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Some Computational Approach for the Solution of the Electro Convection to Fluid Turbulent Flow Problem

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Abstract:
We have steady a perfectly conducting spherical particle. Which is suspended within an electrolyte solution and exposed to uniformly electric field? So we are using the weak field approximation. The electro kinetic flow is analyzed for arbitrary Debye-layar thickness. If the commonly employed thin layer model emerging as a special case. Then we identify a scalar property which quantifies the global strength of the quadrupolar flow structure. The (L.S) function maintains a singed distance function without an auxiliary equation via the practical bashed Lagrangian re-initialization technique. To assess the new hybrid method, numerical simulation of several. Standard interface- moving problem and two fluid laminar and turbulent flows are conducted. The turbulent energy spectral energy density function and the consistency between Eulerian and Lagrangian component. The result, our analysis indicate that the hybrid practical level set method can handle interfaces with complex shape change. The interface values without any significant (unphysical) mass loss or gain even in turbulent flow, The results obtained for isotropic turbulence by new practical level set method are validated by comparison with those obtained by 'zero much number' variable density method. Analysis of the vortices and energy equations indicated that destabilization effect of turbulence. And the stability effects of surface tension of interface motion are strongly dependent on the density and viscosity ratio of the fluids.

Keywords: *Interface tracking, two fluids turbulent flows, Practical level set method*

1. Introduction

We have been an increased interest the induced charge electro kinetic flow about polarizable particles. We unlike the conventional electro kinetic flows (Saville 1977) which require the pressure of an immobile surface change distributions. The induced charge flows can be driven even with, initially uncharged particles. The prototypical scenario involves a perfectly conducting ion-impermeable particle. Which is suspended in electrolyte solutions. An external electro fluid is applied, Faraday current, charge the region adjacent to its surface, thereby the generating a polarished Debye -layer. The partial itself polorised. An electric field exerts lorents body forces itself Debye cloud. Which is clearly non linear in the applied filed density.

1.1. Euler's Equation of Motion

Let a closed surface S enclosing a volume V of a non-viscous fluid be moving with the fluid so that S contains the same number of fluid particles at any time t. Consider a point P inside S. Let ρ be the fluid density, q the fluid velocity and dV the elementary volume enclosing P. Since the mass ρ dV remains unchanged throughout the motion so that that

$$\frac{d}{dt}(\rho dV) = 0 \qquad \ldots\ldots\ldots\ldots\ldots (1)$$

The entire momentum M of the volume V is

$$M = \int_V q \;\; \rho \; dV \text{ or momentum = mass x velocity}$$

$$\therefore \; \frac{dM}{dt} = \int \left[\frac{dq}{dt}\rho dV + \frac{d}{dV}(\rho dV) q \right]$$

Using (1), $$\frac{dM}{dt} = \int \frac{dq}{dt} \rho \, dV. \qquad \text{................ (2)}$$

Let n be the unit outward normal vector on the surface element dS. Suppose F is the F is the external force per unit mass acting on the fluid and ρ the pressure at any point on the element ds. Total surface force is

$$\int_V F \quad \rho \int_S \rho(-n) dS$$

[For pressure acts longs inward normal]

$$= \int_V F \quad \rho \, dV + \int_V -\nabla p \quad dV,$$

by Gauss Theorem

$$= \int_V (F\rho - \nabla p) dV. \qquad \text{........................ (3)}$$

By Network's second law of motion.
rate of change of momentum = total applied force

i.e. $$\int \frac{dq}{dt} \rho \, dV = \int (F\rho - \nabla p) \, dV, \qquad \text{by (2) and (3)}$$

or $$\int \left[\frac{dq}{dt} \rho - F\rho + \nabla p \right] dV = 0$$

Since S is arbitrary and so V is arbitrary so that the integrand of the last integral vanishes,

$$\frac{dq}{dt} \rho - F\rho + \nabla p = 0$$

i.e., $$\frac{dq}{dt} = F - \frac{1}{\rho} \nabla p \qquad \text{.............. (4)}$$

This equation is known as Euler's as Euler's equation of motion.

