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# A Bathtub Model of Downtown Traffic Congestion and Its Application in KISII Town

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***Abstract:***
*Traffic congestion in surface transport causes serious economical and environmental problems. In urban areas more especially during the rush hours. A person living in a major town will appreciate that congestion is a significant issue. In this project bathtub traffic flow model was studied to show how congestion can be managed in our major towns in Kenya. The partial differential equations with appropriate initial and boundary conditions governing this problem were solved using the finite difference method. The parameters affecting dynamic velocity and traffic flow density namely; were investigated. The numerical results were presented graphically. It was found from this project that the results were consistent with traffic flow behavior. This model was tested in Kisii CBD and may be extended to other downtown areas.*

***Keywords:*** *bathtub traffic flow, The partial differential equations, finite difference method, vehicle flow density and dynamic vehicle velocity*

## 1. Introduction

Traffic congestion is one of the greatest problems in Kenya like some other countries of the world. In this respect, countries managing traffic in congested networks requires a clear understanding of traffic flow operations. The aims of this project are principally represented by the maximization of vehicles flow, and the minimization of traffic congestions, accidents and pollutions among others. Macroscopic fluid-dynamic model which is characterized by representations of a bathtub traffic flow model was focused. Vehicles in traffic flow are considered as particles in fluid Haberman *et al* (1977). Further the behavior of traffic flow is modeled by the method of fluid-dynamics and formulated by hyperbolic partial differential equations (PDE).

Some purely data-driven models have already been developed to improve the nature of traffic state. An example, a hybrid method combining Kohomen maps with ARIMA time series models which was developed to forecast short time traffic flow Der roort *et al* (1996). Suggestions have been given that different model specifications should be set for different time periods of the day Stathopoulos and Karlaftis (2003).Other than time series models, neutral network could model undefined and complex non-linear surface Smith and Demetsky (1994). A framework of traffic flow model combining wavelet transform was found useful to eliminate noise caused by random travel conditions Viao *et al* ( 2003).

Bathtub model of traffic congestion in downtown areas builds on ideas put forward by William Vickrey (1969). Water flowing into the bathtub corresponds to cars entering the traffic stream, water flowing out of the bathtub to cars exiting from it and the height of the water in the bathtub to traffic density and outflow is proportional to the product of density and velocity. Above critical density, outflow falls as density increases (traffic jam situation).

When demand is high relative to capacity, applying optimal time varying toll generates benefits that may be considerably larger than those obtained from other models and that exceeds the toll revenue collected. The results of these empirical studies were broadly supported by the current generation of bathtub traffic simulation model, which incorporates elements permitting flow to fall as density increases, Gonzales *et al* (2011). While the project restricts attention on traffic congestion, the bathtub model of traffic congestion can be adopted to other congestible facilities for which heavy loading results in decreased output such as brown-outs and black-outs in electrical systems and jammed switches in telephone circuits.

*1.1. Literature Review*

The smooth traffic flow is important for healthy business and community development in downtown areas. Traffic congestion haunts big towns and communities from various perspectives. It inflicts uncertainties, drains resources, reduces productivity, stresses commuters and harms environment. For instance we think of a downtown network of one way street with signalized intersections. Assuming that each intersection operates at capacity, if the queue in both directions of traffic are sufficiently long. As demand increases, an increasingly high proportion of intersections satisfy this condition and in the limit all intersections operate at capacity. In this simple model of downtown traffic congestion, flow increases as demand increases.

One way of reducing congestion is to increase the capacity of existing roads by addition of more lanes Kimathi and Sigey (2006).This method is long term and very expensive, due to continuous deterioration of urban traffic conditions more models have been developed and implemented to help manage the highway and make the usage of the whole traffic flow more efficient.

For example ramp metering system Arnold (1998), advice usually consisting of traffic light together with a signal control installed on the lamp has been implemented on some of the cities to control traffic flow.

Vickrey (1969) describes congestion as occurring at a bottleneck, which is a point where traffic can only pass at a certain rate. Vickrey actually preceded Daganzo working on what he called a bathtub model of congestion, which is based on the same intuition. The present model is called a bathtub model since it uses sum an aggregate speed-density to describe urban congestion. Another important distinction between the Vickrey bottleneck model and the present model was the role played by distance which was essentially ignored by the other model.

The relationship between speed and density is well established for a single point on a road Greenshields (1935). The speed was a strictly decreasing function of density. The fundamental identity of a traffic flow holds that flow equals speed times density. All forms of congested traffic seem to have almost universal properties which are largely independent of the initial conditions and the average density. This universality arises from the highly correlated state of motion produced by traffic congestions. In particular, the out flow is related to the time interval between successive departures from traffic jam. Therefore, the outflow is almost independent of the kind and density of congested traffic.

Arnott (2013) developed a version of bathtub model which is similar to the present model. In comparison, this project uses a more general congestion technology but of the same type as Vickrey. The present project also uses scheduling preferences that compose of the $\lambda, \beta, \tau$ scheduling preferences of Vickrey as a limiting case. The main difference concerns the treatment of distance Arnott (2013), makes an assumption that can be interpreted as saying each driver's distance was random with a certain distribution and that drivers do not know their distance at the time they make their departure decisions.

