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A Study on the Mathematical Analysis of Beacon Message Dissemination in the DSRC using SMP Model

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Abstract:
Beacon messages, consisting the position, velocity, direction, etc, are broadcasted by every moving vehicle time to time. A semi markov processes (SMP) is developed to analyse the tagged vehicle's messages. In this paper we give the mathematical analysis and the steady state probabilities for each state of the SMP.

1. Introduction

Safety being the prime concern in transportation, successful transmission of beacon messages from each vehicle takes the highest priority. Beacon messages are the messages containing the information about the velocity, position, direction etc. The beacon messages are transmitted in the Dedicated Short Range Communication (DSRC) band on the highway system. These messages are transmitted between the other vehicles, vehicle–to-vehicle (V2V) and the road side units (RSU). On the basis of the beacon message, requirements of safety applications are rendered to the drivers to act accordingly in prevention of collision and other emergencies.

2. The Model

We propose the model in two stages.

- The basic broadcast protocol by IEEE 802.11 DCF
- The SMP, which captures the channel contention and the backoff behaviour for a single vehicle.

2.1. IEEE 802.11 DCF

The DCF [1] basic access method is shortly summarized as follows. A station with a packet to transmit monitors the channel activity until an idle period equal to a distributed interframe space (DIFS) is detected. The time immediately following an idle DIFS is slotted, and a station is allowed to transmit only at the beginning of each slot time, defined as the time needed at any station to detect the transmission of a packet from any other station. It accounts for the propagation delay, for the time needed to switch from the receiving to the transmitting state (RX TX Turnaround Time), and for the time to signal to the MAC layer the state of the channel (busy detect time). After sensing an idle DIFS, the station generates a random backoff interval before transmitting. The backoff time counter is decremented as long as the channel is sensed idle, stopped when a transmission is detected on the channel, and reactivated when the channel is sensed idle again for more than a DIFS (see Fig 1). The station transmits when the backoff time reaches zero. At each transmission, the backoff time is uniformly chosen in the range $(0, w-1)$. At the first transmission attempt, $w = W$, namely the minimum backoff window. After each unsuccessful transmission, w is doubled, up to a maximum value 2nW. Since the CSMA/CA does not rely on the capability of the stations to detect a collision by hearing their own transmission, a positive acknowledgment (ACK) is transmitted by the destination station to signal the successful packet transmission. To allow an immediate response, the ACK is transmitted following the received packet, after a short interframe space (SIFS). If the transmitting station does not receive the ACK within a specified ACK Timeout, or it detects the transmission of a different packet on the channel, it reschedules the packet transmission according to the previous backoff rules.