1.2. Prandtl's Boundary Layer Theory

Same important advances in fluid dynamics where contributed by prandtl 1904. Prandtl's theory divides the motion of fluid around the object into two domains

i. A thin domain very close to the object where the frictional forces (viscous forces) are prominent.
ii. An outer domain where the frictional forces may be neglected in domain fluid is treated as a non-viscous fluid. The first domain is known as the boundary layer and the second one is known as external boundary.

1.3. Different Classes of Fluids

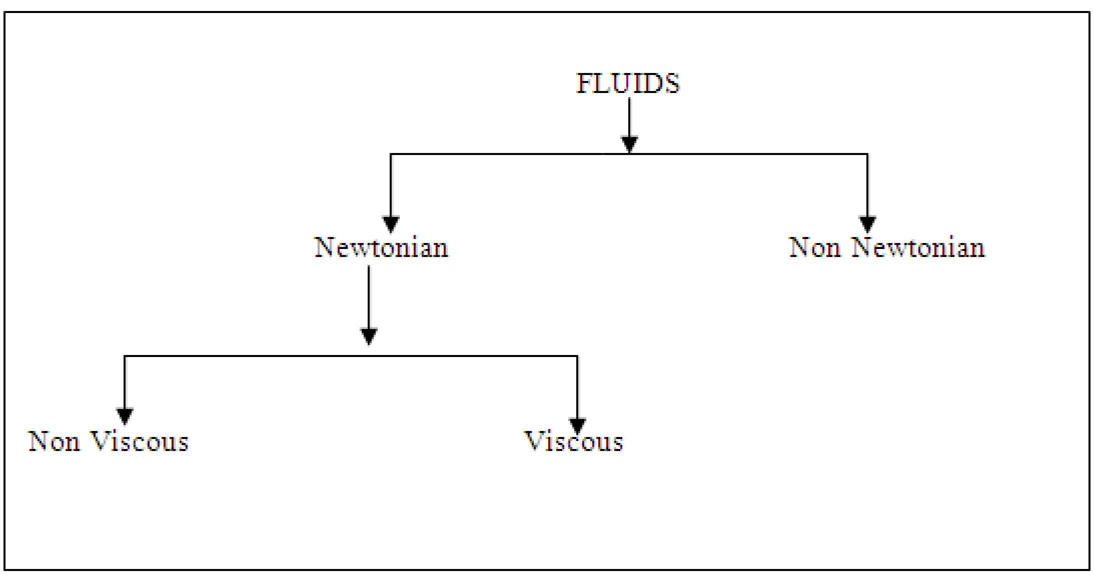

Figure 1

1.4. Viscous Model of Non Newtonian Fluids

Therefore, non Newtonian fluids represented by a curve. The main classes of non Newtonian fluids are Bingham Plastic, Pseudo Plastic and Dilants.