This assumption was the way of obtaining analytical tractability Fosgerau and Small (2013).They also analyzed a bathtub model, in their model all traffic driven the same distance and there was a bottleneck with a capacity that depends on the number of cars queuing.

Tractability was achieved by simplifying the bottleneck capacity to a step function. The present model does not resort to such ad hoc devices and allows for a general distribution of distance. In the bottleneck model there was a wide range of time varying tolls that do not affect the timing of the first and last departures. Since all drivers achieve the same utility in equilibrium, such tolls can be used to extract revenue. Collete (2012) presented and discussed macroscopic traffic models and tried to solve the partial differential equations both analytically and numerically.

Tom L.M (2014) studied how a continous modeling traffic flow can be derived. Jin (2000) looked at the bottleneck as being brought about by change in the number of lanes. Philipe G.(2002) looked at modeling of a car traffic flow using conservation laws and numerically discretization of the resulting hyperbolic differential equations. Zhang, H.M. (1998) discussed the Korens scheme for the partial differential equations that describes both non-linear propagation and the diffusive effects for partial differential equations with Cauchy and Neumann boundary condition.

Olszewski and Suchorzewski (1987) presented a complex macroscopic model of traffic congestion in the city centre of Warsaw that relates aggregate traffic flow to average speed. Ardekani and Herman (1987) estimated the parameters of Herman and Prigogine (1979) macroscopic two fluid model of downtown traffic congestion of Astin and Dallars assuming a stable relation between mean density, mean flow, mean velocity and the fraction of vehicles stopped.

Hall *et al* (1986) studied the flow-density relationship of Canada using an extensive data set collected on the Queen Elizabeth way in Ontario. Ohta and Harata (1989) developed the regression model for the three variable relationships for Japanese cities. Olszewki *et al* (1995) developed an area-wide traffic speed model for the Singapore CBD to get an analytical framework for traffic management measures evaluation. Nielsen *et al* (2008) explored the speed-flow relationship of different lanes under a given traffic condition and general level of service based on Chengdu expressway data of China. However most researchers focus on the character of the network roads of different types and lack to consider the evolution of traffic congestion and jams at the downtown areas that is an essential feature of a bathtub model of downtown traffic congestion.

*1.2. Specific Objectives*

i. To investigate the traffic wave phenomenon found within Kisii CBD using bathtub model.
ii. To use the bathtub model of downtown traffic congestion to investigate the vehicle flow density by applying it in Kisii CBD.
iii. To explain how the county government could minimize congestion by using optimal-time varying tolls.

*1.3. Assumptions of the Study*

To achieve the objectives stated, the following assumptions and approximations have to be made.

i. There traffic flow was considered to be incompressible.
ii. Under the same conditions, drivers were to behave the same way.
iii. The traffic flow was considered to be laminar.
iv. The flow is one dimensional.
v. Pre-braking if the vehicle a head is much slower.
vi. Desire not to exceed significantly the speed limit.

*1.4. Geometry of the Problem*

The Kinematic wave model of a traffic flow theory is the simplest dynamic traffic flow that reproduces propagation of traffic waves. The fundamental diagram wave model relates traffic flow with density as in the figure below.

