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A New Method for Solving Fully Fuzzy Bottleneck-Cost Transportation Problems

A. Akilbasha
Research Scholar, Department of Mathematics
School of Advanced Sciences, VIT University, Vellore-14, Tamil Nadu, India
Dr. G. Natarajan
Professor, Department of Mathematics
School of Advanced Sciences, VIT University, Vellore-14, Tamil Nadu, India
Dr. P. Pandian
Senior Professor, Department of Mathematics
School of Advanced Sciences, VIT University, Vellore-14, Tamil Nadu, India

Abstract:
A new method is proposed for finding an optimal solution to fully fuzzy bottleneck transportation problems which is very different from other existing methods. Then, for finding all efficient solutions of a fully fuzzy bottleneck-cost transportation problem, the proposed method will provide the necessary decision support to decision makers while they are handling time oriented logistic problems. The solution procedure is illustrated with numerical examples.

Keywords: *Fully fuzzy bottleneck transportation problem; blocking method; optimal solution; fully fuzzy bottleneck-cost transportation problem; efficient solution; blocking zero point method*

1. Introduction

The time-minimizing or bottleneck transportation problem (BTP) is a special case of a transportation problem in which a time is associated with each shipping route. Rather than minimizing cost, the objective is to minimize the maximum time to transport all supply to the destinations. In a BTP, the time of the transporting items from origins to destinations is minimized, satisfying certain conditions in respect of availabilities at sources and requirements at the destinations. Many researchers [7, 9, 12, 13, 4, 3, 2] developed various algorithms for solving time minimizing transportation problems. The transportation time is relevant in a variety of real transportation problems, too. In [8], Ilija Nikolić has developed an algorithm to find the minimum of the total time transportation to time transportation problems.

Bottleneck-cost transportation problem (BCTP) is a kind of a bicriteria transportation problem. The bicriteria transportation problem is a particular case of multiobjective transportation problem which had been proposed and also, solved by Aneja and Nair [1] and until, now many researchers [12, 6, 5] also, have great interest in this problem , and some method used their special techniques in finding the solutions for two objective functions approximately approaching to the ideal solution and also P.Pandian and G.Natarajan developed various algorithms for solving time minimizing transportation problems[11]. In this paper, we find optimal solution for fuzzy bottleneck transportation problem using new method where the supply and demand are triangular fuzzy numbers.

In this paper, we first review some basic concepts of fuzzy set theory in section 2, the operations on triangular fuzzy numbers are given in section 3. In section 4, a blocking zero point principles on fuzzy numbers with example is explained. In section 5, the fully fuzzy bottleneck-cost transportation problem is given and solved fully fuzzy bottleneck-cost transportation problem. In section 6, conclusion is given. The proposed methods will provide the necessary decision support to the user while they are handling time oriented logistic problems.

2. Preliminaries

2.1. Definition: Fuzzy Set

A fuzzy set $\tilde{A}$ is defined by $\tilde{A} = \{(x, \mu_A(x)): x \in A, \mu_A(x) \in [0, 1]\}$, in the pair $(x, \mu_A(x))$, the first element x belong to the classical set A, the second element $\mu_A(x)$ belong to the interval [0, 1], called Membership function.

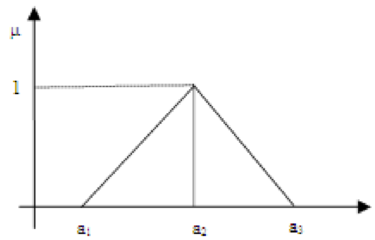
2.2 Definition: Triangular fuzzy number

It is a fuzzy number represented with three points as follows: $\tilde{A} = (a_1, a_2, a_3)$

This representation is interpreted as membership functions and holds the following conditions

- a_1 to a_2 is increasing function
- a_2 to a_3 is decreasing function
- $(a_1 \le a_2 \le a_3$.

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & for \;\; x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & for \;\; a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & for \;\; a_2 \le x \le a_3 \\ 0 & for \;\; x > a_3 \end{cases}$$



Triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$

3. Operation of Triangular Fuzzy Number

The following are the four operations that can be performed on triangular fuzzy numbers: Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ then,

(i) Addition : $\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$.

