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Special Dio- Quadruples with Property D(2)

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Abstract:
This paper concerns with the study of constructing a special Dio quadruples (a,b,c,d) such that the product of any two elements of the set added with their sum and increased by two is a perfect square.

***Key words**: Diophantine Quadruples, Pell equation*
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1. Introduction

The problem of constructing the sets with property that the product of any two of its distinct elements is one less than a square has a very long history and such sets were studied by Diophantus[1]. A set of m positive integers $\{a_1, a_2 ......a_m\}$ is said to have the property D(n), $n \in z - \{0\}$ if $a_i a_j + n$, a perfect square for all $1 \le i \le j \le m$ and such a set is called a Diophantine m-tuples with property D(n). Many mathematicians considered the construction of different formulations of Diophantine quadruples with the property D(n) for any arbitrary integer n and also for any linear polynomials in n. In this context, one may refer [2-13] for an extensive review of various problems on Diophantine quadruples. This paper aims at constructing special dio – quadruples where the special mention is provided because it differs from the earlier one and the special Dio – quadruples is constructed where the product of any two members of the quadruples with the addition of the some members and the addition of two satisfies the required property

2. Special Dio – Quadruple with Property D(2)

Let $a = 3^n$ and $b = 3^n + 2$ be two integers such that $ab + a + b + 2$ is a perfect square.
Let $C_N(n)$ be any non-zero integer such that

$$(3^n + 1)C_N(n) + 3^n + 2 = \alpha_N^2 \quad (1)$$

$$C_N(n)(3^n + 3) + 3^n + 4 = \beta_N^2 \quad (2)$$

Eliminating $C_N(n)$ from (1) and (2), we obtain

$$(3^n + 3)\alpha_N^2 - (3^n + 1)\beta_N^2 = 2 \quad (3)$$

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Setting

$$\alpha_N = X_N + (3^n + 1)T_N \quad (4)$$

$$\beta_N = X_N + (3^n + 3)T_N \quad (5)$$

in (3), we get

$$X_N^2 = (3^{2n} + 4 * 3^n + 3)T_N^2 + 1 \quad (6)$$

whose general solution is

$$\left.\begin{aligned} X_N(n) &= \frac{1}{2}\left\{\left[(3^n+2)+\sqrt{3^{2n}+4*3^n+3}\right]^{N+1}+\left[(3^n+2)-\sqrt{3^{2n}+4*3^n+3}\right]^{N+1}\right\} \\ T_N(n) &= \frac{1}{2\sqrt{3^{2n}+4*3^n+3}}\left\{\left[(3^n+2)+\sqrt{3^{2n}+4*3^n+3}\right]^{N+1}-\left[\left((3^n+2)-\sqrt{3^{2n}+4*3^n+3}\right)\right]^{N+1}\right\} \end{aligned}\right\} \quad (7)$$

Substituting N=1 in (7) and (4) and employing (1), we get,

$C_1(n) = 16*3^{3n} + 96*3^{2n} + 188*3^n + 119$

Note that the triple $(a, b, C_1(n))$ is a special Dio-triple with property D(2)

Again, substituting N=2 in (7) and (8) and employing (1) we get,

$C_2(n) = [(8*3^{3n} + 44*3^{2n} + 76*3^n + 42)(8*3^{2n} + 36*3^n + 40) - 1]$

It is seen that the $(a, b, C_1(n), C_2(n))$ is a special Dio-Quadruple with property D(2). The repetition of the above process leads to the result that the quadruple $(a, b, C_{N-1}(n), C_N(n))$,),

N=1,2,3,4……….. is a special Dio – Quadruple with property D(2).

Some Numerical examples are tabulated below:

n	(a, b, C_0, C_1)	(a, b, C_1, C_2)	(a, b, C_2, C_3)	(a, b, C_3, C_4)
1	(3,5,19,1979)	(3,5,1979,194039)	(3,5,194039,19013959)	(3,5,19013959, 1863174059)
2	(9,11,43,21251)	(9,11,21251,10243463)	(9,11, 10243463, 4937328439)	(9,11,4937328439, 2379782064659)
3	(27,29,115,390107)	(27,29,390107, 1311543095)	(27,29,1311543095, 409407498759)	(27,29,4409407498759, 4824426699288139)

Table 1: Special Dio – Quadruple with property D(2)

3. Conclusion

This paper concerns with the construction of special dio - quadruples with property D(2). One may search for special dio-quadruples consisting of special numbers with suitable property.

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