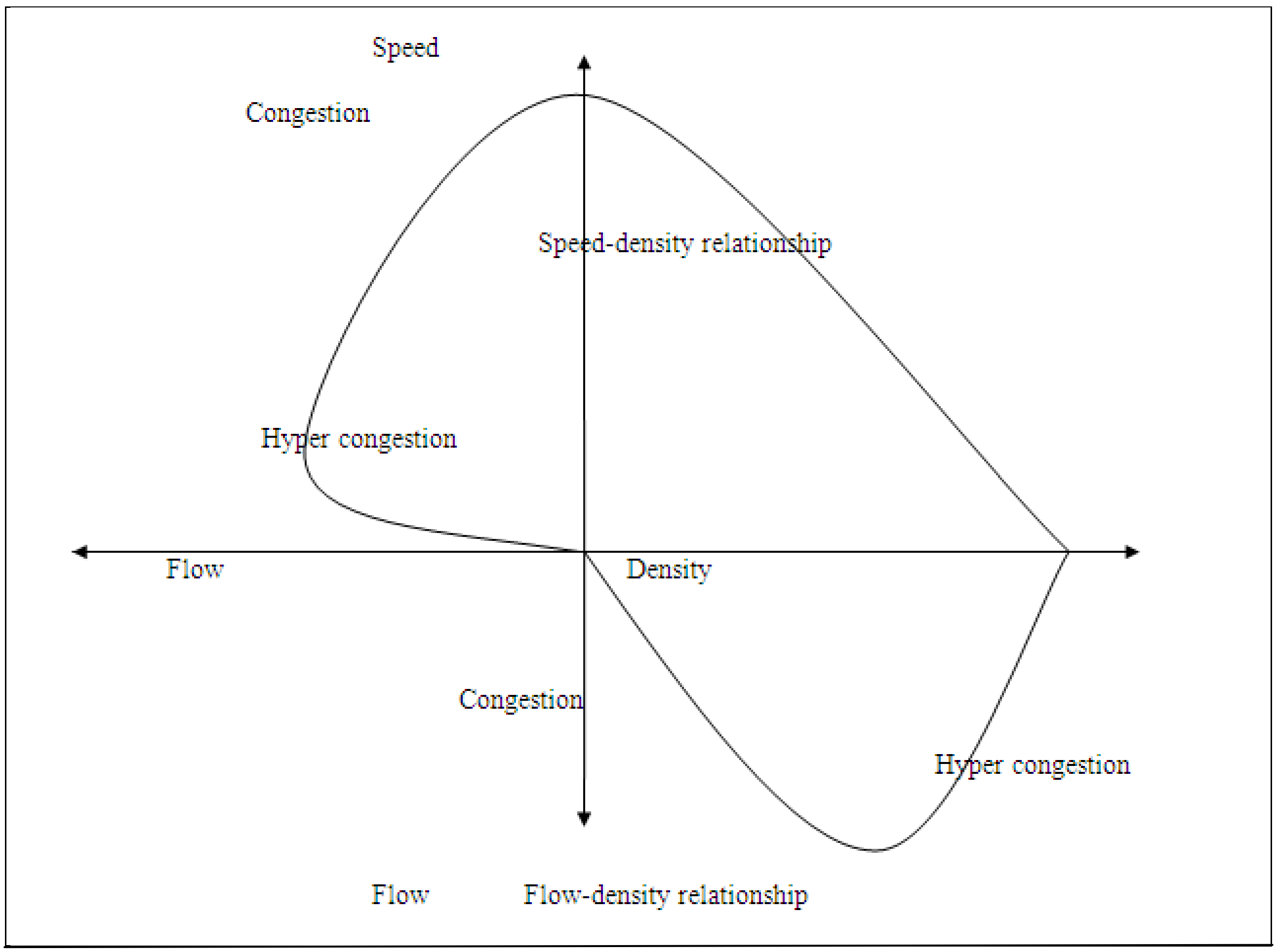


*Figure 1: Fundamental diagram of traffic flow.*

The fundamental diagram shows that flow is an inverse U-shaped function of density. On the upward sloping part we talk about congestion as higher density leads to higher flow but reduced speed. On the downward sloping part we talk about hypercongestion, here higher density is associated with both lower flow and reduced speed. A recent range of contributions have indicated that such a fundamental diagram of traffic flow also applies at the level of an urban neighborhood (Geroliminis and Daganzo, 2008; Daganzo *et al*, 2011). The underlying mechanism is that drivers continuously adapt their route choices to avoid more congested places in the network applied in the Kisii CBD.

In the next section we formulate the mathematical problem of this study and the equations governing the flow are discussed and non-dimensionalised.

**2. Mathematical Formulation**

*2.1. Macroscopic Traffic Models*

At macroscopic level, traffic can be continuum flow because the number of vehicles is conserved. Therefore, macroscopic traffic models are based on the continuity equation for the vehicle density $\rho(x,t)$ per lane at position $x$ and time t:

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho V)}{\partial x}=v^{\pm}(x,t) \quad (1)$$

Where V ($x$,t) is the average vehicle velocity. According to Eq. (1), the temporal change $\partial\rho/\partial t$ of the vehicle density is given by the spatial change $-\partial Q/\partial x$ of the traffic flow Q($x$,t) = $\rho$ ( $x$,t )V($x$,t) and the rate $v^{\pm}$ ( $x$,t) ≥ 0 of vehicles entering (+) or leaving (–) the CBD at on or off ramps. To describe time and spatially varying velocities such as occur in emergent traffic jams and stop-and-go traffic, we need a dynamic velocity equation. For most continuous models, this equation can be written as; (Helbing D.1996)

$$\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial t}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\frac{1}{\tau}(V_e-V) \quad (2)$$

According to Eq. (2), the change $\partial\rho/\partial t$ of the average vehicle velocity is given by three terms. The transport term originates from the propagation of the velocity profile with the velocity V of the vehicles. The pressure term reflects either an anticipation of spatial changes in the traffic situation or dispersion effects due to a finite variance of the vehicle velocities. The relaxation term describes the adaptation to a dynamic equilibrium velocity Ve with a relaxation time $\tau$.

### *2.2. First Order LWR Model*

Lighthill and Whitham (1955) and Richards (1956) considered that traffic was an inviscid but compressible fluid (fluid-dynamic model). Densities, speed values and flows were defined as continuous variables in each point in time and space (continuum, macroscopic model). The first order partial differential equation (PDE) from this model is:

$$\frac{\partial \rho}{\partial t}+\frac{\partial q}{\partial x}=\beta \quad (3)$$

Where $\beta$ is the flux of vehicles entering the traffic or exiting the traffic stream.

Crucial to the approach of Lighthill, Whitham and Richards was the fundamental hypothesis, i.e. flow is a function of density and speed: Greenshields *et al* ( 1935).

