THE INTERNATIONAL JOURNAL OF SCIENCE & TECHNOLEDGE

Generalized 3 – Complement of Set Domination

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Abstract:
Let G=(V,E) be a simple, undirected, finite nontrivial graph. A set $S\subseteq V$ of vertices of a graph G = (V, E) is called a dominating set if every vertex $v\in V$ is either an element of S or is adjacent to an element of S. A set $S\subseteq V$ is a set dominating set if for every set $T\subseteq V-S$, there exists a non-empty set $R\subseteq S$ such that the subgraph <RUT> is connected. The minimum cardinality of a set dominating set is called set domination number and it is denoted by γ_s (G).Let $P=(V_1,V_2,V_3)$ be a partition of V of order 3. Remove the edges between V_i and V_j where $i\neq j$ $(1\leq i,j\leq 3)$ in G and join the edges between V_i and V_j which are not in G. The graph G_3^p thus obtained is called 3-complement of G with respect to 'P'.

Keywords: *Dominating set, Set dominating set, 3-complement of G.*

1. Introduction

Let G=(V,E) be a simple, undirected, finite nontrivial graph with vertex set V and edge set E. And K_n,$K_{m,n}$,C_n,P_n and $K_{1,n}$ denote the complete graph, the complete bipartite graph, the cycle, the path and the star on n-vertices respectively. A nonempty set $S\subseteq V$ of vertices in a graph G=(V,E) is called a dominating set if every vertex $v\varepsilon$ V is either an element of S or is adjacent to an element of S. A set $S\subseteq V$ is a set dominating set if for every set $T\subseteq V-S$, there exists a non-empty set $R\subseteq S$ such that the subgraph <RUT> is connected. The minimum cardinality of a set dominating set is called set domination number and it is denoted by γ_s (G).

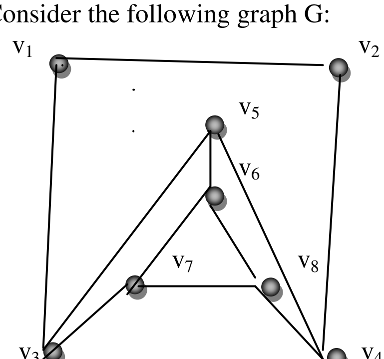
2. Observation

For any connected graph G, γ (G)$\leq$ γ_s (G).

In the following example the set domination number γ_s is calculated.

3. Example

Consider the following graph G:



Let V= {v_1, v_2......v_n} be the vertices of G. S= {v_3,v_4,v_6}.
For every $T\subseteq V-S$ there exists a nonempty set $R\subseteq S$ such that <RUT> is connected.
Here, γ_s (G) = 3.

The 3-complementary of the set domination number of some standard graphs are given below.

4. Theorem

When $G=K_n$ ($n \geq 3$),let (V_1,V_2,V_3) be a partition of G and $|V_1| =k,|V_2|=r,|V_3|=l$ ($k\leq r\leq l$)then
$\gamma_s (G_3^p)= k+r+1$.

Proof:-

Let $V(K_n)= \{v_1,v_2,.....v_n\}$ and (V_1,V_2,V_3) be a partition of G. Suppose $V_1= \{v_1\}$ $V_2= \{v_2\}$ and $V_3=\{ v_3,v_4,\ldots,v_n\}$ then G^3_p is a disconnected graph with 3 components. And v_1 and v_2 are isolated vertices.
Here < $\{v_3, v_4.....v_n\}$> form a complete graph with n-2 vertices in G_3^p.Here a γ_s – set is $\{v_1, v_2,v_3\}$. Therefore
$\gamma_s (G_3^p) = 3$.
If $V_1= \{v_1\},V_2= \{v_2,v_3\}$ and $V_3=\{ v_4,v_5,\ldots,v_n\}$ then G^3_p is a disconnected graph with 3 components. And v_1 is an isolated vertex. And v_2 is adjacent to v_3.Here < $\{v_4, v_5.....v_n\}$> form a complete graph with n-3 vertices in G_3^p.Here a γ_s – set is $\{v_1, v_2,v_3,v_4\}$. Therefore $\gamma_s (G_3^p) = 4$.
Suppose $V_1= \{v_1\},V_2= \{v_2,v_3,v_4\}$ and $V_3=\{ v_5,v_6,\ldots,v_n\}$ then G^3_p is a disconnected graph with 3 components. And v_1 is an isolated vertex. Here < $\{v_5, v_6.....v_n\}$> and < $\{v_2, v_3,v_4\}$> are disjoint and they form a complete graph Here a γ_s – set is $\{v_1, v_2,v_3,v_4,v_5\}$. Therefore $\gamma_s (G_3^p) = 5$.
Suppose $V_1= \{v_1\},V_2= \{v_2,v_3,v_4,v_5\}$ and $V_3=\{ v_6,v_7,\ldots,v_n\}$ then G^3_p is a disconnected graph. And v_1 is an isolated vertex. Here < $\{v_2, v_3,v_4,v_5\}$> and < $\{v_6, v_7,\ldots,v_n\}$> are disjoint and they form a complete graph.Here a γ_s – set is $\{v_1, v_2,v_3,v_4,v_5,v_6\}$. Therefore $\gamma_s (G_3^p) = 6$.
Proceeding like this, Suppose $V_1= \{v_1\},V_2= \{v_2,v_3,\ldots v_{n-1}\}$ and $V_3=\{ v_n\}$ then G^3_p is a disconnected graph. Here a γ_s – set is$\{v_1, v_2,v_n\}$. Therefore $\gamma_s (G_3^p) = 3$.
If $V_1= \{v_1,v_2\},V_2= \{v_3,v_4\}$ and $V_3=\{ v_5,v_6,\ldots,v_n\}$ then G^3_p is a disconnected graph with 3 components. And v_1 is adjacent to v_2, v_3 is adjacent to v_4 and < $\{v_5, v_6,\ldots v_n\}$> form a complete graph with n-5 vertices.Here a γ_s – set is
$\{v_1, v_2,v_3,v_4,v_5\}$. Therefore $\gamma_s (G_3^p) = 5$.
If $V_1= \{v_1,v_2\},V_2= \{v_3,v_4,v_5\}$ and $V_3=\{ v_6,v_7,\ldots,v_n\}$ then G^3_p is a disconnected graph with 3 components. Here $\{<v_1,v_2>\}$ and < $\{v_3, v_4,v_5\}$> and <$v_6,v_7,\ldots v_n\}$> are disjoint. Here v_1 is adjacent to v_2 and <$v_3,v_4,v_5\}$> and <$\{v_6,v_7,\ldots,v_n\}$> form a complete graph. Here a γ_s – set is $\{v_1, v_2,v_3,v_4,v_5,v_6\}$. Therefore $\gamma_s (G_3^p) = 6$.
If $V_1= \{v_1,v_2\},V_2= \{v_3,v_4,v_5,v_6\}$ and $V_3=\{ v_7,v_8,\ldots,v_n\}$ then G^3_p is a disconnected graph with 3 components. Here $\{<v_1,v_2>\}$ and < $\{v_3, v_4,v_5,v_6\}$> and <$v_7,v_8,\ldots v_n\}$> are disjoint. Here v_1 is adjacent to v_2 and <$v_3,v_4,v_5,v_6\}$> and <$\{v_7,v_8,\ldots,v_n\}$> form a complete graph. Here a γ_s – set is $\{v_1, v_2,v_3,v_4,v_5,v_6,v_7\}$. Therefore $\gamma_s (G_3^p) = 7$
Proceeding like this,If $V_1= \{v_1,v_2\},V_2= \{v_3,v_4,\ldots v_{n-1}\}$ and $V_3=\{ v_n\}$ then G^3_p is a disconnected graph. Here v_n is an isolated vertex. And v_1 is adjacent to v_2 and <$v_3,v_4,\ldots v_{n-1}\}$> form a complete graph with n-2 vertices. Here a γ_s – set is $\{v_1, v_2,v_3,v_n\}$. Therefore $\gamma_s (G_3^p) =4$.
All other partitions, we get an isomorphic graph of one of the above cases.