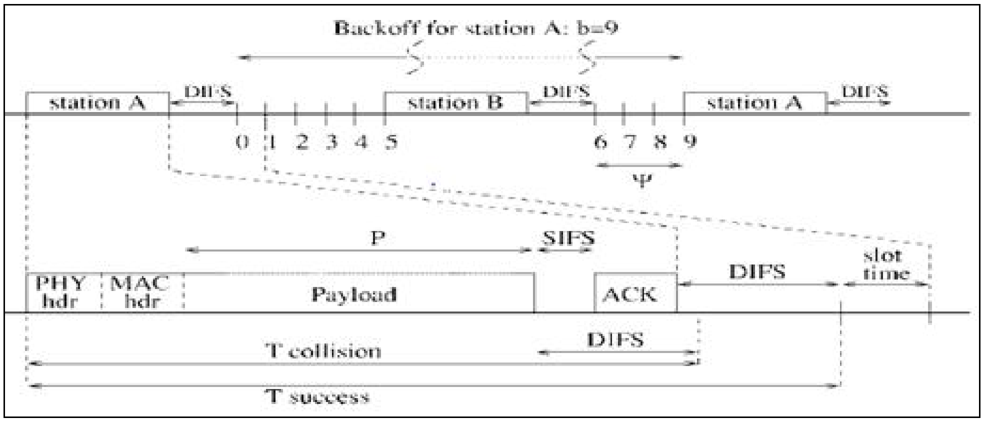

Figure 1

Each vehicle in the network can generate messages and compete for the available channel to transmit their message. If a vehicle does not have any message to transmit, it will wait for a packet to be generated. Then, for a newly generated packet, the vehicle senses the channel activity before it starts to transmit the packet. If the channel is sensed idle for a time period of distributed inter-frame space (*DIFS*), the packet can be directly transmitted. Otherwise, the vehicle continues to monitor the channel until the channel is detected to be idle for DIFS time period. Subsequently, according to the collision avoidance feature of the protocol, the vehicle goes through the backoff process before transmitting the packet. It generates an initial random backoff counter from a uniform probability mass function over the range [0, *CW*], where *CW* represents the contention window. The backoff time counter is decreased by one if the channel is sensed idle for a time slot of duration σ . Otherwise, the counter is frozen and reactivated when the channel is sensed idle again for more than DIFS duration. The packet is transmitted as soon as the backoff counter reaches zero. After this packet is transmitted, if there is no packet left in this vehicle, the process will start over again and the vehicle will wait for the next packet to be generated. Otherwise, if there are packets left, the vehicle repeats the procedure starting with sensing the channel for DIFS duration and going through the backoff procedure before transmitting the next packet. According to the protocol, a vehicle must go through the backoff process between two consecutive packet transmissions even if the channel is sensed idle for the duration of DIFS time for the second packet.

2.2. Semi Markov Process (SMP) Model

The behaviour of a tagged vehicle is characterized using the irreducible SMP [2] model in Fig. 3. The tagged vehicle is in idle state if there is no packet. After a packet is generated, the vehicle senses channel activity for DIFS time period, which is represented by state CS_1. If the channel is detected not busy during this period (with probability 1-qb), the vehicle goes from idle state to TX state, which means that a packet is transmitting.

Otherwise, the vehicle will defer until channel is idle for DIFS duration represented by state D_{CS}. Such deference behaviour for the tagged vehicle includes two parts: waiting for the current packet in the channel finishing transmission and waiting for subsequent transmissions if any from other neighbours within its receiving range.

The self-loop for state D_{CS} represents the phenomena in Fig 2. that the tagged vehicle (vehicle B) waits for the current packet (from vehicle A) in the channel finishing transmission, and then senses the channel for DIFS time, which seizes the transmission from another vehicle (vehicle C) and leads to further deference for backoff procedure of vehicle B. The probability that the tagged vehicle detects another neighbours' transmission during DIFS time is denoted as r_b. If no other neighbours' transmission is detected, the tagged vehicle will start backoff procedure and randomly choose a backoff counter in the range [0, W-1], where W=CW+1 is the backoff window size. The backoff counter will be decreased by one if the channel is detected to be idle for a time slot of duration σ (with probability $1\text{-}p_b$), which is captured by the transition from state W-i to state W-i-1. If the channel is busy during a backoff time slot of duration σ (i.e., another vehicle starts to transmit a packet during this time slot), the backoff counter of the tagged vehicle will be suspended, which represented by the transition from state W-i to $D_{Wi\text{-}1}$ with probability p_b. Similar to state D_{CS}, state $D_{W\text{-}i\text{-}1}$ also contains self-loop because other neighbours transmission can lead to further deference of the tagged vehicle. When the backoff counter reaches zero, the packet will directly be transmitted (an SMP transition occurs from state 0 to state TX with probability one).

In TX state, a packet is transmitting. To capture the out-dated packet replacement behaviour, which can happen during any state except state idle, we simplify the model by considering the total replacement probability and placing it after state TX. If the current packet has not been replaced by the next packet (with probability $1\text{-}p_f$), the SMP goes to state idle. Otherwise, this current packet is out-dated and replaced by the next incoming packet. Such simplification is reasonable since the packet transmission delay is usually much smaller than the packet generation interval and hence the replacement occurs extremely rare.