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho . u_o)}{\partial x}=\beta \quad (4)$$

Lighthill and Whitham assumed that the fundamental hypothesis holds at all traffic densities, not just for light-density traffic but also for congested traffic conditions. Relating the two dependent variables in the left-hand side of (3) (density $\rho$ and flow *q*) to one another, it is possible to solve the partial differential equation, given initial and boundary conditions.

Equation (4) can be made simpler by assuming a constant speed. In this project a constant speed is assumed, i.e. $u=u_o=q'(\rho)$

Therefore, (4) becomes

$$\frac{\partial \rho}{\partial t}+q'(\rho)\frac{\partial \rho}{\partial x}=\beta \quad (5)$$

It is worth mentioning that Lighthill, Whitham and Richards noted that because of the continuity assumption, the theory only holds for a large number of vehicles (long crowded roads).In the next chapter the non- dimensionalised governing equations are discretized and then solved using the finite difference method.

## 3. Method of Solution

In this study we have developed numerical scheme and used finite difference method to solve the model equations i.e (2) and (5). The method obtains a finite system of linear or nonlinear algebraic equations from the PDE by discretizing the given PDE and coming up with the numerical schemes analogues to the equation. We have solved the equations subject to the given boundary conditions. MATLAB software was used to generate solution values in this study.

### *3.1. Discretization of Model Equation*

The finite difference technique basically involves replacing the partial derivatives occurring in the partial differential equation as well as in the boundary and initial conditions by their corresponding finite difference approximations and then solving the resulting linear algebraic system of equations by a direct method or a standard iterative procedure. The numerical values of the dependent variable are obtained at the points of intersection of the parallel lines, called mesh points or nodal point.

### *3.2. The Governing Equations*

The traffic flow theory is the simplest dynamic traffic flow model that reproduces the propagation of traffic waves. It made up of the dynamic velocity equation and the density equation with the initial and boundary conditions.

3.3.1. Dynamic Velocity Equation

$$\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial x}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\frac{1}{\tau}\left(V_e-V\right) \quad (6)$$

The implicit scheme for the equation above is obtained as follows

$$\frac{V_{i,j+1}-V_{i,j}}{(\Delta x)^2}+V\frac{V_{i+1,j}-V_{i-1,j+1}}{(\Delta x)}=-\frac{1}{\rho}\frac{p_{i,j}+p_{i-1,j}}{(\Delta x)}+\frac{1}{\tau}\left(V_e-V\right) \quad (7)$$

Taking $V_e$= 110 Km/hr, V= 30Km/hr, $\tau=32$ seconds $\rho=160$ vehicles/Km and $\Delta t=\Delta x=1$

$$4800V_{i+1,j+1}-9540V_{i,j+1}+4800V_{i-1,j+1}=160V_{i,j}+P_{i,j}-P_{i-1,j}+12800 \quad (8)$$

Taking and i= 1, 2, 3…….8 and j= 1 we form the following systems of linear algebraic equations.

$$4800V_{2,2}-9540V_{1,2}+4800V_{0,2}=160V_{1,1}+P_{1,1}-P_{0,1}+12800$$
$$4800V_{3,2}-9540V_{2,2}+4800V_{1,2}=160V_{2,1}+P_{2,1}-P_{1,1}+12800$$
$$4800V_{4,2}-9540V_{3,2}+4800V_{2,2}=160V_{3,1}+P_{3,1}-P_{2,1}+12800$$
$$4800V_{5,2}-9540V_{4,2}+4800V_{3,2}=160V_{4,1}+P_{4,1}-P_{3,1}+12800$$
$$4800V_{6,2}-9540V_{5,2}+4800V_{4,2}=160V_{5,1}+P_{5,1}-P_{4,1}+12800$$
$$4800V_{7,2}-9540V_{6,2}+4800V_{6,2}=160V_{6,1}+P_{6,1}-P_{5,1}+12800$$
$$4800V_{8,2}-9540V_{7,2}+4800V_{7,2}=160V_{7,1}+P_{7,1}-P_{6,1}+12800$$
$$4800V_{9,2}-9540V_{8,2}+4800V_{8,2}=160V_{8,1}+P_{8,1}-P_{7,1}+12800$$

Taking the initial and boundary conditions

$$V(x,t)=0, P(0,t)=0, P(x,t)=0$$

The above algebraic equations can be written in matrix form as

$$\begin{bmatrix} 9540 & -4800 & 0 & 0 & 0 & 0 & 0 & 0 \\ -4800 & 9540 & -4800 & 0 & 0 & 0 & 0 & 0 \\ 0 & -4800 & 9540 & -4800 & 0 & 0 & 0 & 0 \\ 0 & 0 & -4800 & 9540 & -4800 & 0 & 0 & 0 \\ 0 & 0 & 0 & -4800 & 9540 & -4800 & 0 & 0 \\ 0 & 0 & 0 & 0 & -4800 & 9540 & -4800 & 0 \\ 0 & 0 & 0 & 0 & 0 & -4800 & 9540 & -4800 \\ 0 & 0 & 0 & 0 & 0 & 0 & -4800 & 9540 \end{bmatrix}\begin{bmatrix} V_{1,2} \\ V_{2,2} \\ V_{3,2} \\ V_{4,2} \\ V_{5,2} \\ V_{6,2} \\ V_{7,2} \\ V_{8,2} \end{bmatrix}=\begin{bmatrix} 12800 \\ 12800 \\ 12800 \\ 12800 \\ 12800 \\ 12800 \\ 12800 \\ 12800 \end{bmatrix} \quad (9)$$