5. Theorem

Let G be a Complete bipartite graph with partition (V_1,V_2), where $|V_1|=m$ and $|V_2|=n$ where $m\leq n$. Let (W_1,W_2,W_3)be a partition of $V(G_3^p)$ then

$$\gamma_s (G_3^{\,p}) = \begin{cases} 1 & \text{if } |W_i|=|W_j|=1 \text{ where } i,j=1,2,3 \text{ with } i\neq j \\ 2 & \text{if } W_i=\{u,v\} \text{ where } u\varepsilon V_1 \text{ and } v\varepsilon V_2 \\ m+1 & \text{if } W_i=V_1,\ W_j=\{v\} \text{ where } v\varepsilon V_2, W_k= V\backslash(W_i\cup W_j) \text{ for } i,j=1,2,3 \\ n+1 & \text{if } W_i=V_2,\ W_j=\{u\} \text{ where } v\varepsilon V_1, W_k= V\backslash(W_i\cup W_j) \text{ for } i,j=1,2,3 \\ m+2 & \text{if } W_i=V_1,\ W_j=\{v_p,v_q\} \text{ where } v_p,v_q\ \varepsilon V_2, W_k= V\backslash(W_i\cup W_j) \text{ for } i,j=1,2,3 \\ n+2 & \text{if } W_i=V_2,\ W_j=\{u_p,u_q\} \text{ where } u_p,u_q\ \varepsilon V_1, W_k= V\backslash(W_i\cup W_j) \text{ for } i,j=1,2,3 \end{cases}$$

Proof:

→ Case:1

Let $|W_1| =|V_j| +1$, $|W_2| =1$, $|W_3| =1$for some i. Then G^3_p is a connected graph. In G^3_p, which element is joined to V_i that element is adjacent to all other elements. Therefore a γ_s -set has only one element to satisfy the set domination. Hence $\gamma_s (G_3^p)=1$

→ Case:2

If $V_1= \{u_1\},V_2= \{v_2\}$ and $V_3=V\backslash(V_1\cup V_2)$ then G^3_p is a connected graph. Here a γ_s – set is $\{v_1, u_2\}$. Therefore
$\gamma_s (G_3^p) =2$.
If $V_1= \{u_1\},V_2= \{u_2,v_1\}$ and $V_3=V\backslash(V_1\cup V_2)$ then G^3_p is a connected graph. Here a γ_s – set is $\{v_1, u_2\}$. Therefore
$\gamma_s (G_3^p) =2$.
If $V_1= \{u_1\},V_2= \{u_2,u_3,v_1\}$ and $V_3=V\backslash(V_1\cup V_2)$ then G^3_p is a connected graph. Here a γ_s – set is $\{v_1, u_3\}$. Therefore $\gamma_s (G_3^p) =2$.
If $V_1= \{v_2\},V_2= \{v_3\}$ and $V_3=V\backslash(V_1\cup V_2)$ then G^3_p is a connected graph. Here a γ_s – set is $\{v_1, v_2\}$. Therefore
$\gamma_s (G_3^p) =2$.

If $V_1=\{u_1,v_1\}$,$V_2=\{u_2,u_3\}$ and $V_3=V\backslash(V_1\cup V_2)$ then G^3_p is a connected graph. Here u_1 is adjacent to u_2 and u_3. And v_1 is adjacent to $v_2,v_3,\ldots,v_n$. Therefore a γ_s – set is $\{u_1, v_1\}$. Therefore $\gamma_s(G_3^P)=2$.
If $V_1=\{u_1,v_1,v_2\}$,$V_2=\{u_2,u_3\}$ and $V_3=V\backslash(V_1\cup V_2)$ then G^3_p is a connected graph. Here u_1 is adjacent to $u_2,u_3,\ldots u_n$. And v_1 and v_2 are adjacent to $v_3,v_4,\ldots,v_n$. Therefore a γ_s – set is $\{u_1, v_2\}$. Therefore $\gamma_s(G_3^P)=2$.
If $V_1=\{u_1,v_1,v_2,v_3\}$,$V_2=\{u_2,u_3\}$ and $V_3=V\backslash(V_1\cup V_2)$ then u_1 is adjacent to $u_2,u_3,\ldots u_n,v_1,v_2,v_3$.And v_1,v_2,v_3 are adjacent to $v_4,v_5,\ldots,v_n$. Therefore a γ_s – set is $\{u_1, v_1\}$. Therefore $\gamma_s(G_3^P)=2$.
If $V_1=\{u_1,u_2\}$,$V_2=\{u_3\}$ and $V_3=V\backslash(V_1\cup V_2)$ then G^3_p is a connected graph. Here u_3 is adjacent to $u_1,u_2,u_4,u_5\ldots u_n$. And u_4 is adjacent to $v_1,v_2,\ldots,v_n$. Therefore a γ_s – set is $\{u_3, u_4\}$. Therefore $\gamma_s(G_3^P)=2$.
If $V_1=\{u_1,u_2\}$,$V_2=\{u_3,v_1\}$ and $V_3=V\backslash(V_1\cup V_2)$ then G^3_p is a connected graph. Here u_3 is adjacent to $u_1,u_2,u_3\ldots u_n$. And v_1 is adjacent to $v_2,v_3,\ldots,v_n$. Therefore a γ_s – set is $\{u_3, v_1\}$. Therefore $\gamma_s(G_3^P)=2$.
Proceeding like this, for other similar partitions we get $\gamma_s(G_3^P)=2$.

→ Case:3

In this case, G^3_p is a disconnected graph. Here $u_1,u_2,\ldots,u_m$ are isolated vertices. And $v_i\varepsilon W_j$ is adjacent to $v_1,v_2,\ldots v_n$ except i. Therefore a γ_s – set is $\{u_1,u_2,u_3,\ldots u_m, v_i\}$. Therefore $\gamma_s(G_3^P)=m+1$.

→ Case:4

In this case, G^3_p is a disconnected graph. Here $v_1,v_2,\ldots,v_n$ are isolated vertices. And $u_i\varepsilon W_j$ is adjacent to $u_1,u_2,\ldots u_n$ except i. Therefore a γ_s – set is $\{u_i,v_1,v_2,\ldots v_n\}$. Therefore $\gamma_s(G_3^P)=n+1$.

→ Case:5

In this case, G^3_p is a disconnected graph. Here $u_1,u_2,\ldots,u_m$ are isolated vertices. And $v_p,v_q\varepsilon W_j$ are adjacent to $v_1,v_2,\ldots v_n$. Therefore a γ_s – set is $\{u_1,u_2,u_3,\ldots u_m, v_p,v_q\}$. Therefore $\gamma_s(G_3^P)=m+2$.

→ Case:6

In this case, G^3_p is a disconnected graph. Here $v_1,v_2,\ldots,v_n$ are isolated vertices. And $u_p,u_q\varepsilon W_j$ are adjacent to $u_1,u_2,\ldots u_m$. Therefore a γ_s – set is $\{v_1,v_2,v_3,\ldots v_n, u_p,u_q\}$. Therefore $\gamma_s(G_3^P)=n+2$.

6. Theorem

Let G be a star ($K_{1,n}$ where $n\geq 4$) Let u be the star center and $u_1,u_2,\ldots,u_n$ be the pendant of G. Let (W_1,W_2,W_3) be the partition of G_3^P.

Then $\gamma_s(G_3^P)= \begin{cases} 1 \text{ if } W_k=\{u,u_i\},\ 1\leq i\leq n,\ k=1,2,3 \\ 3 \text{ if } W_1=\{u\},W_2=\{u_i,u_j\},W_3=V\backslash(W_1\cup W_2) \text{ or } W_1=\{u\},\ W_2=\{u_i,u_j,u_k\},\ W_3=V\backslash(W_1\cup W_2) \\ 2 \text{ otherwise} \end{cases}$

Proof:

→ case:1

If $W_1=\{u,u_1\}$, $W_2=\{u_2\}$, $W_3=\{u_3,u_4,\ldots,u_n\}$ then in G_3^P, u_1 is adjacent to all other vertices. Since, u_1 is adjacent to u and all other vertices of W_2 and W_3. Therefore a γ_s-set is$\{u_1\}$. Hence $\gamma_s(G_3^P)=1$.
If $W_1=\{u,u_2\}$, $W_2=\{u_3\}$, $W_3=\{u_1,u_4,u_5,\ldots,u_n\}$ then in G_3^P, u_2 is adjacent to u and all other vertices of W_2 and W_3. Therefore a γ_s-set is$\{u_2\}$. Hence $\gamma_s(G_3^P)=1$.
Proceeding like this, if $W_1=\{u,u_n\}$, $W_2=\{u_{n-1}\}$, $W_3=\{u_1,u_2,\ldots,u_{n-2}\}$ then in G_3^P, u is adjacent to u_2 and all other vertices W_2 and W_3. Therefore a γ_s-set is$\{u_n\}$. Hence $\gamma_s(G_3^P)=1$.