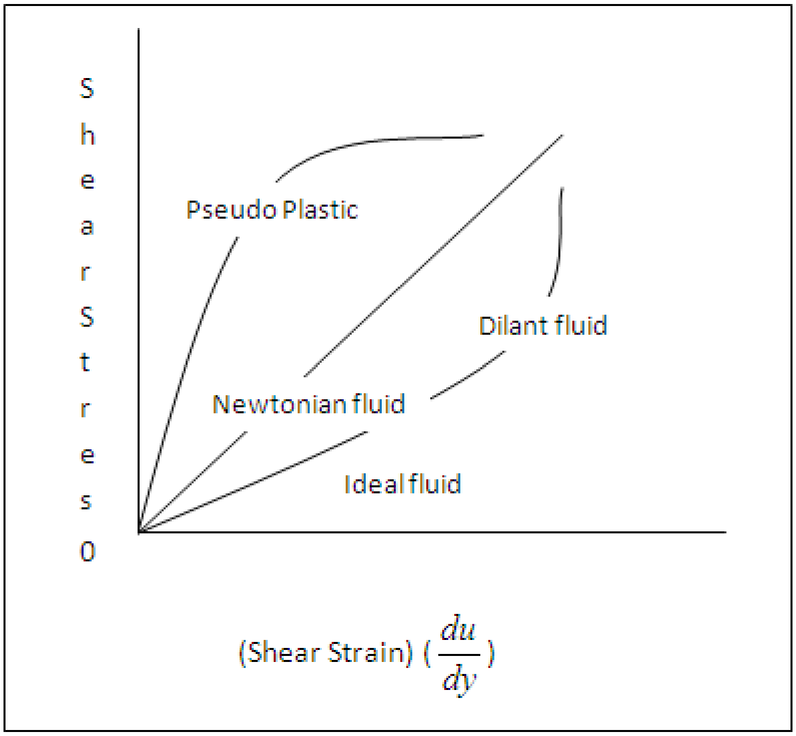

Figure 2

1.5. Steady Laminator Flow through a Tube of Uniform Circular Cross Section Hagen- Poiseuilte Flow

First we know steady flow through a cylindrical pipe equation.

$$\frac{\partial p}{dz} = \mu \left(\frac{\partial 2w}{\partial x2} + \frac{\partial 2w}{\partial y2}\right) = P \quad \ldots\ldots\ldots\ldots\ldots\ldots..(1) \text{ where } p > 0$$

changing this into cylindrical co-ordinates (r, θ, z) by the transformations x =r cos θ , y=rsin θ , z = z, we obtain

$$\frac{\partial p}{dz} = \mu\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) = -P$$

by symmetry w=w (r) so that

$$\frac{dp}{dz} = \mu\left(\frac{\partial^2 w}{\partial r2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) = -p \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots..\ \ldots(2)$$

This $\Rightarrow \frac{dp}{dz} = -p \Rightarrow -pdz \Rightarrow p = -pz \pm A$ …………………… (3)

Where A is constant of integration.

Subjecting (3) to the conditions (i) p = p_1 when z = z_1

(ii) p = p_2 when z = z_2, z_2-z_1 = 1

we get p_1=-pz_1+A, p_2=-px_2+A.

this $p_1 - p_2 = p(z_2 - z_1) = p_1 \Rightarrow \frac{p_1 - p_2}{l} = P$ ……………… (4)

by (2) $\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)=-\frac{p}{\mu} or \frac{d}{dr}\left(r\frac{dw}{dr}\right)=-\frac{\Pr}{\mu}$

integrating, $r\frac{dw}{dr}=-\frac{pr2}{2\varpi}+A or \frac{dw}{dr}=-\frac{pr}{2\mu}+\frac{A}{r}$

again integration, w=- $\frac{\Pr^2}{4\mu}+A\log r+b$ ………………… (5)

Since velocity w is finite along the axis and hence in particular at r =0. This requires A= 0.

$\therefore w=-\frac{\Pr 2}{4\mu}+\beta$ …………………….. (6)

There is no slipping on the tube so that w = 0 when r=a.

Now (6) $\Rightarrow 0=-\frac{\text{P}a^2}{4\mu}+B\Rightarrow w=\frac{p}{4\mu}(a^2-r^2)$.

Rate of flow

$Q=\int_0^a w.2\pi r dr=\frac{\text{P}}{4\mu}.2\pi\int_0^a(a^2r-r^3)$

Q = $\frac{\pi\text{P}}{2\mu}\left(a2.\frac{a2}{2}-\frac{a4}{4}\right)=\frac{\pi\text{P}a2}{8\mu}=\frac{\pi a4}{8\mu l}.(p_1-p_2)$, by (4) …………………..(7)