3.3.2 Traffic Density Equation

$$\frac{\partial \rho}{\partial t}+q^{'}(\rho)\frac{\partial \rho}{\partial x}=\beta \quad (10)$$

$$\frac{\partial \rho}{\partial t}+(1-\lambda)\frac{\partial \rho}{\partial x}=\beta,(where, q'(\rho)=1-\lambda) \quad (11)$$

The implicit scheme for the equation above is obtained as follows

$$\frac{\rho_{i,j+1}-\rho_{i,j}}{(\Delta t)}+\frac{\lambda}{\Delta x}\frac{\rho_{i,j}-\rho_{i-1,j}}{(\Delta x)}=\beta \quad (12)$$

Taking , $\lambda=0.4, 0.3, 0.2$ seconds $\beta=5$ vehicles/Km and $\Delta t=60$

$$\rho_{i,j+1}=(1-\lambda)\rho_{i,j}+\lambda\rho_{i-1,j}+\beta\Delta t \quad (13)$$

Taking and i=1, 2, 3…….8 and j=1we form the following systems of linear algebraic equations

$$\rho_{1,2} = 0.6\rho_{1,1} + 0.4\rho_{0,1} + 330$$
$$\rho_{2,2} = 0.6\rho_{2,1} + 0.4\rho_{1,1} + 300$$
$$\rho_{3,2} = 0.6\rho_{3,1} + 0.4\rho_{2,1} + 300$$
$$\rho_{4,2} = 0.6\rho_{4,1} + 0.4\rho_{3,1} + 300$$
$$\rho_{5,2} = 0.6\rho_{5,1} + 0.4\rho_{4,1} + 300$$
$$\rho_{6,2} = 0.6\rho_{6,1} + 0.4\rho_{5,1} + 300$$
$$\rho_{7,2} = 0.6\rho_{7,1} + 0.4\rho_{6,1} + 300$$
$$\rho_{8,2} = 0.6\rho_{8,1} + 0.4\rho_{7,1} + 300$$

Taking the initial and boundary conditions,

$$\rho(0,x) = 25\sin\left(\tfrac{x}{4}\right) + 30 \quad \rho(t,0) = 21/0.1Km$$

The above algebraic equations can be written in matrix form as,

$$\begin{bmatrix} 0.6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.4 & 0.6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.4 & 0.6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.4 & 0.6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.4 & 0.6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.4 & 0.6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.4 & 0.6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.4 & 0.6 \end{bmatrix} \begin{bmatrix} \rho_{1,2} \\ \rho_{2,2} \\ \rho_{3,2} \\ \rho_{4,2} \\ \rho_{5,2} \\ \rho_{6,2} \\ \rho_{7,2} \\ \rho_{8,2} \end{bmatrix} = \begin{bmatrix} 330 \\ 300 \\ 300 \\ 300 \\ 300 \\ 300 \\ 300 \\ 300 \end{bmatrix} \qquad (14)$$

The results obtained are discussed in the next section.

**4. Results and Discussion**

The model was tested within the CBD in a selected section. The selected section was 1km long and was partitioned as a piece of roadway.

*4.1. Effect of vehicle flow density on Dynamic velocity profiles*

We solve equation (14) using Matlab and get the results as in table one below

| | $\rho = 120$ | $\rho = 160$ | $\rho = 200$ |
|---|---|---|---|
| X=0 | 11.781 | 13.824 | 14.34023 |
| X=1 | 20.74808 | 21.5482 | 25.53579 |
| X=2 | 26.78913 | 27.98913 | 33.21348 |
| X=3 | 29.82866 | 30.98913 | 37.11739 |
| X=4 | 29.82866 | 30.98913 | 37.11739 |
| X=6 | 26.78913 | 27.98913 | 33.21348 |
| X=7 | 20.74808 | 21.5482 | 25.53579 |
| X=8 | 11.781 | 13.824 | 14.34023 |

*Table 1: Dynamic velocity profiles $V(x,t)$ values for varying vehicle density $\rho$ at constant and* $V_e = 0.45$

The results in the table 1 are represented graphically as seen in Fig.2 in the next page.