Next, the tagged vehicle starts the service for the next packet immediately and senses the channel for DIFS time (state CS_2). A new backoff procedure is started subsequently for the new packet instead of inheriting the backoff state of the old message. This is mainly because the out-dated message may finish the backoff procedure and is replaced during its transmission. This newly proposed SMP model captures more detailed DCF behaviour for periodic beacon message transmission by adding more states and self-loop structure. In addition, out-dated message replacement behaviour is incorporated into the model by the newly introduced model parameter Pf. The sojourn times and steady-state solutions are computed here.

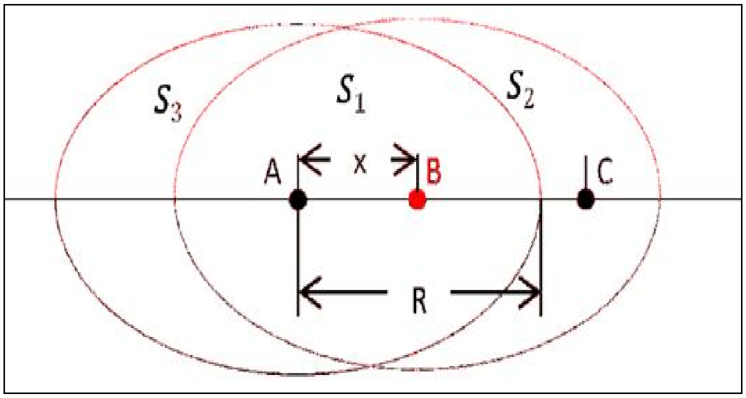

Figure 2

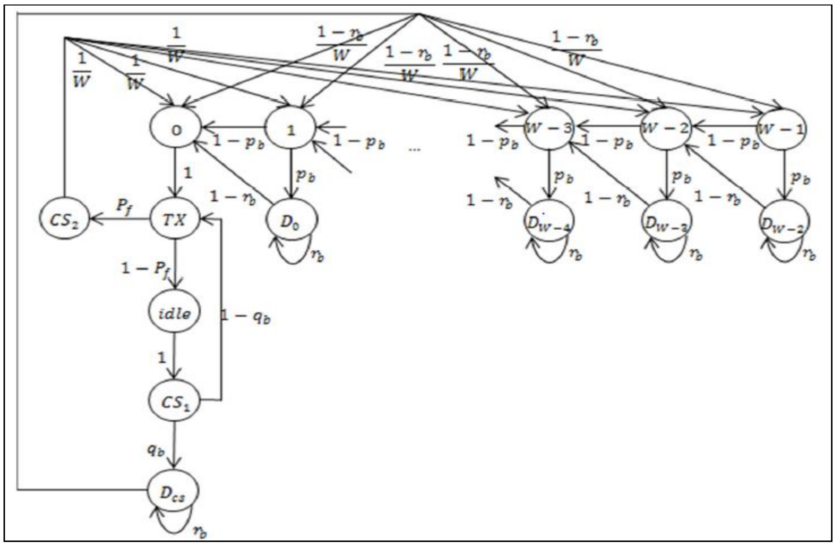

Figure 3

3. Computation of the Steady-State Probabilities for Each State

For the state D_{SC}

$$(1-r_b)v_{D_{CS}} + v_{D_{CS}} r_b = v_{D_{CS}} r_b + v_{CS_1} q_b$$

$$\therefore\ v_{D_{CS}} = \frac{v_{CS_1}}{(1-r_b)} q_b \qquad (1)$$

For The State CS_1

$$v_{CS_1} q_b + v_{CS_1}(1-q_b) = v_{idle}.1$$

$$\therefore v_{CS_1} = v_{idle} \qquad (2)$$

For The State idle

$$v_{idle}.1 = v_{TX}(1-p_f)$$

$$v_{idle} = v_{TX}(1-p_f) \qquad (3)$$

For The State TX

$$v_{TX}(1-p_f) + v_{TX} p_f = v_{CS_1}(1-q_b) + v_0.1$$

$$v_{TX} = v_{CS_1}(1-q_b) + v_0 \qquad (4)$$

For The State CS_2

$$v_{CS_2}.1 = v_{TX}\, p_f$$

$$\therefore v_{CS_2} = v_{TX}\, p_f \qquad (5)$$

For The States 0 to $(w-1)$

$$v_0 = v_{D_{CS}}\left(\frac{1-r_b}{w}\right) + v_{CS_2}\frac{1}{w} + v_1(1-p_b) + v_{D_0}(1-r_b)$$

$$v_1 = v_{D_{CS}}\left(\frac{1-r_b}{w}\right) + v_{CS_2}\frac{1}{w} + v_2(1-p_b) + v_{D_1}(1-r_b)$$