2. Brief Survey of Literature

Since the inception in the field of fluid dynamics, the study the viscous flow and bio magnetic fluid has been remaining the area of interest of research workers. Here we are going to maintain a few research works performed in the area of bio magnetic fluid flow and viscous in compare sable fluid flows, as it is not an easy job to maintain all the research works. In 1962, Prentice- Hall, a book on the title "Physicochemical Hydrodynamics", written by Levich**.** Abramowitz, M. & Ate gun I.A. wrote a book "Hand Book for mathematical function" which was published by National Bureau of Standards in 1964. Happel, J. & Brenner, H 1965 wrote a book " Low Reynolds number Hydrodynamics" by Prentice Hall. In 1969 Cambridge University Press published a book " Theory of Rotating fluids", written by Green Span, H.P. Burry D. Bergeles G. wrote a book " Dispersion of Particals or droplets dispersion and on droplets vaporization in turbulent airflow" in 1973 which was published by Hammamet Tunisia. The book on title "Waves in the atmosphere" written by Gossard, E.E.& Hooke, W.H. in 1975 at Elsevior. Adams RA wrote a book "Sabolev Spaces," was published by Academic Press New York. In 1975. In 1977 University of New Delhi a book on title "Effects of thermal buoyancy on the stability properties of boundary layer flow" written by Mureithi E.W. Light Hill, D wrote a book " Waves in fluids" which was published by Cambridge University Press in 1978. In 1978 Tabata M. wrote a book on the title of "Uniform Convergence of the upwind finite eliment approximation for Simi linear Parabolic problem," which was published by Kyoto University. Ciarlet Pg. wrote a book "The finite element method for elliptic problems," was published by North- Holland Amsterdom in 1978. A book on the title " Numerical Solution of Particles equation" written by Smith G.D. in 1978 which was published by Clerendom Press Oxford. Sykes, R.I. wrote a book on the title of " Stratified effects in the boundary layer flow over hills" in 1978 was published by R Society London. In 1980, McGrow-Hill New York a book on the title "Numerical Heat Transfer and fluid flow", written by Patankar V. Sykes, R.I. wrote a book "On three diamantine boundary layer flow over surface irregularities," which was published by R. Society London in 1980. In 1980, Kelvin, Lord wrote a book with the title of " Vibrations of Colummervertex" published by Phil Mog. Tabata M. wrote a book on the title of "Consarvabune upwind finite element Approximations, and its applications," in year 1981 published by North – Holland Amsterdam. Drazin, P.G. & REID, W.H. wrote a book on the title of "Hydrodynamic Stability" which was published by Combridge University Press in 1981. In 1982 Academic Press New York a book on title "The time dependent multi material flow with large fluid distortion," written by Young's D.L. Liang D. A. wrote a book " Kind upwind schemes for Convocation- diffusion equation," which was published by Mathematics Numerical Sinica in 1991.

In 1997, Pilliod JE. Puckett EG. wrote a book on the title of " Second order accurate volume of fluid algorithms for tracking material interface" published by Berkeley National Laboratory. Li RH, Chen Zy Wu W. wrote a book "Numerical Method of finite volume methods," in 2000 published by Marcel-Dekker New York. In 2002, Cimne Barcelona Spain published a book on title of " A New Parametric instability in rotating Cylinder flow", written by Graftieaux, L.LB Penven, L. Scott, J.F. & Gro Sjeann. Eloy, C.LE. Gal, P, & LE Dized, S wrote a book on the title of "Ecliptic and triangular instabilities in rooting cylinder" in year 2003. In 2003, Q. J. R. met Soc. A book on the title " abuser ration and simulation of the waves case," written by Smith, S. A. Lis, Shimuta M, Xiao F.A. wrote a book "fourth order and single cell based advection scheme on unstructured grids using multi moments," which was published by Computer physics comma captions in 2005. In 2006 Hammamet Tunisia a book on the title “Effect of Co-flow on particles or droplets dispersion and on droplets vaporization in turbulent airflow” written by Boughattas N., Gazzah M.H., Said R. Boughattas N., Gazzah M.H., Said R. wrote a book “Lagrangian Prediction of the particulate two phase flow”, which was published by Monstir Tunisia in 2007. In 2008 J. fluid mechanics a book on the title “ Instability of stratified boundary layer and its Coupling with internal gravity waves,” written by Wu, X. & Zhang J.

3. Objective and Methodology

i. To find the interface is identified based on the locations of notional (Lagrangian) particles and the geometrical information concerning the interface.
ii. To find the interface values without any significant (unphysical) mass loss or gain even in turbulent flow.