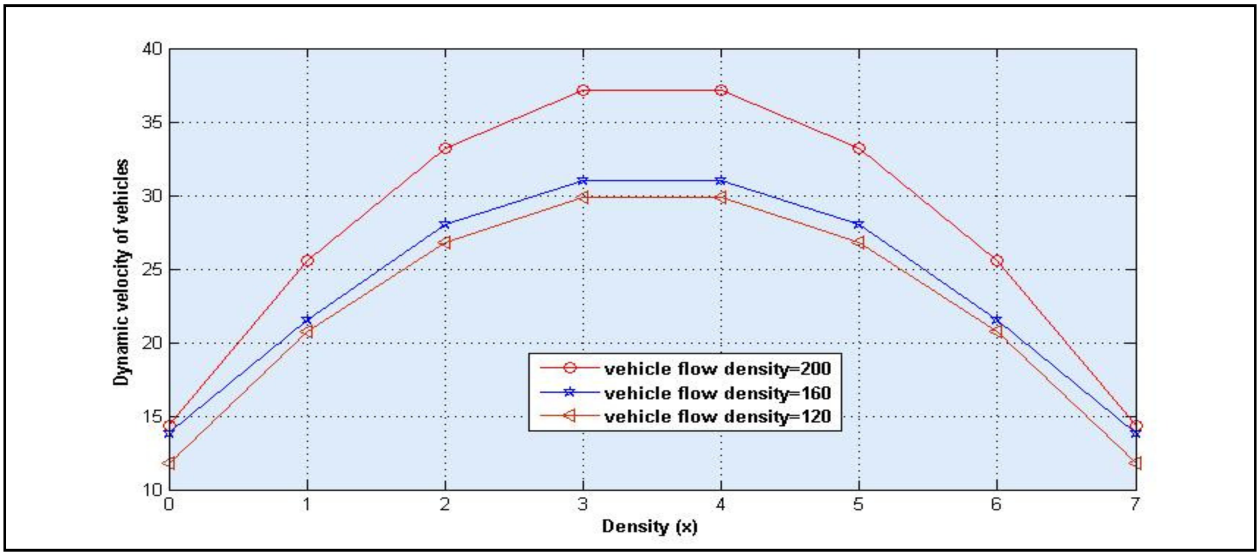


*Figure 2: The changes in dynamic velocity with distance for varying vehicle flow density.*

Figure 2 displays dynamic velocity as a function of distance varying vehicle flow density. Dynamic velocity increase with increase in distance until it reaches a maximum point called the critical point .At this point dynamic velocity starts to decrease with increase in distance .We also observe that with vehicle flow density of 200veh/km, the dynamic velocity at the peak is 37.11739km/hr. For vehicle flow density of 160veh/km and 120veh/ we experience a denser traffic flow, thus increase in vehicle flow density but reduced speed. After the critical point, higher vehicle flow density is associated with both lower traffic flow and reduced speed.

*4.2. Effect of vehicle velocity on Dynamic velocity profiles*

| | V=30km/h | V=40km/h | V=50km/h |
|---|---|---|---|
| X=0 | 14.34023 | 8.651823 | 5.661522 |
| X=1 | 25.53579 | 15.33735 | 10.00981 |
| X=2 | 33.21348 | 19.88944 | 12.95791 |
| X=3 | 37.11739 | 22.1943 | 14.44685 |
| X=4 | 37.11739 | 22.1943 | 14.44685 |
| X=6 | 33.21348 | 19.88944 | 12.95791 |
| X=7 | 25.53579 | 15.33735 | 10.00981 |
| X=8 | 14.34023 | 8.651823 | 5.661522 |

*Table 2: Dynamic velocity profiles V( x, t) values for varying vehicle*

The results in the table 2 above is represented graphically as seen in Fig.3 below

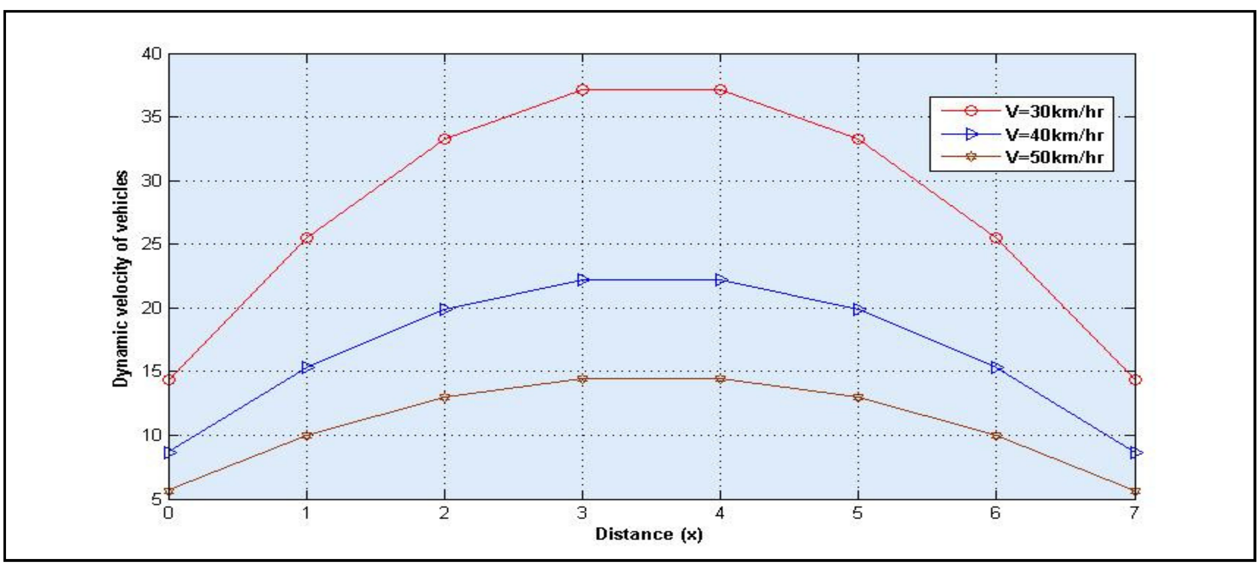


*Figure 3: The changes in dyanamic velocity with distance for varying vehicle velocity.*