(ii) Subtraction : $\tilde{A} - \tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$.

(iii) Multiplication : $\tilde{A} \times \tilde{B} = (\min(a_1 b_1, a_1 b_3, a_3 b_1, a_3 b_3), a_2 b_2, \max(a_1 b_1, a_1 b_3, a_3 b_1, a_3 b_3))$.

(iv) Division : $\tilde{A} / \tilde{B} = (\min(a_1/b_1, a_1/b_3, a_3/b_1, a_3/b_3), a_2/b_2, \max(a_1/b_1, a_1/b_3, a_3/b_1, a_3/b_3))$.

(v) Mag : $(\tilde{A}) = (a_1 + 10 \times a_2 + a_3)/12$

4. Fuzzy Bottleneck transportation problem

Consider the following BTP:

(P) Minimize $\tilde{T} = [\underset{(i,j)}{\text{Maximize}}\; \tilde{t}_{ij} / x_{ij} > 0]$

Subject to

$$\sum_{j=1}^{n} a_{ij} x_{ij} = a_i, \; i = 1, 2, \ldots, m \qquad (1)$$

$$\sum_{i=1}^{m} a_{ij} x_{ij} = b_j, \; j = 1, 2, \ldots, n \qquad (2)$$

$x_{ij} \ge 0$, for all i and j and integers, (3)

Where m = the number of supply points; n = the number of demand points;

x_{ij} = the number of units shipped from supply point i to demand point j;

$\tilde{t}_{ij}$ = the fuzzy time of transporting goods from supply point i to demand point j:

a_i = the supply at supply point i and b_j = the demand at demand point j.

In a fuzzy BTP, time matrix $[\tilde{t}_{ij}]$ is given where $\tilde{t}_{ij}$ is the fuzzy time of transporting goods from the ith origin to the jth destination. For any given feasible solution $X = \{x_{ij} : i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n\}$ of the problem (P), the time transportation is the maximum of $\tilde{t}_{ij}$'s among the cells in which there are positive allocations. This time of the transportation remains independent of the amount of commodity sent so long as $x_{ij} > 0$.

4.1. Blocking method

The blocking method for finding an optimal solution to fuzzy bottleneck transportation problems.

The blocking method proceeds as follows.

Algorithm:

- **Step 1:** Find the maximum of the minimum of each row and column of the given transportation table. Say, T.
- **Step 2:** Construct a reduced transportation table from the given table by blocking all cells having time more than T.
- **Step 3:** Check if each column demand is less than to the sum of the supplies in the reduced transportation problem obtained from the Step 2.. Also, check if each row supply is less than to sum of the column demands in the reduced transportation problem obtained from the Step 2.. If so, go to Step 6. (Such reduced transportation table is called the active transportation table).
  If not, go to Step 4.
- **Step 4:** Find a time which is immediately next to the time T. Say U.
- **Step 5:** Construct a reduced transportation table from the given transportation table by blocking all cells having time more than U
  and then, go to the Step 3..
- **Step 6:** Do allocation according to the following rules:
  allot the maximum possible to a cell which is only one cell in the row / column. Then, modify the active transportation table and then, repeat the process till it is possible or all allocations are completed.
  If (a) is not possible, select a row / a column having minimum number of unblocked cell and allot maximum possible to a cell which helps to reduce the large supply and/ or large demand of the cell.
- **Step 7:** This allotment yields a solution to the given bottleneck transportation problem.

4.2. Numerical Example 1

Consider the following 3X4 fuzzy bottleneck transportation problem.

