent if

$$a_0 = 1, a_1 = -a_5 = -\frac{1}{2n}, a_2 = a_3 = a_4 = a_6 = a_7 = 0$$

(IX) an extended generalized W_0^* - ϕ - recurrent if

$$a_0 = 1, a_1 = -a_5 = \frac{1}{2n}, a_2 = a_3 = a_4 = a_6 = a_7 = 0$$

(X) an extended generalized W_1 - ϕ - recurrent if

$$a_0 = 1, a_1 = -a_2 = \frac{1}{2n}, a_3 = a_4 = a_5 = a_6 = a_7 = 0$$

(XI) an extended generalized W_1^* - ϕ - recurrent if

$$a_0 = 1, a_1 = -a_2 = -\frac{1}{2n}, a_3 = a_4 = a_5 = a_6 = a_7 = 0$$

(XII) an extended generalized W_2^* - ϕ - recurrent if

$$a_0 = 1, a_4 = -a_5 = -\frac{1}{2n}, a_1 = a_2 = a_3 = a_6 = a_7 = 0$$

(XIII) an extended generalized W_3 - ϕ - recurrent if

$$a_0 = 1, a_2 = -a_4 = -\frac{1}{2n}, a_1 = a_3 = a_5 = a_6 = a_7 = 0$$

(XIV) an extended generalized W_4 - ϕ - recurrent if

$$a_0 = 1, a_5 = -a_6 = -\frac{1}{2n}, a_1 = a_2 = a_3 = a_6 = a_7 = 0$$

(XV) an extended generalized W_5 - ϕ - recurrent if

$$a_0 = 1, a_2 = -a_5 = -\frac{1}{2n}, a_1 = a_3 = a_4 = a_6 = a_7 = 0$$

(XVI) an extended generalized W_6 - ϕ - recurrent if

$$a_0 = 1, a_1 = -a_6 = -\frac{1}{2n}, a_2 = a_3 = a_4 = a_5 = a_7 = 0$$

(XVII) an extended generalized W_7 - ϕ - recurrent if

$$a_0 = 1, a_1 = -a_4 = -\frac{1}{2n}, a_2 = a_3 = a_5 = a_6 = a_7 = 0$$

(XVIII) an extended generalized W_8 - ϕ - recurrent if

$$a_0 = 1, a_1 = -a_3 = \frac{1}{2n}, a_2 = a_4 = a_5 = a_6 = a_7 = 0$$

(XIX) an extended generalized W_9 - ϕ - recurrent if

$$a_0 = 1, a_3 = -a_4 = \frac{1}{2n}, a_1 = a_2 = a_5 = a_6 = a_7 = 0$$

- Theorem 4. If a $(2n+1)$-dimensional para Sasakian manifold $M^{2n+1}(\phi, \xi, \eta, g)$, $n \geq 1$ is an extended generalized T - ϕ - recurrent such that $a_0 + 2na_1 + a_2 + a_3 \neq 0$, then M^{2n+1} is generalized Ricci-recurrent if and only if the following relation holds:

$$\frac{[B(W) + A(W)\{2n(a_2 + a_3 + a_5 + a_6) + a_0 + ra_7\} - a_7 dr(W)]}{a_0 + 2na_1 + a_2 + a_3}\eta(Y)\eta(Z) \qquad (4.3)$$

$$-\frac{(a_5 + a_6)}{2(a_0 + 2na_1 + a_2 + a_3)} \times g((\nabla_W Q)Y, Z) - \eta((\nabla_W Q)Y)\eta(Z)$$

$$+ g((\nabla_W Q)Z, Y) - \eta((\nabla_W Q)Z)\eta(Y)]$$

$$+ \frac{(a_2 + a_3)}{2(a_0 + 2na_1 + a_2 + a_3)} \times \{S(\phi W, Z) + 2N(\phi W, Z)\}\eta(Y)$$

$$+\{S(\phi W,Y)+2n(\phi W,Y)\}\eta(Z)]=0$$

- Proof.Let us consider an extended generalized T-ϕ recurrent para Sasakian manifold. Then by virtue of (2.1) it follows from (4.2) that

$$\nabla_W(X,Y)Z-\eta((\nabla_W T)(X,Y)Z)\xi = A(W)[T(X,Y)Z-\eta(T(X,Y)Z)\xi]$$
$$+B(W)[G(X,Y)Z-\eta(G(X,Y)Z)\xi]$$