The new methodology has been assessed by applying it to several standard interface-moving and two-fluid laminar problems. It has also been used for simulation of two-fluid isotropic turbulent flows under various flow /fluid conditions. The numerical results are evaluated by monitoring the mass conservation law, the turbulence energy spectral density function and the consistency between Eulerian and Lagrangian components of the method. It is shown that the IPLS method can handle interfaces with substantial topological complexity, and accurately predict the interface evolution even at the 'sub-grid range' as long as the velocity field is well resolved. The accuracy of the Navier-Stokes flow solver in the IPLS method is established first by comparing the results obtained by the IPLS method for a single-fluid isotropic turbulent flow with those obtained *via* high-order incompressible and compressible pseudQ-spectral numerical schemes. The IPLS results for a two-fluid isotropic turbulent flow are also compared with those predicted by a high-order zero Mach number variable density flow solver. Despite the differences in the equations and the numerical schemes, we have *found* that the IPLS method generates virtually identical results with other methods. The similarity of the results confirm the accuracy of the IPLS flow solver in single-fluid and two-fluid turbulent systems.

4. Analysis

The Lagrangian particles can accurately represent the interface evolution without any numerical mass loss or gain and the level set function provides smooth geometrical information concerning the interface and its effects on the flow.

4.1. Flow Field

Since V is solenoidel, the momentum equation (1) can be recast in the from

$$\nabla P = -\nabla \times \omega - \frac{1}{4\lambda^2} Q(\nabla x - \nabla Q). \qquad \text{(I)}$$

where $\omega = \nabla \times V$ is the fluid vorticity. Forming the curl of this equation yields the vorticity equation:

$$\nabla X(\nabla X \omega) = -\frac{1}{4\lambda^2} \nabla Q \times \nabla x. \qquad \text{(II)}$$

then, the lift hand side of (II) may be expressed in terms of ψ :

$$\nabla X(\nabla X \omega) = -e\,\varpi \frac{E^4 \psi}{e \sin\theta} \qquad \text{(III)}$$

here

$$E^2 = \frac{\partial}{\partial r} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\right). \qquad \text{(IV)}$$

In (III) into the vorticity equation (II) yields the inhomogeneous equation for ψ :

$$E^4\psi = \frac{3}{\lambda^3(1+\lambda)} e^{(1-r)/\lambda} \left(1 + \frac{3\lambda}{r} + \frac{3\lambda^2}{r^2} + \frac{1}{2r^3}\right) b^{-1/2} t\,im\,(\cos\theta) \qquad \text{(V)}$$

here, $b^{-1/2}(n) = n(1 - n^2)/2$ is the Gene Bauer function of order 3 and degree -1/2. This equation is to be solved subject to a zero-velocity condition at the particle surface,

$$r = 1: \quad \psi = \frac{\partial\psi}{\partial r} = 0 \qquad \text{(VI)}$$

$\log \psi = r$

and at large distances,

$$r \to \infty : \log \psi / r = 0 \qquad \text{(VII)}$$

We seek a solution which possesses the quadrupolar structure,

$$\psi = h(r)\, b_3^{-1/2} (\cos\theta). \qquad \text{(VIII)}$$

This equation is solved subject to the vanishing of h and its derivative at r = 1, together with requirement that h is 0 (r^2) at larger r. The Solution is

$$\begin{aligned}(1+\lambda)h(r) &= \frac{3\lambda}{2} + \frac{3}{2} + \frac{1}{20\lambda^2} + \frac{9-7\lambda}{160\lambda^4} + \frac{\lambda-1}{320\lambda^6} + e^{(1-r)/\lambda} \\ &\left(\frac{3}{70r^2} + \frac{9\lambda^3}{r^2} + \frac{3}{70\lambda r} + \frac{9\lambda^2}{r} + 3\lambda - \frac{1}{35\lambda^2} + \frac{11r\lambda - 13r^2}{560\lambda^4} + \frac{r^4 - \lambda r^3}{1120\lambda^6} \right) + \frac{1}{r^2} \\ &\left(\frac{1-\lambda}{448\lambda^6} + \frac{27\lambda - 37}{1120\lambda^4} - \frac{3}{140\lambda^2} - \frac{3}{70\lambda} - \frac{54}{35} - \frac{9\lambda}{2} - 9\lambda^2 - 9\lambda^3 \right) + e^{1/\lambda} \\ &\left[E_1(r/\lambda)\left(\frac{r^3}{40\lambda^5} - \frac{r^5}{1120\lambda^7} \right) + E_1(1/\lambda)\left(\frac{1}{320\lambda^7} - \frac{1}{448\lambda^7 r^2} - \frac{1}{16\lambda^5} + \frac{3}{80\lambda^5 r^2} \right) \right] \qquad \text{(IX)}\end{aligned}$$