→ Case:2

If $W_1=\{u\}$, $W_2=\{u1,u_2\}$, $W_3=\{u_3,u_4,\ldots,u_n\}$ then in G_3^P, u is an isolated vertex. And u_1 is adjacent to $u_3,u_4,\ldots,u_n$, u_2 is adjacent to $u_3,u_4,\ldots,u_n$. Therefore a γ_s-set is$\{u,u_1,u_2\}$. Hence $\gamma_s(G_3^P)=3$.
If $W_1=\{u\}$, $W_2=\{u_1,u_2,u_3\}$, $W_3=V|(W_1\cup W_2)$ then in G_3^P, u is an isolated vertex. And u_1 is adjacent to $u_4,u_5,\ldots,u_n$, u_2 is adjacent to $u_4,u_5,\ldots,u_n$ and also u_4 is adjacent to u_1,u_2,u_3. Therefore a γ_s-set is$\{u,u_1,u_4\}$. Hence $\gamma_s(G_3^P)=3$.
If $W_1=\{u\}$, $W_2=\{u_1,u_2,\ldots,u_{n-2}\}$, $W_3=\{u_{n-1},u_n\}$ then in G_3^P, u is an isolated vertex. And u_1 is adjacent to u_{n-1},u_n and also u_{n-1} is adjacent to u_1,u_2,u_{n-2}. Therefore a γ_s-set is$\{u,u_1,u_{n-1}\}$. Hence $\gamma_s(G_3^P)=3$.
If $W_1=\{u\}$, $W_2=\{u_1,u_2,\ldots,u_{n-3}\}$, $W_3=\{u_{n-2},u_{n-1},u_n\}$ then in G_3^P, u is an isolated vertex. And u_1 is adjacent to u_{n-2},u_{n-1},u_n,also u_{n-2} is adjacent to $u_1,u_2,\ldots,u_{n-3}$. Therefore a γ_s-set is$\{u,u_1,u_{n-2}\}$. Hence $\gamma_s(G_3^P)=3$.

→ Case:3

If $W_1=\{u\}$, $W_2=\{u_1\}$, $W_3=V|(W_1\cup W_2)$ then in G_3^P, u is an isolated vertex. And u_1 is adjacent to $u_2,u_3,\ldots,u_n$. Therefore a γ_s-set is$\{u,u_1\}$. Hence $\gamma_s(G_3^P)=2$.
If $W_1=\{u\}$, $W_2=\{u_n\}$, $W_3=V|(W_1\cup W_2)$ then in G_3^P, u is an isolated vertex. And u_n is adjacent to $u_1,u_2,\ldots,u_{n-1}$. Therefore a γ_s-set is$\{u,u_n\}$. Hence $\gamma_s(G_3^P)=2$.
If $W_1=\{u,u_1,u_2\}$, $W_2=\{u_3\}$, $W_3=V|(W_1\cup W_2)$ then in G_3^P, u is adjacent to u_1,u_2. And u_1 is adjacent to all other vertices of W_2 and W_3. Therefore a γ_s-set is$\{u,u_1\}$. Hence $\gamma_s(G_3^P)=2$.

If $W_1=\{u,u_1,u_2\}$, $W_2=\{u_3,u_4,\ldots,u_{n-2}\}$, $W_3=\{u_{n-1},u_n\}$ then in G_3^p, u is adjacent to u_1,u_2. And u_1 is adjacent to all other vertices of W_2 and W_3. Therefore a γ_s-set is$\{u,u_1\}$. Hence $\gamma_s(G_3^p)=2$.
If $W_1=\{u,u_1,u_2,\ldots,u_{n-4}\}$, $W_2=\{u_{n-3},u_{n-2}\}$, $W_3=\{u_{n-1},u_n\}$ then in G_3^p, u is adjacent to $u_1,u_2,\ldots,u_{n-4}$. And u_1 is adjacent to all other vertices of W_2 and W_3. Therefore a γ_s-set is$\{u,u_1\}$. Hence $\gamma_s(G_3^p)=2$.
If $W_1=\{u,u_1,u_2,\ldots,u_{n-3}\}$, $W_2=\{u_{n-2}\}$, $W_3=\{u_{n-1},u_n\}$ then in G_3^p, u is adjacent to $u_1,u_2,\ldots,u_{n-3}$. And u_1 is adjacent to u_{n-2},u_{n-1},u_n. Therefore a γ_s-set is$\{u,u_1\}$. Hence $\gamma_s(G_3^p)=2$.
Proceeding like this, If $W_1=\{u,u_1,u_2,\ldots,u_{n-2}\}$, $W_2=\{u_{n-1}\}$, $W_3=\{u_n\}$ then in G_3^p, u is adjacent to $u_1,u_2,\ldots,u_{n-2}$. And u_1 is adjacent toun-1,un. Therefore a γ_s-set is$\{u,u_1\}$. Hence $\gamma_s(G_3^p)=2$.

7. Theorem

Let G be a star ($K_{1,n}$ where n=3) Let u be the star center and u_1,u_2,u_3 be the pendant of G. Let (W_1,W_2,W_3) be the partition of G_3^p.

$$\text{Then } \gamma_s(G_3^p)=\begin{cases} 1 & \text{if } W_k=\{u,u_i\},\ 1\le i\le n,\ k=1,2,3 \\ 2 & \text{if } W_i=\{u\},W_j=\{u_i\},W_3=V\setminus(W_1\cup W_2) \end{cases}$$

Proof:

If $W_1=\{u,u_1\}$, $W_2=\{u_2\}$, $W_3=\{u_3\}$ then in G_3^p, u_1 is adjacent to all other vertices. Therefore a γ_s-set is$\{u_1\}$. Hence $\gamma_s(G_3^p)=1$.
If $W_1=\{u,u_2\}$, $W_2=\{u_1\}$, $W_3=\{u_3\}$ then in G_3^p, u_2 is adjacent to all other vertices. Therefore a γ_s-set is$\{u_2\}$. Hence $\gamma_s(G_3^p)=1$.
If $W_1=\{u,u_3\}$, $W_2=\{u_1\}$, $W_3=\{u_2\}$ then in G_3^p, u_3 is adjacent to all other vertices. Therefore a γ_s-set is$\{u_3\}$. Hence $\gamma_s(G_3^p)=1$.

→ Case:2

If $W_1=\{u\}$, $W_2=\{u_1\}$, $W_3=\{u_2,u_3\}$ then in G_3^p, u is an isolated vertex. And u_1 is adjacent to u_2,u_3. Therefore a γ_s-set is$\{u,u_1\}$. Hence $\gamma_s(G_3^p)=2$.
If $W_1=\{u\}$, $W_2=\{u_2\}$, $W_3=\{u_1,u_3\}$ then in G_3^p, u is an isolated vertex. And u_2 is adjacent to u_1,u_3. Therefore a γ_s-set is$\{u,u_2\}$. Hence $\gamma_s(G_3^p)=2$.
If $W_1=\{u\}$, $W_2=\{u_3\}$, $W_3=\{u_1,u_2\}$ then in G_3^p, u is an isolated vertex. And u_3 is adjacent to u_1,u_2. Therefore a γ_s-set is$\{u,u_3\}$. Hence $\gamma_s(G_3^p)=2$.

8. Note

If W_k has only a star u then G_k^p is a disconnected graph for k=1,2,3.

9. Theorem

Let G be a path on n vertices (n≥5) say $v_1, v_2.........v_n$.Let v_1 and v_n are pendant vertices and $v_2,v_3,\ldots,v_{n-1}$ are vertices of degree 2 then

$$\text{Then } \gamma_s(G_3^p)=\begin{cases} 1 & \text{if } W_k=\{v_j\cup N(v_j)\},\ j=1,n,\ k=1,2,3 \text{ or } W_k=\{v_s,v_{s+1},v_{s+2}\}, \text{where } 1\le s\le n. \\ 3 & \text{if } W_i=\{v_r\},W_j=\{v_s\},W_k=V\setminus(W_i\cup W_j) \text{ where } v_r \text{ and } v_s \text{ are alternative, non pendant vertices.} \\ 2 & \text{otherwise} \end{cases}$$

Proof:

→ case :1

If $W_1=\{v_1,v_2\}$,$W_2=\{v_3,v_4\}$ and $W_3=V\setminus(W_1\cup W_2)$ then in $G^3{}_p$, v_1 is adjacent to to all other vertices. Therefore γ_s – set is $\{v_1\}$. Hence $\gamma_s(G_3^p)=1$.
If $W_1=\{v_1,v_2\}$,$W_2=\{v_3,v_4,v_5\}$ and $W_3=V\setminus(W_1\cup W_2)$ then in $G^3{}_p$, v_1 is adjacent to to all other vertices. Therefore γ_s – set is $\{v_1\}$. Hence $\gamma_s(G_3^p)=1$.
If $W_1=\{v_1,v_2\}$,$W_2=\{v_3,v_4,v_5,v_6\}$ and $W_3=V\setminus(W_1\cup W_2)$ then in $G^3{}_p$, v_1 is adjacent to to all other vertices. Therefore γ_s – set is $\{v_1\}$. Hence $\gamma_s(G_3^p)=1$.
Proceeding like this, If $W_1=\{v_1,v_2\}$,$W_2=\{v_3,v_4,\ldots,v_{n-1}\}$ and $W_3=\{v_n\}$ then in $G^3{}_p$, v_1 is adjacent to $v_2,v_3,\ldots,v_n$. Therefore γ_s – set is $\{v_1\}$. Hence $\gamma_s(G_3^p)=1$.
If $W_1=\{v_1\}$,$W_2=\{v_2,v_3,\ldots,v_{n-2}\}$ and $W_3=\{v_{n-1},v_n\}$ then in $G^3{}_p$, v_n is adjacent to to all other vertices. Therefore γ_s – set is $\{v_n\}$. Hence $\gamma_s(G_3^p)=1$.
If $W_1=\{v_1\}$,$W_2=\{v_2,v_3,v_4\}$ and $W_3=V\setminus(W_1\cup W_2)$ then in $G^3{}_p$, v_3 is adjacent to to all other vertices. Therefore γ_s – set is $\{v_3\}$. Hence $\gamma_s(G_3^p)=1$.

Proceeding like this,
If $W_1=\{v_1\}, W_2=\{v_2,v_3,\ldots,v_{n-3}\}$ and $W_3=\{v_{n-2},v_{n-1},v_n\}$ then in G^3_p, v_{n-1} is adjacent to to all other vertices. Therefore γ_s – set is $\{v_{n-1}\}$. Hence $\gamma_s(G_3^p)=1$.
And,if $W_1=\{v_1,v_3,v_4,v_5,\ldots,v_{n-10},v_{n-8},v_{n-7},v_{n-6}\}, W_2=\{v_2,v_6,v_7,v_8,\ldots,v_{n-9},v_{n-5},v_{n-4},v_{n-3}\}$ and $W_3=\{v_{n-2},v_{n-1},v_n\}$ then in G^3_p, v_{n-1} is adjacent to to all other vertices. Therefore γ_s – set is $\{v_{n-1}\}$. Hence $\gamma_s(G_3^p)=1$.

→ Case:2

If $W_1=\{v_3\}, W_2=\{v_5\}$ and $W_3=V\backslash(W_1\cup W_2)$ then in G^3_p, v_4 is an isolated vertex. And v_3 is adjacent to $v_1,v_5,v_6,\ldots,v_n$. Also v_5 is adjacent to $v_1,v_2,v_7,v_8,\ldots v_n$. Therefore γ_s – set is $\{v_3,v_4,v_5\}$. Hence $\gamma_s(G_3^p)=3$.
If $W_1=\{v_3\}, W_2=\{v_5\}$ and $W_3=V\backslash(W_1\cup W_2)$ then in G^3_p, v_4 is an isolated vertex. And v_3 is adjacent to $v_1,v_5,v_6,\ldots,v_n$. Also v_5 is adjacent to $v_1,v_2,v_7,v_8,\ldots v_n$. Therefore γ_s – set is $\{v_3,v_4,v_5\}$. Hence $\gamma_s(G_3^p)=3$.
If $W_1=\{v_2\}, W_2=\{v_4\}$ and $W_3=V\backslash(W_1\cup W_2)$ then in G^3_p, v_3 is an isolated vertex. And v_2 is adjacent to $v_4,v_5,v_6,\ldots,v_n$. Also v_4 is adjacent to $v_1,v_6,v_7,v_8,\ldots v_n$. Therefore γ_s – set is $\{v_2,v_3,v_4\}$. Hence $\gamma_s(G_3^p)=3$.
Proceeding like this, If $W_1=\{v_{n-1}\}, W_2=\{v_{n-3}\}$ and $W_3=V\backslash(W_1\cup W_2)$ then in G^3_p, v_{n-2} is an isolated vertex. And v_{n-3} is adjacent to $v_1,v_{2\ldots},v_{n-5},v_{n-1},v_n$. Also v_{n-1} is adjacent to $v_1,v_2,\ldots,v_{n-5},v_{n-4}$.Therefore γ_s – set is $\{v_{n-2},v_{n-3},v_{n-1}\}$. Hence
$\gamma_s(G_3^p)=3$.
If $W_1=\{v_{n-2}\}, W_2=\{v_n\}$ and $W_3=V\backslash(W_1\cup W_2)$ then in G^3_p, v_{n-1} is an isolated vertex. And v_{n-2} is adjacent to $v_1,v_2,\ldots v_{n-5},v_{n-4}$. Also v_n is adjacent to $v_1,v_2,\ldots,v_{n-4},v_{n-3}$.Therefore γ_s – set is $\{v_{n-1},v_{n-2},v_n\}$. Hence $\gamma_s(G_3^p)=3$.

→ Case:3

If $W_1=\{v_1\}, W_2=\{v_2\}$ and $W_3=V\backslash(W_1\cup W_2)$ then in G^3_p, v_1 is adjacent to $v_3,v_4,\ldots,v_n$. Also v_2 is adjacent to $v_4,v_5,\ldots v_n$. Therefore γ_s – set is $\{v_1,v_2\}$. Hence
$\gamma_s(G_3^p)=2$.
If $W_1=\{v_1\}, W_2=\{v_2,v3\}$ and $W_3=V\backslash(W_1\cup W_2)$ then in G^3_p, v_1 is adjacent to $v_3,v_4,\ldots,v_n$. Also v_2 is adjacent to $v_3,v_4,\ldots v_n$. Therefore a γ_s – set is $\{v_1,v_2\}$. Hence $\gamma_s(G_3^p)=2$.
If $W_1=\{v_1\}, W_2=\{v_2,v_3,v_4,v_5\}$ and $W_3=V\backslash(W_1\cup W_2)$ then in G^3_p, v_1 is adjacent to $v_3,v_4,\ldots,v_n$. Also v_2 is adjacent to $v_6,v_7,\ldots v_n$. Also there exists a path from v_2 to v_5 and v_6 to v_n. Therefore γ_s – set is $\{v_1,v_2\}$. Hence $\gamma_s(G_3^p)=2$.
If $W_1=\{v_1\}, W_2=\{v_2,v_3,\ldots v_{n-1}\}$ and $W_3=\{v_n\}$ then in G^3_p, v_1 is adjacent to $v_3,v_4,\ldots,v_n$. Also v_n is adjacent to $v_1,v_2,\ldots v_{n-2}$. Also, there exists a path from v_2 to v_{n-2}. Therefore γ_s – set is $\{v_1,v_2\}$. Hence $\gamma_s(G_3^p)=2$.
If $W_1=\{v_1\}, W_2=\{v_3\}$ and $W_3=V\backslash(W_1\cup W_2)$ then in G^3_p is a disconnected graphwith two components. Here v_2 is an isolated vertex. Also v_1 is adjacent to $v_3,v_4,\ldots v_n$. Also, there exists a path from v_4 to v_{11}. Therefore γ_s – set is $\{v_1,v_2\}$. Hence $\gamma_s(G_3^p)=2$.
If $W_1=\{v_1,v_3\}, W_2=\{v_5,v_7\}$ and $W_3=V\backslash(W_1\cup W_2)$ then in G^3_p, v_1 is adjacent to $v_4,v_5,\ldots,v_n$. Also v_5 is adjacent to $v_2,v_3,v_7,v_8,\ldots,v_n$. Therefore γ_s – set is $\{v_1,v_5\}$. Hence $\gamma_s(G_3^p)=2$.
Proceeding like this,if $W_1=\{v_1,v_3,v_5,\ldots,v_{n-10},v_{n-8},v_{n-6}\}, W_2=\{v_7,v_9,v_{11},\ldots v_{n-4},v_{n-2},v_n\}$ and $W_3=V\backslash(W_1\cup W_2)$ then in G^3_p, v_1 is adjacent to $v_4,v_6,v_8,v_{10}.,v_7,v_9,v_{11},v_{n-4},v_{n-2},v_n,v_{n-9},v_{n-7},v_{n-5},v_{n-3},v_{n-1}$. Also v_n is adjacent to $v_1,v_2,\ldots v_{n-2}$. Therefore γ_s – set is $\{v_1,v_n\}$. Hence $\gamma_s(G_3^p)=2$.
If $W_1=\{v_2,v_4,v_6\}, W_2=\{v_{n-3},v_{n-1}\}$ and $W_3=V\backslash(W_1\cup W_2)$ then in G^3_p, v_2 is adjacent to $v_5,v_7,v_8\ldots,v_{n-2},v_{n-1},v_n$. Also v_{n-1} is adjacent to $v_1,v_2,\ldots v_{n-2}$. Therefore γ_s – set is $\{v_2,v_{n-1}\}$. Hence $\gamma_s(G_3^p)=2$.