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$$v_j = v_{D_{CS}}\left(\frac{1-r_b}{w}\right) + v_{CS_2}\frac{1}{w} + v_{j+1}(1-p_b) + v_{D_j}(1-r_b) \qquad (6)$$

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$$v_{w-1} = v_{D_{CS}}\left(\frac{1-r_b}{w}\right) + v_{CS_2}\frac{1}{w} \qquad (7)$$

Substituting eqn (7) in (6) we get

$$v_j - v_{j+1} = v_{w-1} - p_b v_{j+1} + (1-r_b)v_{D_j}, \qquad j = 0,1,2,\ldots,w-2 \qquad (8)$$

Substituting $j = 0,1,2,\ldots,w-2$ in (8) and summing up we get

$$v_0 - v_1 = v_{w-1} - p_b v_1 + (1-r_b)v_{D_0}$$

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$$v_{w-2} - v_{w-1} = v_{w-1} - p_b v_{w-1} + (1-r_b)v_{D_{w-2}}$$

$$v_0 - v_{w-1} = (w-1)v_{w-1} - p_b\left(v_1 + \ldots + v_{w-1}\right) + (1-r_b)\left(v_{D_0} + \ldots + v_{D_{w-2}}\right) \qquad (9)$$

For The States D_0 to D_{w-2}

$$(1-r_b)v_{D_0} + r_b v_{D_0} = v_{D_0} r_b + v_1 p_b$$

$$v_{D_0} = v_{D_0} r_b + v_1 p_b$$

$$v_{D_1} = v_{D_1} r_b + v_2 p_b$$

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$$v_{D_j} = v_{D_j} r_b + v_{j+1} p_b, \quad \text{i.e., } (1-r_b)v_{D_j} = v_{j+1} p_b \qquad (10)$$

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$$v_{D_{w-2}} = v_{D_{w-2}} r_b + v_{w-1} p_b$$

Summing up the above equations we get

$$\left(v_{D_0} + v_{D_1} + \ldots + v_{D_{w-2}}\right) = \left(v_{D_0} + v_{D_1} + \ldots + v_{D_{w-2}}\right) r_b + \left(v_1 + v_2 + \ldots + v_{w-1}\right) p_b$$

$$\therefore (1-r_b)\left(v_{D_0} + v_{D_1} + \ldots + v_{D_{w-2}}\right) = \left(v_1 + v_2 + \ldots + v_{w-1}\right) p_b \qquad (11)$$

Substituting (11) in (9), we get,

$$v_0 - v_{w-1} = (w-1)v_{w-1}$$

$$\therefore v_0 = w v_{w-1} \qquad (12)$$

Using eqn (10) in eqn (8) we get,

$$v_k - v_{k+1} = v_{w-1}, \quad k = 0,1,2,\ldots,w-2,$$

Expanding k and summing up, we get,

$$v_0 - v_1 = v_{w-1}$$
$$v_1 - v_2 = v_{w-1}$$
$$\ldots\ldots\ldots\ldots\ldots\ldots$$
$$v_{j-1} - v_j = v_{w-1}$$
$$v_0 - v_j = j.v_{w-1}$$

i.e., $v_j = v_0 - j.v_{w-1}$

from (12)

$$v_j = (w-j)v_{w-1}, \quad j = 0,1,2,\ldots,w-1 \tag{13}$$

We know that from (10),

$$(1-r_b)v_{D_j} = v_{j+1}p_b, \quad j = 0,1,2,\ldots,w-2,$$

and also $v_j = (w-j)v_{w-1}$,

put $j = j+1$, we get, $v_{j+1} = [w-(j+1)]v_{w-1}$

$$\therefore v_{D_j} = \frac{[w-j-1]p_b}{(1-r_b)}v_{w-1}, \quad j = 0,1,2,\ldots,w-2 \tag{14}$$