From which it follows that

$$g((\nabla_W T)(X,Y)Z,U)-\eta((\nabla_W T)(X,Y)Z)\eta(U) = A(W)[g(T(X,Y)Z,U)-\eta(T(X,Y)Z)\eta(U)] \quad (4.4)$$
$$+B(W)[g(G(X,Y)Z,U)-\eta(G(X,Y)Z)\eta(U)]$$

Let $\{e_i : i=1,2,3......,2n+1\}$ be an orthonormal basis of the manifold.Setting $X=U=e_i$ in (4.4) and taking summation over i ,$1\le i\le 2n+1$, then using (1.2) and (4.1), we get

$$\{a_0+(2n+1)a_1+a_2+a_3\}\nabla_w S(Y,Z)+\{a_4+2na_7\}dr(w).g(Y,Z) \quad (4.5)$$
$$+a_5 g((\nabla_W Q)Y,Z)+a_6 g((\nabla_W Q)Z,Y)-a_0 g((\nabla_W r)(\xi,Y)Z,\xi)$$
$$-a_1(\nabla_W S)(Y,Z)-a_2(\nabla_W S)(\xi,Z)\eta(Y)-a_3(\nabla_W S)(Y,\xi)\eta(Z)$$
$$-a_4 g(Y,Z)\eta((\nabla_W Q)\xi)-a_5\eta((\nabla_W Q)Y)\eta(Z)-a_6\eta((\nabla_W Q)Z)\eta(Y)$$
$$-a_7 dr(W)\{g(Y,Z)-\eta(Y)\eta(Z)\}$$
$$=A(W)[\{a_0+(2n+1)a_1+a_2+a_3+a_5+a_6\}S(Y,Z)$$
$$+\{a_4+2na_7\}rg(Y,Z)-a_0\eta(R(\xi,Y)Z)-a_1 S(Y,Z)$$
$$-\{a_2+a_6\}S(\xi,Z)\eta(Y)-(a_3+a_5)S(Y,\xi)\eta(Z)$$
$$-a_4 S(\xi,\xi)g(Y,Z)-a_7 r\{g(Y,Z)-\eta(Y).\eta(Z)\}]$$
$$B(W)\{(2n-1)g(Y,Z)+\eta(Y)\eta(Z)\}$$

Using $(2.8),(2.9)$ and (2.11) and the relation $g(\nabla_W R)(X,Y)Z,U)=-g((\nabla_W R)$ $(X,Y)U,Z)$ We have

$$(a_0+2na_1+a_2+a_3)(\nabla_W S)(Y,Z) \quad (4.6)$$
$$=A(W)\{a_0+2na_1+a_2+a_3+a_5+a_6\}S(Y,Z)$$
$$+[(2n-1)B(W)+\{a_4+2na_7\}\{A(W)r-dr(W)\}$$
$$+A(W)\{-a_0+2na_4-ra_7\}+a_7 dr(W)]g(Y,Z)$$
$$+[B(W)+A(W)\{2n(a_2+a_3+a_5+a_6)+a_0+ra_7\}$$
$$-a_7 dr(W)]\eta(Y)\eta(Z)-a_5[g((\nabla_W Q\}Y,Z)-\eta((\nabla_W Q)Y)\eta(Z)]$$
$$-a_6[g((\nabla_W Q)Z,Y)-\eta((\nabla_W Q)Z)\eta(Y)]$$
$$+a_2[S(\phi W,Z)+2ng(\phi W,Z)]\eta(Y)+[S(\phi W,Y)+2\eta(\phi W,Y)\eta(Z)]$$

Interchanging Y and Z in (4.6), and then substracting the resultant from (4.6), we obtain by symmetric property of S that

$$(\nabla_W S)(Y,Z)=A(W)\left[1+\frac{a_5+a_6}{a_0+2na_1+a_2+a_3}S(Y,Z)\right] \quad (4.7)$$
$$+\left[\frac{(2n-1)B(W)+\{a_4+2na_7\}\{A(W)r-dr(W)\} +A(W)\{-a_0+2na_4-ra_7\}+a_7 dr(W)}{a_0+2na_1+a_2+a_3}\right]g(Y,Z)$$

$$+\left[\frac{B(W)+A(W)\{2n(a_2+a_3+a_5+a_6)+a_0+ra_7\}-a_7dr(W)]}{a_0+2na_1+a_2+a_3}\right]\eta(Y)\eta(Z)$$