Wherein $E_1(s) = \int_s^\infty e^{-t}/t\, dt$ is the exponential integral. The streamlines ψ = constant for $\lambda = 1$ are depicted in figure 2 (a). Evaluation of h for other λ values indicates that the quadrupolar flow pattern possesses the same attributes as its thin – layer counterpart, namely open – ended streamlines without points.

The solution (VIII) describes a flow mechanism which pumps, liquid from the region about the $\theta = 0$ (fore) and $\theta = \pi$ (aft) axes and ejects it along the $\theta = \pi/2$ equatorial plane. (Indeed, such flows were suggested by Bazant & Squires (2004) to be used as micro fluidic pumps.) These regions can be precisely defined: from (VIII) it is evident that the radial velocity of outflow is $\theta_0 < \theta < \pi - \theta_0$, where $\cos\theta_0 = 1/\sqrt{3}$ ($\theta_0 \cong 54.7^0$). The regions of inflow are within the 'front cone' $\theta < \theta_0$ and the 'back cone' $\theta > \pi - \theta_0$.the region of outflow is $\theta_0 < \theta < \pi - \theta_0$.

Here, we propose using a new scalar estimate of the 'pumping rate', to be defined as the net volumetric flux F (normalized with) $\beta^2 r_0^2$ which enters the back cone at large distance form the particle. Given the flux – measure property of the stream.

4.2. Numerical Solution of IPLS Equations

The solution for the velocity and pressure calculations in the IPLS method is based on a variable density projection algorithm. In this algorithm, the momentum equation

$$\frac{\partial U}{\partial t} + \frac{\nabla p}{\rho(\phi)} = W \qquad \text{……………………..(1)}$$

where

$$W = -(U.\nabla)U + \frac{1}{\mathrm{Re}} \frac{\nabla.(2\mu(\phi)D)}{\rho(\phi)} - \frac{1}{We} \frac{k(\phi)\nabla H(\phi)}{\rho(\phi)} - \frac{1}{Fr}\frac{y}{|y|}$$

By taking the divergence of equation (1) and using the continuity equation, one can readily derive the Poisson equation for pressure p.

$$\nabla . \left[\frac{\partial U}{\partial t} + \frac{\nabla p}{\rho(\phi)} \right] = \nabla . W$$

if $$\nabla . \frac{\partial U}{\partial t} = 0$$

then $$\nabla . \frac{\nabla p}{\rho(\phi)} = \nabla . W \qquad \text{……………………………(2)}$$

once the pressure field p is determined using Equation (2), U is updated from the following equation:

$$\frac{\partial U}{\partial t} = -\frac{\nabla p}{\rho(\phi)} + W \qquad \text{……………………………(3)}$$

4.2.1. Temporal Discretization

The time differencing in the IPLS equation is based on the second-order Adams- Bashforth scheme that yields the following difference equations:

$$\Phi^{n+1} = \phi^{n} - dt \text{ X } \left(\frac{3}{2}(U^{n}.\nabla)\ \phi^{n} - \frac{1}{2}(U^{n-1}.\nabla)\phi^{n-1}\right) \qquad \text{………..(4)}$$

$$x_p^{n+1} = x_p^{n} + dt \text{ X } \left(\frac{3}{2}V(x_p^{n}) - \frac{1}{2}V(x_p^{n-1})\right) \qquad \text{……….(5)}$$

$$U^* = U^n + dt \text{ X } \frac{3}{2}W^{n} - \frac{1}{2}W^{n-1}) \qquad \text{……….(6)}$$

$$\frac{U^{n+1} - U^*}{dt} = -\frac{\nabla p^{n+1}}{\rho^{n}} \qquad \text{……….(7)}$$

With the known intermedial velocity.U* and the time step, dt, Equation (7) is solved together with the following Poisson equation for U^{n+1} and P^{n+1} with an interative method.

