THE INTERNATIONAL JOURNAL OF SCIENCE & TECHNOLEDGE

On the Heptic Diophantine Equation with Three Unknowns $3(x^2+y^2)-5xy=15z^7$

Manjusomanath
Department of Mathematics, National College, Trichy, India
G. Sangeetha
Department of Mathematics, Indra Ganesan College Of Engineering, Trichy, India
M. A. Gopalan
Department of Mathematics, Shrimati Indira Gandhi College , Trichy, India

Abstract:
We obtain three different patterns of non-zero integral solutions of the Heptic Diophantine equation with three unknowns $3(x^2+y^2)-5xy=15z^7$ *by employing suitable transformations.*

Key words: *Heptic equation with three unknowns, Integral solutions*

1. Introduction

Diophantine equations, homogeneous and non- homogeneous have aroused the interest of numerous mathematicians since antiquity as can be seen from [1-2]. The problem of finding all integer solutions of a Diophantine equation with three or more variables and degree at least three, in general presents a good deal of difficulties. There is a vast general theory of homogeneous quadratic equations with three variables [1-5]. Cubic equations in two variables fall into the theory of elliptic curves which is a very developed theory but still an important topic of current research [6, 7, 8]. A lot is known about equations in two variables in higher degrees. For equations with more than three variables and degree at least three very little is known. It is worth to note that undesirability appears in equations, even perhaps at degree four with fairly small co-efficients. It seems that much work has not been done in solving higher order Diophantine equations. In [1, 2, 9-14] a few higher order equations are considered for integral solutions. In this communication a seventh degree equation with three variables represented by $3(x^2+y^2)-5xy=15z^7$ is considered and three different patterns of non-zero integral solutions are presented.

2. Method of Analysis

The equation under consideration is

$$3(x^2+y^2)-5xy=15z^7 \tag{1}$$

Assuming the transformations

$$x=u+v,\ y=u-v,\ z=a^2+11b^2 \tag{2}$$

in (1), we get,

$$u^2+11v^2=15(a^2+11b^2)^7 \tag{3}$$

Pattern 1:

Let $$15=(2+i\sqrt{11})(2-i\sqrt{11}) \tag{4}$$

By employing (4) in (3) and by the method of factorization and equating the positive factors

$$\left.\begin{aligned} u&=2a^7-462a^5b^2+8470a^3b^4-18634ab^6-77a^6b+4235a^4b^3-27951a^2b^5+14641b^7\\ v&=a^7-231a^5b^2+4235a^3b^4-9317ab^6-14a^6b-770a^4b^3+5082a^2b^5-2662b^7\end{aligned}\right\} \tag{5}$$

By substituting (5) in (2) and hence the nonzero integral solutions of (1) are

$$x = 3a^7 - 63a^6b - 693a^5b^2 + 3465a^4b^3 + 12705a^3b^4 - 22869a^2b^5 - 27951ab^6 + 11979b^7$$
$$y = a^7 - 91a^6b - 231a^5b^2 + 5005a^4b^3 + 4235a^3b^4 - 33033a^2b^5 - 9317ab^6 + 17303b^7$$
$$z = a^2 + 11b^2$$

Pattern 2:

Let $$15 = \frac{(7+i\sqrt{11})(7-i\sqrt{11})}{4} \qquad (6)$$

Using the procedure as mentioned above we get

$$u = \frac{7a^7}{2} - \frac{77}{2}a^6b - \frac{1617}{2}a^5b^2 + \frac{4235}{2}a^4b^3 + \frac{29645}{2}a^3b^4 - \frac{27951}{2}a^2b^5 - \frac{65219}{2}ab^6 + \frac{14641}{2}b^7$$
$$v = \frac{a^7}{2} + \frac{49}{2}a^6b - \frac{231}{2}a^5b^2 - \frac{2695}{2}a^4b^3 + \frac{4235}{2}a^3b^4 + \frac{17787}{2}a^2b^5 - \frac{9317}{2}ab^6 - \frac{9317}{2}b^7$$

And hence by employing u and v in (2) and hence the solutions of (1) are found to be

$$x = 4a^7 - 14a^6b - 924a^5b^2 + 770a^4b^3 + 16940a^3b^4 - 5082a^2b^5 - 37268ab^6 + 2662b^7$$
$$y = 3a^7 - 63a^6b - 693a^5b^2 + 3465a^4b^3 + 12705a^3b^4 - 22869a^2b^5 - 27951ab^6 + 11979b^7$$
$$z = a^2 + 11b^2$$

Pattern 3:

Let $$15 = \frac{(23+i\sqrt{11})(23-i\sqrt{11})}{36} \qquad (7)$$

Following the same procedure as in pattern 1, the non-zero integral solutions of (1) are

$$x = 4a^7 + 14a^6b - 924a^5b^2 - 770a^4b^3 + 16940a^3b^4 + 5082a^2b^5 - 37268ab^6 - 2662b^7$$
$$y = \frac{22}{6}a^7 - \frac{238}{6}a^6b - 847a^5b^2 + \frac{13090}{6}a^4b^3 + \frac{93170}{6}a^3b^4 - 14399a^2b^5 - \frac{204974}{6}ab^6 + \frac{42254}{6}b^7$$
$$z = a^2 + 11b^2$$

3. Conclusion

In this paper we have presented three different patterns of non-zero integral solutions of the heptic Diophantine equation with three unknowns(1). One may search for other patterns of solutions and their corresponding properties.

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