10. Theorem

Let G be a path on n vertices with n=3 say $v_1, v_2.........v_n$ then $\gamma_s(G_3^p)=2$.

Proof:
If $W_1=\{v_1\}, W_2=\{v_2\}$ and $W_3=\{v_3\}$ then G^3_p is a disconnected graph with two components. Here v_2 is an isolated vertex. Also v_1 is adjacent to v_3. Therefore γ_s – set is $\{v_1,v_2\}$. Hence $\gamma_s(G_3^p)=2$.
If $W_1=\{v_2\}, W_2=\{v_3\}$ and $W_3=\{v_1\}$ then G^3_p is a disconnected graph with two components. Here v_2 is an isolated vertex. Also v_1 is adjacent to v_3. Therefore γ_s – set is $\{v_1,v_2\}$. Hence $\gamma_s(G_3^p)=2$.
If $W_1=\{v_3\}, W_2=\{v_1\}$ and $W_3=\{v_2\}$ then G^3_p is a disconnected graph with two components. Here v_2 is an isolated vertex. Also v_1 is adjacent to v_3. Therefore γ_s – set is $\{v_1,v_2\}$. Hence $\gamma_s(G_3^p)=2$.

11. Theorem

Let G be a path on n vertices (n=4) say $v_1, v_2.........v_n$.Let v_1 and v_n are pendant vertices and $v_2,v_3,\ldots,v_{n-1}$ are vertices of degree 2 then

Then $\gamma_s(G_3{}^p)=\begin{cases} 1 & \text{if } W_k=\{v_j\cup N(v_j)\},\ 1\le j\le n \text{ or } W_k=\{v_r\}, \text{where } k\ne i,\ W_l=\{v_{s+1}\} \text{and vice versa.} \\ 3 & \text{if } W_k=\{v_1,v_n\} \text{where } k=1,2. \end{cases}$

Proof:
If $W_1=\{v_1\}, W_2=\{v_2\}$ and $W_3=\{v_3,v_4\}$ then in G^3_p, v_4 is adjacent to all other vertices. Therefore γ_s – set is $\{v_4\}$. Hence $\gamma_s(G_3^p)=1$.

If $W_1=\{v_3\}, W_2=\{v_4\}$ and $W_3=\{v_1,v_2\}$ then in G^3_p, v_1 is adjacent to all other vertices. Therefore γ_s – set is $\{v_1\}$. Hence $\gamma_s(G_3^p)=1$.

→ Case:2

If $W_1=\{v_2\}, W_2=\{v_3\}$ and $W_3=\{v_2,v_4\}$then in G^3_p is a disconnected graph with two components. Here v_1 is adjacent to v_3. Also v_2 is adjacent to v_4. Therefore γ_s – set is $\{v_1,v_2,v_3\}$. Hence $\gamma_s(G_3^p)=3$

→ Case:3

If $W_1=\{v_1\}, W_2=\{v_4\}$ and $W_3=\{v_2,v_3\}$then in G^3_p, v_2 is adjacent to v_3,v_4. Also v_1 is adjacent to v_3 and v_4. Therefore γ_s – set is $\{v_2,v_3\}$. Hence $\gamma_s(G_3^p)=2$.

If $W_1=\{v_2\}, W_2=\{v_4\}$ and $W_3=\{v_1,v_3\}$then G^3_p is a disconnected graph with two components. In G^3_p, v_3 is an isolated vertex. And v_4 is adjacent to v_1,v_2. Therefore γ_s – set is $\{v_3,v_4\}$. Hence $\gamma_s(G_3^p)=2$.

12. Theorem

Let G be a cycle with n vertices with n=4 say $v_1, v_2 ....v_n$ then

$\gamma_s(G_3^{\ p})=$
- 2 if $W_i=\{v_j \cup N(v_j)\}$, $1 \le j \le n$, $W_j=\{v_s\}, W_k=\{v_{s+1}\}$ for some s and vice versa.
- 3 if $W_i=\{v_j \cup N(v_j)\}$, $1 \le j \le n$, $W_j=\{v_r\}, W_3=\{v_s\}$where v_r and v_s are non- adjacent vertices or $W_1=\{v_p,v_q\}$ where v_p,v_q are non adjacent vertices,$W_2=\{v_r\}, W_3=\{v_s\}$ where v_r and v_s are non – adjacent vertices and vice versa.

Proof:

If $W_1=\{v_1\}, W_2=\{v_2\}$ and $W_3=\{v_3,v_4\}$then v_3 is adjacent to v_2 and v_4.And v_4 is adjacent to v_1,v_3. Therefore γ_s – set is $\{v_3,v_4\}$. Hence $\gamma_s(G_3^p)=2$.

If $W_1=\{v_2\}, W_2=\{v_4\}$ and $W_3=\{v_1,v_3\}$then v_1 is adjacent to v_3 and v_4.And v_3 is adjacent to v_1,v_2. Therefore γ_s – set is $\{v_1,v_3\}$. Hence $\gamma_s(G_3^p)=2$.

If $W_1=\{v_3\}, W_2=\{v_4\}$ and $W_3=\{v_1,v_2\}$then v_1 is adjacent to v_2 and v_4.And v_2 is adjacent to v_1,v_3. Therefore γ_s – set is $\{v_3,v_4\}$. Hence $\gamma_s(G_3^p)=2$.

If $W_1=\{v_2\}, W_2=\{v_4\}$ and $W_3=\{v_1,v_3\}$then v_1 is adjacent to v_3 and v_4.And v_2 is adjacent to v_3. Therefore γ_s – set is $\{v_1,v_2\}$. Hence $\gamma_s(G_3^p)=2$.

→ Case:2

If $W_1=\{v_2,v_3\}, W_2=\{v_1\}$ and $W_3=\{v_4\}$then G^3_p is a disconnected graph with three components. Here v_2 and v_3 are isolated vertices. Also v_1 is adjacent to v_4. Therefore γ_s – set is $\{v_1,v_2,v_3\}$. Hence $\gamma_s(G_3^p)=3$

If $W_1=\{v_1\}, W_2=\{v_4\}$ and $W_3=\{v_2,v_3\}$then G^3_p is a disconnected graph with three components. Here v_2 and v_3 are isolated vertices. Also v_1 is adjacent to v_4. Therefore γ_s – set is $\{v_2,v_3,v_4\}$. Hence $\gamma_s(G_3^p)=3$

If $W_1=\{v_2\}, W_2=\{v_3\}$ and $W_3=\{v_1,v_4\}$then G^3_p is a disconnected graph with three components. Here v_1 and v_4 are isolated vertices. Also v_2 is adjacent to v_3. Therefore γ_s – set is $\{v_1,v_2,v_4\}$. Hence $\gamma_s(G_3^p)=3$.

13. Theorem

Let G be a cycle with n vertices with n=5 say $v_1, v_2 ....v_n$ then

Then $\gamma_s(G_3^{\ p})=$
- 1 if $W_k=\{v_s,v_{s+1},v_{s+2}\}$ for k=1,2,3.
- 2 if $W_i=\{v_j \cup N(v_j)\}$, $1 \le j \le n$, $W_j=\{v_p\}$ for some p, $W_k=V \backslash (W_i \cup W_j)$ and vice versa.
- 3 if $W_i=\{v_k\}, W_j=\{v_l\}$ where v_k and v_l are no adjacent vertices or $W_i=\{v_p\}$where $1 \le p \le n$, $W_j=\{v_j,v_k\}$ where $i \ne j$ $W_k=V \backslash (W_i \cup W_j)$ where v_j and v_k are non adjacent vertices.

Proof:

If $W_1=\{v_1\}, W_2=\{v_2\}$ and $W_3=\{v_3,v_4,v_5\}$then v_4 is adjacent to all other vertices. Therefore γ_s – set is $\{v_4\}$. Hence $\gamma_s(G_3^p)=1$.