$$-\frac{(a_5+a_6)}{2(a_0+2na_1+a_2+a_3)}\times g((\nabla_W Q)Y,Z)-\eta((\nabla_W Q)Y)\eta(Z)$$

$$+g((\nabla_W Q)Z,Y)-\eta((\nabla_W Q)Z)\eta(Y)]$$

$$+\frac{(a_2+a_3)}{2(a_0+2na_1+a_2+a_3)}\times\{S(\phi W,Z)+2n(\phi W,Z)\}\eta(Y)$$

$$+\{S(\phi W,Y)+2n(\phi W,Y)\}\eta(Z)]$$

If relation (4.3) holds,then above relation can be reduced to

$$\nabla S=A'\otimes S+B'\otimes g$$

where

$$A'=A(W)\left[1+\frac{a_5+a_6}{a_0+2na_1+a_2+a_3}\right]$$

$$B'=\left[\frac{(2n-1)B(W)+\{a_4+2na_7\}\{A(W)r-dr(W)\}+A(W)\{-a_0+2na_4-ra_7\}+a_7dr(W)}{a_0+2na_1+a_2+a_3}\right]g(Y,Z)$$

This M^{2n+1} is generalized Ricci-recurrent.

- Theorem 5. An extended generalized T -ϕ- recurrentpara Sasakian manifold $M^{2n+1}(\phi,\xi,\eta,g), n\geq 1$, such that $\frac{2(a_0+2na_1)+3(a_2+a_3)}{2(a_0+2na_1+a_2+a_3)}\neq 0$ is an Einstein manifold.
- Proof. Substituting $Z=\xi$ in (4.7) then using $(2.2(b))$ and (2.9) we get

$$(\nabla_W S)(Y,Z)=\left[\frac{A(W)[2n\{-a_0-2na_1+a_4\}+r\{a_4+2na_7\}]+2nB(W)-\{a_4+2na_7\}dr(W)}{a_0+2na_1+a_2+a_3}\right]\eta(Y)$$

$$+\frac{(a_2+a_3)}{2(a_0+2na_1+a_2+a_3)}\{S(\phi W,Y)+2ng(\phi W,Y)\} \quad (4.8)$$

Replacing Y by ϕY in (4.8) and then using $(2.1(b))$ we have

$$(\nabla_W S)(\phi Y,\xi)=\frac{(a_2+a_3)}{2(a_0+2na_1+a_2+a_3)}\{S(\phi W,\phi Y)+2ng(\phi W,\phi Y)\}$$

Using (3.10) we obtain from above relation that

$$\left\{\frac{2(a_0+2na_1)+3(a_2+a_3)}{2(a_0+2na_1+a_2+a_3)}\right\}\{S(\phi W,\phi Y)+2ng(\phi W,\phi Y)\}=0 \quad (4.9)$$

If $\frac{2(a_0+2na_1)+3(a_2+a_3)}{2(a_0+2na_1+a_2+a_3)}\neq 0$ then by virtue of (2.3) and (2.10) ,relation (4.9) yields

$$S(Y,Z)=-2ng(Y,W) \quad (4.10)$$

Corollary 1.*Let* M^{2n+1} *be a* $(2n+1)$ *- dimensional* $n \geq 1$*extended generalized* T *-* $\phi -$ *recurrent para sasakian manifold such that* $a_0 + 2na_1 + a_2 + a_3 \neq 0$ *.Then the associated 1-form* A *and* B *are related by*

$$B(W) = A(W)\left[a_0 + 2na_1 - a_4\left(1+\frac{r}{2n}\right) - ra_7\right] - \frac{1}{2n}\left[(a_4 + 2na_7)dr(W)\right] \tag{4.11}$$

For any vector field $W \in \chi(M)$ *.*

- Proof. By plugging Y by ξ in (4.8), we have (4.11).

It is also observed that from above corollary that,in an extended generalized T -ϕ-recurrent para sasakian manifold if T is equal to $C, P, M, W_0, W_1^*, W_6, W_8$, Then the 1 -form B vanishes(that is $B = 0$),which is not possible. Hence we can state the following:

- Theorem 6 There exists no extended generalized $\{C, P, M, W_0, W_1^*, W_6, W_8\}$ -ϕ -recurrent para Sasakian manifold.

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