5. Conclusions and Future Scope

A hybrid Lagrangian-Eulerian interfacial particle-level set (lPLS) method is developed for numerical simulation of two-fluid turbulent flows. In this method, the interface is identified based on the locations of notional (Lagrangian) particles and the geometrical information concerning the interface and the fluid properties are obtained from the (Eulerian) level set function. The Lagrangian particles can accurately represent the interface evolution without any numerical mass loss or gain and the level set function provides smooth geometrical information concerning the interface and its effects on the flow.

Analysis of the velocity statistics and vorticity/interface contours in turbulent flows indicates that the destabilization effect of turbulence and the stability effect of surface tension on the interface motion are strongly dependent on the fluid density and viscosity ratios. As expected, the interface becomes more stable and less 'active' as the surface tension increases or the Weber number decreases. The turbulence can completely break up a thin layer of oil in water. However, *for* an initially thick layer, turbulence can only pinch off small ligaments or droplets from the layer. Despite the complex and sometimes very significant effects that the turbulence and the interface have on each other, it is shown that the IPLS method can correctly capture the interface evolution and the effect of turbulence on the interface in all the cases considered in this study.

6. Acknowledgement

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7. References

i. Levich, V.G. (1962) Physicochemical Hydrodynamics" Prentice- Hall
ii. Abramowitz, M. & Ate gun I.A. (1964) "Hand Book for mathematical function", National Bureau of Standards .
iii. Happel, J. & Brenner, H (1965), " Low Reynolds number Hydrodynamics", Prentice Hall.
iv. Baines P.G. (1967),"Forced oscillations of an ereloshed rotating fluid,"
v. Green Span, H.P. (1969) "Theory of Rotating fluids", Cambridge University Press.
vi. Burry D. Bergeles G. (1973), " Dispersion of Particals or droplets dispersion and on droplets vaporization in turbulent airflow" Hammamet Tunisia.
vii. Gossard, E.E. & Hooke, W.H. (1975), "Waves in the atmosphere". Elsevior.

viii. Adams RA (1975), "Sabolev Spaces," Academic Press New York
ix. Mureithi E.W. (1977), "Effects of thermal buoyancy on the stability properties of boundary layer flow" University of New Delhi.
x. Light Hill, D (1978) " Waves in fluids", Cambridge University Press.
xi. Tabata M. (1978), "Uniform Convergence of the upwind finite eliment approximation for Simi linear Parabolic problem," Kyoto University.
xii. Ciarlet Pg. (1978) "The finite element method for elliptic problems,"North- Holland Amsterdom.
xiii. Smith G.D. (1978), "Numerical Solution of Particles equation". Clerendom Press Oxford.
xiv. Sykes, R.I. (1978)," Stratified effects in boundary layer flow over hills"
xv. Patankar V. (1980), "Numerical Heat Transfer and fluid flow", McGrow-Hill New York.
xvi. Sykes, R.I. (1980), "On three diamantine boundary layer flow over surface irregularities,"R. Society London.
xvii. Kelvin, Lord (1980), "Vibrations of Colummervertex" Phil. Mog
xviii. Tabata M. (1981), "Consarvabune up wind finite element Approximations, and its applications,".North – Holland Amsterdam.
xix. Drazin, P.G. & REID, W.H. (1981), "Hydrodynamic Stability", Cambridge University Press.
xx. Roe PL. (1981), "Approximate Riemann solvers, parameter vectors and difference schemes.
xxi. Young's D.L. (1982.) "The time dependent multi material flow with large fluid distortion," Academic Press New York.
xxii. Reitz RD, Bracco FV, (1986) "mechanisms of breakup of round of liquid jets; Houston Guff.
xxiii. Stull, .B. (1988), "An introduction to boundary layer meteorology,"Kluwer.
xxiv. Shu CW. (1988), "Totol variatiol diminishing time discrete zations," SIAM stat tactical computing.
xxv. Liang D. A. (1991)," Kind upwind schemes for Convocation- diffusion equation," Mathematics Numerical Sinica.