	S1	S2	S3	S4	**Supply**
D1	(8,10,12)	(67,68,69)	(71,73,75)	(51,52,53)	8
D2	(65,66,67)	(93,95,97)	(29,30,31)	(19,21,23)	19
D3	(95,97,99)	(62,63,64)	(17,19,21)	(22,23,24)	17
Demand	11	3	14	16	

The magnitude of fuzzy bottleneck transportation table is given below

	S1	S2	S3	S4	Supply
D1	10	68	73	52	8
D2	66	95	30	21	19
D3	97	63	19	23	17
Demand	11	3	14	16	

Now, using the Step 1. to the Step 5., we have the following complete allocation table:

	S1	S2	S3	S4	Supply
D1	10			52	8
D2	66		30	21	19
D3		63	19	23	17
Demand	11	3	14	16	

Now, using the Step 6., the optimal solution to the fuzzy bottleneck problem is given below:

	S1	S2	S3	S4	Supply
D1	10(8)				8
D2	66(3)			21(16)	19
D3		63(3)	19(14)		17
Demand	11	3	14	16	

Using the Blocking method, we have that the optimal solution of the time Transportation problem of fuzzy BTP is (65,66,67).

5. Fully Fuzzy Bottleneck-Cost Transportation Problem

Consider the following BCTP:

Minimize $\tilde{z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \otimes \tilde{x}_{ij}$

Minimize $\tilde{T} = [\underset{(i,j)}{\text{Maximize}}\ \tilde{t}_{ij} / \tilde{x}_{ij} > 0]$

Subject to

$\sum_{j=1}^{n} \tilde{a}_{ij} \otimes \tilde{x}_{ij} = \tilde{a}_i$, i = 1, 2, …, m (1)

$\sum_{i=1}^{m} \tilde{a}_{ij} \otimes \tilde{x}_{ij} = \tilde{b}_j$, j = 1,2,...….,n (2)

$\tilde{x}_{ij} \geq 0$, for all i and j and integers, (3)

Where m = the number of supply points; n = the number of demand points;

$\tilde{x}_{ij}$ = the number of fuzzy units shipped from supply point i to demand point j;

$\tilde{t}_{ij}$ = the time of transporting goods from supply point i to demand point j;

$\tilde{a}_i$ = the fuzzy supply at supply point i and $\tilde{b}_j$ = the fuzzy demand at demand point j.

In Fuzzy BCTP, faster/ slower mode of the transportation results in lesser/ higher time of transportation but involves a high/low cost of transportation. This implies that the commissioning of a project is influenced by the bottleneck. Thus, for a sequence of various times in a specified time interval, one can have a sequence of less-cost transportation schedules. This type of analysis enables the decision makers to select an appropriate transportation schedule, depending on his financial position and the extent of bottleneck that they can afford

Definition 5.1: A point (X,T) where $X = \{x_{ij}; i = 1,2,...,m \text{ and } j = 1,2,3,...,n\}$ and T is a time, is said to be a feasible solution of (MP) if *X* satisfies the conditions (1), (2) and (3).

Definition 5.2: A feasible point (X_0,T_0) is said to be efficient for (MP) if there exists no other feasible point (X,T) in (MP) such that $z_1(X) \leq z_1(X_0)$ and $z_2(T) < z_2(T_0)$ or $z_1(X) < z_1(X_0)$ and $z_2(T) \leq z_2(T_0)$.

Definition 5.3: A cost transportation problem of a BCTP is said to be active for any time M if the minimum time transportation corresponding to the cost transportation problem is M.

5.4 Blocking Zero point method

Now the blocking zero point method for finding all efficient solutions to the Fuzzy BCTP.

The blocking zero point method proceeds as follows.

Algorithm:

Step 1: Construct the time transportation problem from the given BCTP.

Step 2: Solve the time transportation problem by the blocking method. Let the optimal solution be T_0.

Step 3: Construct the cost transportation problem from the given BCTP.

Step 4: Solve the cost transportation problem by the zero point method and also, find the corresponding time transportation. Let it be T_m.

Step 5: For each time M in $[T_0, T_m]$, compute $\alpha = \dfrac{T_m - M}{T_m - T_0}$ which is the level of time satisfaction for the time *M*.

Step 6: Construct the active cost transportation problem for each time M in $[T_0, T_m]$ and solve it by zero point method.

Step 7: For each time M, an optimal solution to the cost transportation problem, X is obtained from the Step 6 with the level of time satisfaction α . Then, the vector (X, M) is an efficient solution to BCP.