If $W_1=\{v_1\}, W_2=\{v_5\}$ and $W_3=\{v_2,v_3,v_4\}$then v_3 is adjacent to all other vertices. Therefore γ_s – set is $\{v_3\}$. Hence $\gamma_s(G_3^p)=1$.

If $W_1=\{v_2\}, W_2=\{v_3\}$ and $W_3=\{v_1,v_4,v_5\}$then v_5 is adjacent to all other vertices. Therefore γ_s – set is $\{v_5\}$. Hence $\gamma_s(G_3^p)=1$.

If $W_1=\{v_3\}, W_2=\{v_4\}$ and $W_3=\{v_1,v_2,v_5\}$then v_1 is adjacent to all other vertices. Therefore γ_s – set is $\{v_1\}$. Hence $\gamma_s(G_3^p)=1$.

If $W_1=\{v_4\}, W_2=\{v_5\}$ and $W_3=\{v_1,v_2,v_3\}$then v_2 is adjacent to all other vertices. Therefore γ_s – set is $\{v_2\}$. Hence $\gamma_s(G_3^p)=1$.

→ Case:2

If $W_1=\{v_1\}, W_2=\{v_2,v_3\}$ and $W_3=\{v_4,v_5\}$then v_2 is adjacent tov_3,v_4,v_5.And v_3 is adjacent to v_1,v_5. Therefore γ_s – set is $\{v_2,v_3\}$. Hence $\gamma_s(G_3^p)=2$.

If $W_1=\{v_2\}, W_2=\{v_1,v_3\}$ and $W_3=\{v_4,v_5\}$then v_4 is adjacent tov_1,v_2,v_5.And v_5 is adjacent to v_2,v_3,v_4. Therefore γ_s – set is $\{v_4,v_5\}$. Hence $\gamma_s(G_3^p)=2$.

If $W_1= \{v_2\}$,$W_2= \{v_1,v_5\}$ and $W_3=\{v_3,v_4\}$then v_1 is adjacent tov_3,v_4.And v_5 is adjacent to v_2,v_3. Therefore γ_s – set is $\{v_1,v_5\}$. Hence $\gamma_s (G_3^p) =2$.
If $W_1= \{v_3\}$,$W_2= \{v_1,v_2\}$ and $W_3=\{v_4,v_5\}$then v_1 is adjacent tov_2,v_3,v_4.And v_2 is adjacent to v_1,v_4,v_5. Therefore γ_s – set is $\{v_1,v_2\}$. Hence $\gamma_s (G_3^p) =2$.
If $W_1= \{v_4\}$,$W_2= \{v_1,v_2\}$ and $W_3=\{v_3,v_5\}$then v_1 is adjacent tov_2,v_3,v_4.And v_2 is adjacent to v_1,v_4,v_5. Therefore γ_s – set is $\{v_1,v_2\}$. Hence $\gamma_s (G_3^p) =2$.
If $W_1= \{v_5\}$,$W_2= \{v_1,v_2\}$ and $W_3=\{v_3,v_4\}$then v_1 is adjacent tov_2,v_3,v_4. And v_2 is adjacent to v_1,v4,v_5. Therefore γ_s – set is $\{v_1,v_2\}$. Hence $\gamma_s (G_3^p) =2$.
If $W_1= \{v_5\}$,$W_2= \{v_1,v_4\}$ and $W_3=\{v_2,v_3\}$then v_2 is adjacent to v_3,v_4,v_5.And v_3 is adjacent to v_1,v_2,v_5. Therefore γ_s – set is $\{v_2,v_3\}$. Hence $\gamma_s (G_3^p) =2$.
If $W_1= \{v_5\}$,$W_2= \{v_1,v_4\}$ and $W_3=\{v_2,v_3\}$then v_2 is adjacent tov_3,v_4,v_5.And v_3 is adjacent to v_1,v_2,v_5. Therefore γ_s – set is $\{v_2,v_3\}$. Hence $\gamma_s (G_3^p) =2$.

→ Case:3

If $W_1= \{v_1\}$,$W_2= \{v_3\}$ and $W_3=\{v_2,v_4,v_5\}$then G^3_p is a disconnected graph with two components. Here v_2 is an isolated vertex. And v_1 is adjacent to v_4,v_5.And v_3 is adjacent to v_1,v_5. Therefore γ_s – set is $\{v_1,v_2,v_3\}$. Hence $\gamma_s (G_3^p) =3$.
If $W_1= \{v_1\}$,$W_2= \{v_4\}$ and $W_3=\{v_2,v_3,v_5\}$then G^3_p is a disconnected graph with two components. Here v_5 is an isolated vertex. And v_2 is adjacent to v_3,v_4. And v_3 is adjacent to v_1,v_2. Therefore γ_s – set is $\{v_2,v_3,v_5\}$. Hence $\gamma_s (G_3^p) =3$.
If $W_1= \{v_2\}$,$W_2= \{v_4\}$ and $W_3=\{v_1,v_3,v_5\}$then G^3_p is a disconnected graph with two components. Here v_3 is an isolated vertex. And v_1 is adjacent to v_4,v_5.And v_2 is adjacent to v_4,v_5. Therefore γ_s – set is $\{v_1,v_2,v_3\}$. Hence $\gamma_s (G_3^p) =3$.
If $W_1= \{v_2\}$,$W_2= \{v_5\}$ and $W_3=\{v_1,v_3,v_4\}$then G^3_p is a disconnected graph with two components. Here v_1 is an isolated vertex. And v_2 is adjacent to v_4,v_5. And v_3 is adjacent to v_4,v_5. Therefore γ_s – set is $\{v_1,v_2,v_3\}$. Hence $\gamma_s (G_3^p) =3$.
If $W_1= \{v_3\}$,$W_2= \{v_5\}$ and $W_3=\{v_1,v_2,v_4\}$then G^3_p is a disconnected graph with two components. Here v_4 is an isolated vertex. And v_1 is adjacent to v_2,v_3.And v_2 is adjacent to v_1,v_5. Therefore γ_s – set is $\{v_1,v_2,v_4\}$. Hence $\gamma_s (G_3^p) =3$.
If $W_1= \{v_2\}$,$W_2= \{v_1,v_4\}$ and $W_3=\{v_3,v_5\}$then G^3_p is a disconnected graph with two components. In one component v_1 is adjacent to v_3.And in the other component v_2 is adjacent to v_4,v_5. Therefore γ_s – set is $\{v_1,v_2,v_3\}$. Hence $\gamma_s (G_3^p) =3$.
If $W_1= \{v_3\}$,$W_2= \{v_1,v_4\}$ and $W_3=\{v_2,v_5\}$then G^3_p is a disconnected graph with two components. In one component v_2 is adjacent to v_4.And in the other component v_3 is adjacent to v_1,v_5. Therefore γ_s – set is $\{v_2,v_3,v_4\}$. Hence $\gamma_s (G_3^p) =3$.
If $W_1= \{v_5\}$,$W_2= \{v_1,v_3\}$ and $W_3=\{v_2,v_4\}$then G^3_p is a disconnected graph with two components. In one component v_1 is adjacent to v_4.And in the other component v_5 is adjacent to v_2,v_3. Therefore γ_s – set is $\{v_1,v_4,v_5\}$. Hence $\gamma_s (G_3^p) =3$.

14. Theorem

Let G be a cycle with n vertices with n=6 say v_1, v_2.........v_n

Then $\gamma_s (G_3{}^p)= \begin{cases} 1 & \text{if } W_k=\{v_j \cup N(v_j)\},\ 1 \le j \le n, \text{ or } W_k=\{v_s,v_{s+1},v_{s+2}\} \text{ for } k=1,2,3. \\ 3 & \text{if } W_k=\{v_i,v_j,v_l\} \text{where } W_k \text{ contains all alternative vertices for } k=1,2,3. \\ 2 & \text{otherwise} \end{cases}$

Proof:

→ Case:1

If $W_1= \{v_1,v_2,v_3\}$,$W_2= \{v_4,v_5\}$ and $W_3=\{v_6\}$then in G^3_p v_2 is adjacent to all other vertices. Therefore γ_s – set is $\{v_2\}$. Hence $\gamma_s (G_3^p) =1$.
If $W_1= \{v_2,v_3,v_4\}$,$W_2= \{v_1\}$ and $W_3=\{v_5,v_6\}$then in G^3_p v_3 is adjacent to all other vertices. Therefore γ_s – set is $\{v_3\}$. Hence $\gamma_s (G_3^p) =1$.
If $W_1= \{v_3,v_4,v_5\}$,$W_2= \{v_1,v_2\}$ and $W_3=\{v_6\}$then in G^3_p v_4 is adjacent to all other vertices. Therefore γ_s – set is $\{v_4\}$. Hence $\gamma_s (G_3^p) =1$.
Proceeding like this, If $W_1= \{v_1,v_2\}$,$W_2= \{v_3\}$ and $W_3=\{v_4,v_5,v_6\}$then in G^3_p v_5 is adjacent to all other vertices. Therefore γ_s – set is $\{v_5\}$. Hence $\gamma_s (G_3^p) =1$.