To express all the states in terms of v_{w-1}

W know that the total Probability = 1, Therefore,

$$\therefore v_{D_{CS}} + v_{CS_1} + v_{idle} + v_{TX} + v_{CS_2} + \sum_{j=0}^{w-1} v_j + \sum_{j=0}^{w-2} v_{D_j} = 1 \tag{15}$$

To find $\sum_{j=0}^{w-1} v_j$

$$\sum_{j=0}^{w-1} v_j = \sum_{j=0}^{w-1}(w-j)v_{w-1} = v_{w-1}\left[w^2 - \left(0+1+2+\ldots+(w-1)\right)\right],$$

from (13)

$$\sum_{j=0}^{w-1} v_j = \left[\frac{w(w+1)}{2}\right]v_{w-1} \tag{16}$$

To find $\sum_{j=0}^{w-2} v_{D_j}$

$$(1-r_b)v_{D_j} = v_{j+1}p_b, \quad j = 0,1,2,\ldots,w-2$$

From (10)

Summing up we get,

$$(1-r_b)\sum_{j=0}^{w-2} v_{D_j} = p_b \sum_{j=0}^{w-2} v_{j+1} = p_b\left[v_1 + v_2 + \ldots + v_{w-1}\right]$$

$$\sum_{j=0}^{w-2} v_{D_j} = \frac{p_b}{(1-r_b)}\left[\frac{w(w-1)}{2}\right]v_{w-1} \tag{17}$$

To find v_{TX}

$$v_{TX} = v_{CS_1}(1-q_b) + v_0 = v_{idle}(1-q_b) + v_0,$$ From (4)

$$v_{TX} = \frac{w}{\left[p_f + q_b\left(1 - p_f\right)\right]} v_{w-1} \tag{18}$$

To find v_{CS_1}

$v_{CS_1} = v_{idle} = v_{TX}(1 - p_f)$

From (2) and (3)

$$v_{CS_1} = \frac{(1 - p_f)w}{\left[p_f + q_b(1 - p_f)\right]} v_{w-1} \tag{19}$$

To find v_{idle}

$$v_{CS_1} = v_{idle} = \frac{(1 - p_f)w}{\left[p_f + q_b(1 - p_f)\right]} v_{w-1}, \text{ From (2)} \tag{20}$$

To find v_{CS_2}

$$v_{CS_2} = v_{TX}\, p_f = \frac{w.p_f}{\left[p_f + q_b(1 - p_f)\right]} v_{w-1} \tag{21}$$

To find $v_{D_{SC}}$

$$v_{D_{SC}} = v_{CS_1} \frac{q_b}{(1 - r_b)} = \frac{q_b\left(1 - p_f\right)w}{\left[p_f + (1 - q_b)\right](1 - r_b)} v_{w-1} \tag{22}$$

Substituting Eqns (16), (17), (18), (19), (20), (21) and (22) in Eqn (15), we get,

$$v_{w-1} = \frac{2\left[p_f + q_b(1 - p_f)\right](1 - r_b)}{\left[(w+1)(1 - r_b) + p_b(w-1)\right]\left[p_f + q_b(1 - p_f)\right]w + 2\left[\left(3 - p_f\right)(1 - r_b) + q_b\left(1 - p_f\right)\right]w} \tag{23}$$

For the states 0,1,2,...,n , The steady-state probabilities of the SMP are

$v_j = \Pr\{X = j\}$

$$\pi_j = \frac{v_j \tau_j}{\sum v_j \tau_j} \tag{24}$$

PL represents the packet length.
Rd represents the data rate.

Therefore $\frac{PL}{Rd}$ is the time to transmit the packet.