Figure 3 displays dynamic velocity as a function of distance varying vehicle velocity. We note that decrease in vehicle velocity leads to increase in dynamic velocity to a maximum point being 37.11739km/hr for a vehicle velocity of 30km/hr. Whereas with the higher vehicle velocity of 50km/hr, dynamic velocity is at a point of 14.44685km/hr. Figure 3 gives the same information as that of Fig. 2. but illustrates how less denser traffic flow is when varying vehicle velocity.

*4.3. Effect of $\lambda$ on Vehicle flow density*

We solve equation (14) using matlab and get the results as in table three below

| | $\lambda = 0.4$ | $\lambda = 0.3$ | $\lambda = 0.2$ |
|---|---|---|---|
| x=0 | 550 | 471.4286 | 412.5 |
| x=1 | 133.3333 | 226.5306 | 271.875 |
| x=2 | 411.1111 | 331.4869 | 307.0313 |
| x=3 | 225.9259 | 286.5056 | 298.2422 |
| x=4 | 349.3827 | 305.7833 | 300.4395 |
| x=6 | 267.0782 | 297.5215 | 299.8901 |
| x=7 | 321.9479 | 301.0622 | 300.0275 |
| x=8 | 285.3681 | 299.5448 | 299.9931 |

*Table3: Vehicle flow density $\rho(x,t)$ values for varying $\lambda$ at constant $\beta = 5$*

The results in the table 3 above is represented graphically as seen in Fig. 4 below

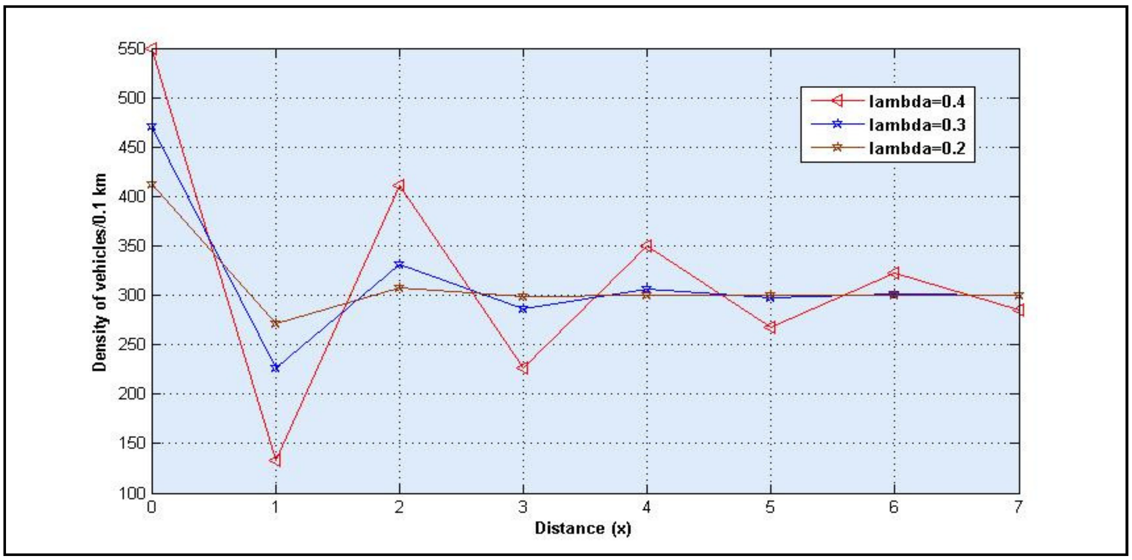


*Figure 4: The changes in density of vehicles with distance for varying lambda*

Figure 4 displays density of vehicles as a function of distance varying lambda. Density of vehicles decreases rapidly reaching a minimum value of 133.3333veh/0.1km when lambda is 0.4 seconds. Vehicle density again it increases as it continuously slightly fluctuates with the minimal transition of density values oscillating around 285.3681, 299.5448 and 299.9931 respectively indicating that when the vehicles accelerates out of a congested region and density decreases smoothly.