→ Case:2

If $W_1= \{v_1\}$,$W_2= \{v_3,v_5\}$ and $W_3=\{v_2,v_4,v_6\}$then in G^3_p,v_1 is adjacent to v_3,v_4,v_5.And v_3 is adjacent to v_1,v_6.Also v_5 is adjacent to v_1,v_2. Therefore γ_s – set is $\{v_1,v_3,v_5\}$. Hence $\gamma_s (G_3^p) =3$.

If $W_1 = \{v_2\}, W_2 = \{v_4, v_6\}$ and $W_3 = \{v_1, v_3, v_5\}$ then in G^3_p, v_2 is adjacent to v_4, v_5, v_6. And v_4 is adjacent to v_1, v_2. Also v_6 is adjacent to v_2, v_3. Therefore γ_s – set is $\{v_2, v_4, v_6\}$. Hence $\gamma_s (G_3^p) = 3$.
If $W_1 = \{v_2, v_4\}, W_2 = \{v_6\}$ and $W_3 = \{v_1, v_3, v_5\}$ then in G^3_p, v_6 is adjacent to v_2, v_3, v_4. And v_4 is adjacent to v_1, v_6. Also v_2 is adjacent to v_5, v_6. Therefore γ_s – set is $\{v_2, v_4, v_6\}$. Hence $\gamma_s (G_3^p) = 3$.

→ Case:3

If $W_1 = \{v_1\}, W_2 = \{v_2\}$ and $W_3 = \{v_3, v_4, \ldots, v_n\}$ then in G^3_p, v_1 is adjacent to $v_3, v_4, v_5 \ldots, v_{n-1}$. And v_2 is adjacent to $v_4, v_5, \ldots, v_n$. Therefore γ_s – set is $\{v_1, v_2\}$. Hence $\gamma_s (G_3^p) = 2$.
If $W_1 = \{v_1\}, W_2 = \{v_3\}$ and $W_3 = \{v_2, v_4, v_5, v_6\}$ then G^3_p is a disconnected graph with two components. Here v_2 is an isolated vertex. And in the other component v5 is adjacent to all other vertices. Therefore γ_s – set is $\{v_2, v_5\}$. Hence $\gamma_s (G_3^p) = 2$.
If $W_1 = \{v_1\}, W_2 = \{v_4\}$ and $W_3 = \{v_2, v_3, v_5, v_6\}$ then in G^3_p, v_1 is adjacent to v_3, v_4, v_5. And v_4 is adjacent to v_1, v_2, v_6. Therefore γ_s – set is $\{v_1, v_4\}$. Hence $\gamma_s (G_3^p) = 2$.
If $W_1 = \{v_1\}, W_2 = \{v_5\}$ and $W_3 = \{v_2, v_3, v_4, v_6\}$ then G^3_p is a disconnected graph with two components. Here v_6 is an isolated vertex. And in the other component v_3 is adjacent to v_1, v_2, v_4, v_5. Therefore γ_s – set is $\{v_3, v_6\}$. Hence
$\gamma_s (G_3^p) = 2$.
If $W_1 = \{v_1\}, W_2 = \{v_6\}$ and $W_3 = \{v_2, v_3, v_4, v_5\}$ then in G^3_p, v_1 is adjacent to v_3, v_4, v_5. And v_6 is adjacent to v_2, v_3, v_4. And there exists a path from v_2 to v_5. Therefore γ_s – set is $\{v_1, v_6\}$. Hence $\gamma_s (G_3^p) = 2$.
Proceeding like this, if $W_1 = \{v_1, v_2, v_3, \ldots v_{n-2}\}, W_2 = \{v_{n-1}\}$ and $W_3 = \{v_n\}$ then in G^3_p, v_n is adjacent to $v_2, v_3, \ldots, v_{n-2}$. And v_1 is adjacent to v_2, v_{n-1}, v_n. And there exists a path from v_1 to v_{n-2}. Therefore γ_s – set is $\{v_1, v_n\}$. Hence
$\gamma_s (G_3^p) = 2$.
Also if $W_1 = \{v_1, v_3\}, W_2 = \{v_2, v_4\}$ and $W_3 = \{v_5, v_6\}$ then in G^3_p, v_5 is adjacent to v_2, v_3, v_6. And v_6 is adjacent to v_2, v_3, v_4. Therefore γ_s – set is $\{v_5, v_6\}$. Hence $\gamma_s (G_3^p) = 2$.

15. Theorem

Let G be a cycle with $n \geq 7$ vertices say $v_1, v_2 \ldots\ldots\ldots v_n$ then

Then $\gamma_s (G_3^{\,p}) = $
- 1 if $W_k = \{v_s, v_{s+1}, v_{s+2}\}$ for k=1,2,3.
- 3 if $W_i = \{v_k\}$, $1 \leq k \leq n$, $W_j = \{v_l\}$ where v_l is any vertex and v_l is an alternative vertex of v_k or W_k contains 2 or 3 alternative vertices.
- 2 otherwise.

Proof:
If $W_1 = \{v_1, v_2, v_3\}, W_2 = \{v_4, v_5\}$ and $W_3 = \{v_6, v_7, \ldots, v_n\}$ then in G^3_p, v_2 is adjacent to all other vertices. Therefore γ_s – set is $\{v_2\}$. Hence $\gamma_s (G_3^p) = 1$.
If $W_1 = \{v_1, v_2, v_3\}, W_2 = \{v_4, v_5, v_6\}$ and $W_3 = \{v_7, v_8, \ldots, v_n\}$ then in G^3_p, v_2 is adjacent to all other vertices. Therefore γ_s – set is $\{v_3\}$. Hence $\gamma_s (G_3^p) = 1$.
Proceeding like this, if $W_1 = \{v_1, v_2, v_3\}, W_2 = \{v_4, v_5, \ldots, v_{n-3}\}$ and $W_3 = \{v_{n-2}, v_{n-}, v_n\}$ then in G^3_p, v_{n-1} is adjacent to all other vertices. Therefore γ_s – set is $\{v_{n-1}\}$. Hence $\gamma_s (G_3^p) = 1$.
If $W_1 = \{v_1, v_2\}, W_2 = \{v_3, v_4, v_5\}$ and $W_3 = \{v_6, v_7, \ldots, v_n\}$ then in G^3_p, v_4 is adjacent to all other vertices. Therefore γ_s – set is $\{v_4\}$. Hence $\gamma_s (G_3^p) = 1$.
If $W_1 = \{v_1, v_2, v_3, v_4\}, W_2 = \{v_5, v_6, v_7\}$ and $W_3 = \{v_8, v_9, \ldots, v_n\}$ then in G^3_p, v_6 is adjacent to all other vertices. Therefore γ_s – set is $\{v_6\}$. Hence $\gamma_s (G_3^p) = 1$.
Proceeding like this, if $W_1 = \{v_1, v_2, v_3, v_4, v_5\}, W_2 = \{v_6, v_7, \ldots v_{n-3}\}$ and $W_3 = \{v_{n-2}, v_{n-1}, v_n\}$ then in G^3_p, v_{n-1} is adjacent to all other vertices. Therefore γ_s – set is $\{v_{n-1}\}$. Hence $\gamma_s (G_3^p) = 1$.
If $W_1 = \{v_1, v_2, \ldots, v_{n-8}\}, W_2 = \{v_{n-7}, v_{n-6}, v_{n-5}, v_{n-4}, v_{n-3}\}$ and $W_3 = \{v_{n-2}, v_{n-1}, v_n\}$ then in G^3_p, v_{n-1} is adjacent to all other vertices. Therefore γ_s – set is $\{v_{n-1}\}$. Hence $\gamma_s (G_3^p) = 1$.