T_H represents the time to transmit the packet header including physical layer header and MAC layer.

$$A_1 = \frac{PL}{Rd} + T_H = E\left[TX\right] = \tau_{TX}$$

Calculation of $v\tau$

$$v_j \tau_j = (w - j)v_{w-1}\sigma, \quad j = 0,1,2,...,w-1 \tag{25}$$

$$\nu_{D_j}\tau_{D_j} = \frac{(w-j-1)p_b}{(1-r_b)}\nu_{w-1}A_5, \quad j = D_0, D_1, D_2, \ldots, D_{w-2} \tag{26}$$

$$\nu_{D_{CS}}\tau_{D_{CS}} = \frac{q_b(1-p_f)w}{\left[p_f + q_b(1-p_f)\right](1-r_b)}\nu_{w-1}A_4 \tag{27}$$

$$\nu_{CS_1}\tau_{CS_1} = \frac{(1-p_f)w}{\left[p_f + q_b(1-p_f)\right]}\nu_{w-1}A_3 \tag{28}$$

$$\nu_{idle}\tau_{idle} = \frac{(1-p_f)w}{\left[p_f + q_b(1-p_f)\right]}\nu_{w-1}A_2 \tag{29}$$

$$\nu_{TX}\tau_{TX} = \frac{w}{\left[p_f + q_b(1-p_f)\right]}\nu_{w-1}A_1 \tag{30}$$

$$\nu_{CS_2}\tau_{CS_2} = \frac{p_f w}{\left[p_f + q_b(1-p_f)\right]}\nu_{w-1}A_3 \tag{31}$$

$$\sum_{j=0}^{w-1}\nu_j\tau_j = \sum_{j=0}^{w-1}(w-j)\nu_{w-1}\sigma$$

$$\sum_{j=0}^{w-1}\nu_j\tau_j = \frac{w(w+1)}{2}\nu_{w-1}\sigma \tag{32}$$

$$\sum_{j=0}^{w-2}\nu_{D_j}\tau_{D_j} = \frac{p_b}{(1-r_b)}\frac{w(w-1)}{2}\nu_{w-1}A_5 \tag{33}$$

Summing up Eqns (27), (22), (29), (30), (31), (32) and (33), we get,

$$\sum_{\alpha\in StateSpace}\nu_\alpha\tau_\alpha = \frac{q_b(1-p_f)w}{\left[p_f + q_b(1-p_f)\right](1-r_b)}\nu_{w-1}A_4 + \frac{(1-p_f)w}{\left[p_f + q_b(1-p_f)\right]}\nu_{w-1}A_3 + \frac{(1-p_f)w}{\left[p_f + q_b(1-p_f)\right]}\nu_{w-1}A_2 + \frac{w}{\left[p_f + q_b(1-p_f)\right]}\nu_{w-1}A_1 + \frac{p_f w}{\left[p_f + q_b(1-p_f)\right]}\nu_{w-1}A_3 + \frac{w(w+1)}{2}\nu_{w-1}\sigma + \frac{p_b}{(1-r_b)}\frac{w(w-1)}{2}\nu_{w-1}A_5$$

$$\pi_{TX} = \frac{\nu_{TX}\tau_{TX}}{\sum\nu_{TX}\tau_{TX}}$$

$$\pi_{TX} = \frac{2A_1}{\left\{\left[p_f + q_b(1-p_f)\right]\left\{(w+1)\sigma + \frac{p_b}{(1-r_b)}(w-1)A_5\right\} + 2\left[A_1 + A_3 + (1-p_f)\left(A_2 + \frac{q_b}{(1-r_b)}A_4\right)\right]\right\}}$$

Where

- p_f: the probability that the beacon message will be updated or replaced by the next beacon message.
- p_b: the probability that the channel is detected busy (transmitting messages from other vehicles) in one time slot by the tagged vehicle.
- q_b: the probability that the channel is detected busy (transmitting messages from other vehicles) in DIFS time by the tagged vehicle.
- r_b: the probability that the channel is detected busy in DIFS time by the tagged vehicle after another neighbour of the tagged vehicle finishes transmission.

4. Conclusion

The mathematical analysis for each state of the semi markov process is completely evaluated and the behaviour of the beacon message contenting for the channel resource is modelled. Finally the steady state probability for the vehicle in transaction state is derived.

5. References

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