*4.4. Effect of flux of vehicles $\beta$ entering or exiting congestion point on Vehicle flow density*

| | $\beta = 5$ | $\beta = 10$ | $\beta = 15$ |
|---|---|---|---|
| X=0 | 412.5 | 787.5 | 1162.5 |
| X=1 | 271.875 | 553.125 | 834.375 |
| X=2 | 307.0313 | 611.7188 | 916.4063 |
| X=3 | 298.2422 | 597.0703 | 895.8984 |
| X=4 | 300.4395 | 600.7324 | 901.0254 |
| X=6 | 299.8901 | 599.8169 | 899.7437 |
| X=7 | 300.0275 | 600.0458 | 900.0641 |
| X=8 | 299.9931 | 599.9885 | 899.984 |

*Table 4: Vehicle flow density $\rho(x,t)$ values for varying $\beta$ at constant $\lambda = 0.2$*

The results in the table 4 above is represented graphically as seen in figure 5 below

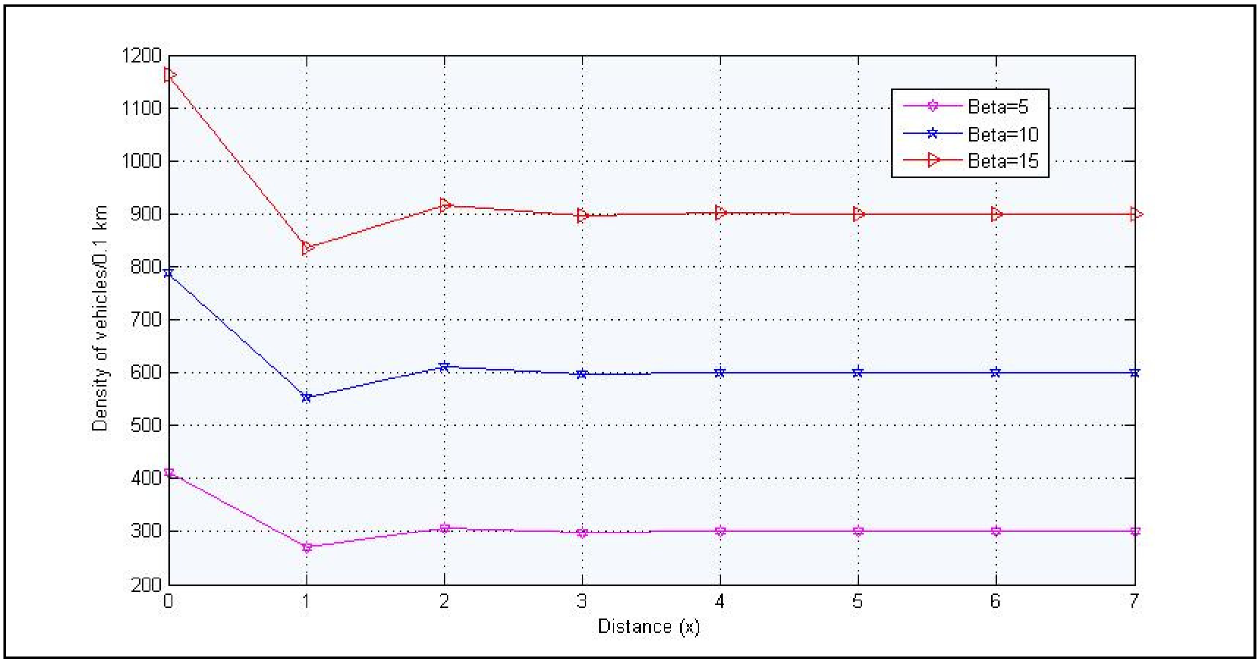


*Figure 5: The changes in density of vehicles with distance varying for varying beta.*

Figure 5 displays density of vehicles/0.1km as a function of distance, varying Beta. Density of vehicles drops and then slightly increases to a given level where it continuously remains almost in the same situation. When the number of vehicles entering the congestion point increases, density becomes too high the vehicles cannot move fast, so traffic flow is slower, when the number of vehicles is small density is too lower, so the traffic flow is also lower.Therefore, we experience very minimal fluctuations in the vehicle flow density. Finally a standing wave on the road hence a situation where vehicles are moving smoothly at almost a constant speed.

The conclusions and recommendations for further research are presented in the next chapter.

## 5. Conclusion and Recommendations

### *5.1. Conclusion*

This project presented a modified model of traffic congestion called the bathtub model of downtown traffic congestion that is particularly well suited to congested downtown areas. The fundamental diagram in traffic flow theory describes the relation between the macroscopic flow variables of flow, density and speed profiles at certain points within the CBD using assumed initial and the density at the boundary.The vehicle flow density and dynamic velocity profiles were analylised by varying various parameters. The outcome from these results are consistent with the values of the model presented. This model was applied in Kisii CBD. The model was studied and implemented numerically using a system of equations and it was observed that the dynamic velocity profiles depends on the vehicle flow density on the road. This project also indicated that, when demand is high relative to capacity downtown traffic congestion relief policy should focus on preventing traffic jams, which can be achieved through time varying tolls converting traffic jams into queues and regulating the entry rate of traffic into downtown areas.

### *5.2. Recommendations*

The bathtub model provides a modified perspective on downtown traffic congestion that is consistent with recent theoretical and empirical development in transportation science. It is therefore recommended that;

i. The development of traffic micro-simulation model that can do a better job of modeling the onset, growth and dissipation of traffic jams on experimentation with alternative policies as well as enrichment of the bathtub model of downtown traffic congestion.
ii. Other extensions can be, provide an integrated treatment of the three stage journey to work, sub-urban access, freeway travel to be modeled as a bathtub connecting to a neighborhood.

## 6. References


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