→ Case:2

If $W_1 = \{v_1\}, W_2 = \{v_2\}$ and $W_3 = \{v_3, v_4, \ldots, v_n\}$ then in G^3_p, v_1 is adjacent to $v_3, v_4, v_5 \ldots, v_n$ and v_2 is adjacent to $v_4, v_5, \ldots v_n$. And there exists a path from v_3 to v_n. Therefore γ_s – set is $\{v_1, v_2\}$. Hence $\gamma_s (G_3^p) = 2$.
If $W_1 = \{v_1, v_2\}, W_2 = \{v_3. v_4\}$ and $W_3 = \{v_5, v_6, \ldots, v_n\}$ then in G^3_p, v_1 is adjacent to $v_2, v_3, v_4 \ldots, v_{n-1}$ and v_2 is adjacent to $v_4, v_5, \ldots v_n$. And there exists a path from v_5 to v_n. Therefore γ_s – set is $\{v_1, v_2\}$. Hence $\gamma_s (G_3^p) = 2$.
If $W_1 = \{v_1, v_2, v_3, v_4\}, W_2 = \{v_5. v_6\}$ and $W_3 = \{v_7, v_8, \ldots, v_n\}$ then in G^3_p, v_5 is adjacent to $v_1, v_2, v_3, v_6, v_7, \ldots\ldots, v_n$ and v_6 is adjacent to $v_1, v_2, v_3, v_4, v_5, v_8, v_9, \ldots v_n$. Therefore γ_s – set is $\{v_5, v_6\}$. Hence $\gamma_s (G_3^p) = 2$.
Proceeding like this, if $W_1 = \{v_1, v_2, v_3, \ldots v_{n-3}\}, W_2 = \{v_{n-2}\}$ and $W_3 = \{v_{n-1}, v_n\}$ then in G^3_p, v_{n-1} is adjacent to $v_1, v_2, \ldots, v_{n-3}, v_n$ and v_n is adjacent to $v_2, v_3, v_4 \ldots, v_{n-1}$. Therefore γ_s – set is $\{v_{n-1}, v_n\}$. Hence $\gamma_s (G_3^p) = 2$.
If $W_1 = \{v_1\}, W_2 = \{v_4\}$ and $W_3 = \{v_7, v_8, \ldots, v_n\}$ then G^3_p is a connected graph. And v_1 is adjacent to $v_3, v_4, \ldots\ldots, v_{n-1}$ and v_4 is adjacent to $v_1, v_2, v_6, v_7, \ldots v_n$. Therefore γ_s – set is $\{v_1, v_4\}$. Hence $\gamma_s (G_3^p) = 2$.

If $W_1=\{v_1\}, W_2=\{v_5\}$ and $W_3=\{v_7,v_8,\dots,v_n\}$ then v_1 is adjacent to $v_3,v_4,\dots,v_{n-1}$ and v_5 is adjacent to $v_2,v_3,v_7,v_8,\dots v_n$. Therefore γ_s – set is $\{v_1,v_5\}$. Hence $\gamma_s(G_3^p)=2$.
Proceeding like this, if $W_1=\{v_1\}, W_2=\{v_n\}$ and $W_3=\{v_2,v_3,\dots,v_{n-1}\}$ then v_1 is adjacent to $v_3,v_4,\dots,v_{n-1}$ and v_n is adjacent to $v_2,v_3,\dots v_{n-2}$. Also there exists a path from v_2 to v_{n-1}. Therefore γ_s – set is $\{v_1,v_n\}$. Hence $\gamma_s(G_3^p)=2$.
If $W_1=\{v_1,v_2,v_3,v_4\}, W_2=\{v_5.v_6,v7,v8\}$ and $W_3=\{v_9,v_{10},\dots,v_n\}$ then v_2 is adjacent to $v_1,v_3,v_5,v_6,v_7,\dots,v_n$ and v_3 is adjacent to $v_2,v_4,v_5,\dots v_n$. Also there exists a path from v_1 to v_4, v_5 to v_8 and v_9 to v_n. Therefore γ_s – set is $\{v_2,v_3\}$. Hence $\gamma_s(G_3^p)=2$.
If $W_1=\{v_1,v_3\}, W_2=\{v_5.v_7\}$ and $W_3=V\backslash(W_1\cup W_2)$ then v_1 is adjacent to $v_4,v_5,\dots,v_{n-1}$ and v_7 is adjacent to $v_1,v_2,\dots,v_5,v_9,v_{10},\dots v_n$. Therefore γ_s – set is $\{v_1,v_7\}$. Hence $\gamma_s(G_3^p)=2$.
If $W_1=\{v_3,v_5\}, W_2=\{v_7.v_9,v_{11}\}$ and $W_3=V\backslash(W_1\cup W_2)$ then V_3 is adjacent to $v_1,v_2,v_6,v_7,\dots,v_n$ and v_7 is adjacent to $v_1,v_2,\dots,v_5,v9,v10,\dots v_n$. Therefore γ_s – set is $\{v_3,v_7\}$. Hence $\gamma_s(G_3^p)=2$.
Proceeding like this, if $W_1=\{v_1,v_3,v_5,\dots v_{2n-1}\}, W_2=\{v_2.v_4,v_6,v_8\}$ and $W_3=V\backslash(W_1\cup W_2)$ then v_2 is adjacent to $v_5,v_6,\dots,v_{2n-1},v_{2n}$ and v_{10} is adjacent to $v_1,v_2,v_3,v_4,v_6,v_8,v_9,\dots v_{2n-1}$. Therefore γ_s – set is $\{v_2,v_{10}\}$. Hence $\gamma_s(G_3^p)=2$.

→ Case:3

If $W_1=\{v_1\}, W_2=\{v_3\}$ and $W_3=V\backslash(W_1\cup W_2)$ then G_3^p is a disconnected graph with two components. v_2 is an isolated vertex. v_1 is adjacent to $v_3,v_4,\dots,v_{n-1}$ and v_3 is adjacent to $v_5,v_6,\dots v_n$. Also there exists a path from v_4 to v_n. Therefore γ_s – set is $\{v_1,v_2,v_3\}$. Hence $\gamma_s(G_3^p)=3$.
If $W_1=\{v_2\}, W_2=\{v_4\}$ and $W_3=V\backslash(W_1\cup W_2)$ then G_3^p is a disconnected graph with two components. v_3 is an isolated vertex. v_2 is adjacent to $v_5,v_6,\dots,v_n$ and v_4 is adjacent to $v_1,v_6,v_7\dots v_n$. Therefore γ_s – set is $\{v_2,v_3,v_4\}$. Hence
$\gamma_s(G_3^p)=3$.
Proceeding like this, if $W_1=\{v_{n-3}\}, W_2=\{v_{n-1}\}$ and $W_3=V\backslash(W_1\cup W_2)$ then in G_3^p. v_{n-2} is an isolated vertex. v_{n-3} is adjacent to $v_1,v_2,\dots,v_{n-4},v_n$ and v_{n-1} is adjacent to $v_1,v_2,\dots v_{n-4}$. Therefore γ_s – set is $\{v_{n-1},v_{n-2},v_{n-3}\}$. Hence
$\gamma_s(G_3^p)=3$.
If $W_1=\{v_{n-2}\}, W_2=\{v_n\}$ and $W_3=V\backslash(W_1\cup W_2)$ then G_3^p is a disconnected graph with two components. v_{n-1} is an isolated vertex. v_{n-2} is adjacent to $v_1,v_2,\dots,v_{n-4}$ and v_n is adjacent to $v_1,v_2,\dots v_{n-3}$. Therefore
γ_s – set is $\{v_{n-2},v_{n-1},v_n\}$. Hence $\gamma_s(G_3^p)=3$.

16. Conjecture

For any complete graph $3\le\gamma_s(G_3^p)\le n-1$, the lower bound is attained when $|W_1|=|W_2|=1$ and the upper bound is attained when n=6.

17. Theorem

If G and G_3^p are connected graphs then $2\le\gamma_s(G)+\gamma_s(G_3^p)\le n+1$.

Proof:
For any connected graph G and G_3^p, $\gamma_s(G)\ge1$ and $\gamma_s(G_3^p)\ge1$. Therefore $2\le\gamma_s(G)+\gamma_s(G_2^p)$.
Also we can justify $\gamma_s(G)+\gamma_s(G_2^p)\le n+1$ with the following examples.

Example:

The upper bound is attained at n=8 with a path. Here $\gamma_s(G)=7$ and $\gamma_s(G_3^p)\le2$ for a connected graph G^3_p.
Therefore $\gamma_s(G)+\gamma_s(G_3^p)\le9=n+1$.

18. Conjecture

If any n partitions contain an isolated vertex then $\gamma_s(G_3^p)\ge n+1$.
Proof:
If any two partition contains an isolated vertex then $\gamma_s(G_3^p)\ge2$.
If any two partitions contain an isolated vertex then $\gamma_s(G_3^p)\ge3$.
In general, If any n partitions contain an isolated vertex then $\gamma_s(G_3^p)\ge n+1$

19. References

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