5.5. Numerical Example 2

Consider the following 3X4 fully fuzzy bottleneck-cost transportation problem. The upper left corner in each cell indicates the time of fuzzy transportation on the corresponding route and the lower right corner in each cell indicates the unit fuzzy transportation cost per unit on that route

	S1	S2	S3	S4	Supply
D1	(8,10,12) **(4,5,6)**	(67,68,69) **(4,6,8)**	(71,73,75) **(9,10,11)**	(51,52,53) **(9,11,13)**	(6,8,10)
D2	(65,66,67) **(4,6,8)**	(93,95,97) **(6,7,8)**	(29,30,31) **(10,12,14)**	(19,21,23) **(13,14,15)**	(17,19,21)
D3	(95,97,99) **(13,14,15)**	(62,63,64) **(9,11,13)**	(17,19,21) **(8,9,10)**	(22,23,24) **(5,7,9)**	(15,17,19)
Demand	(10,11,12)	(2,3,4)	(12,14,16)	(14,16,18)	

Now, the time transportation problem of fully fuzzy BCTP is given below:

	S1	S2	S3	S4	Supply
D1	(8,10,12)	(67,68,69)	(71,73,75)	(51,52,53)	(6,8,10)
D2	(65,66,67)	(93,95,97)	(29,30,31)	(19,21,23)	(17,19,21)
D3	(95,97,99)	(62,63,64)	(17,19,21)	(22,23,24)	(15,17,19)
Demand	(10,11,12)	(2,3,4)	(12,14,16)	(14,16,18)	

Using the Blocking method, we have that the optimal solution of the time Transportation problem of fully fuzzy BCTP is (65,66,67).
Now, the cost transportation table of fully fuzzy BCTP is given below

	S1	S2	S3	S4	Supply
D1	(4,5,6)	(4,6,8)	(9,10,11)	(9,11,13)	(6,8,10)
D2	(4,6,8)	(6,7,8)	(10,12,14)	(13,14,15)	(17,19,21)
D3	(13,14,15)	(9,11,13)	(8,9,10)	(5,7,9)	(15,17,19)
Demand	(10,11,12)	(2,3,4)	(12,14,16)	(14,16,18)	

By zero point method, we get the allotment table

	S1	S2	S3	S4	Supply
D1			(9,10,11) **(6,8,10)**		(6,8,10)
D2	(4,6,8) **(10,11,12)**	(6,7,8) **(2,3,4)**	(10,12,14) **(1,5,9)**		(17,19,21)
D3			(8,9,10) **(-3,1,5)**	(5,7,9) **(14,16,18)**	(15,17,19)

Demand	(10,11,12)	(2,3,4)	(12,14,16)	(14,16,18)	

By zero point method, the optimal solution is $\tilde{x}_{13} = (6,8,10)$; $\tilde{x}_{21} = (10,11,12)$; $\tilde{x}_{22} = (2,3,4)$; $\tilde{x}_{23} = (1,5,9)$; $\tilde{x}_{33} = (-3,1,5)$ and $\tilde{x}_{34} = (14,16,18)$ with the minimum transportation cost is (162,348,576) and the minimum time transportation is (93,95,97).

Now, we have $T_0 = (65,66,67)$; $T_m = (93,95,97)$ and the time M={ (65,66,67), (67,68,69), (71,73,75), (93,95,97) }

Now, the active cost transportation problem of fuzzy BCTP for M = (65,66,67) is given below:

	S1	**S2**	**S3**	**S4**	**Supply**
D1	(4,5,6)	-	-	(9,11,13)	(6,8,10)
D2	(4,6,8)	-	(10,12,14)	(13,14,15)	(17,19,21)
D3	-	(9,11,13)	(8,9,10)	(5,7,9)	(15,17,19)
Demand	(10,11,12)	(2,3,4)	(12,14,16)	(14,16,18)	

By zero point method, we get the allotment table

	S1	**S2**	**S3**	**S4**	**Supply**
D1	(4,5,6) **(-1,6,13)**			(9,11,13) **(-3,2,7)**	(6,8,10)
D2	(4,6,8) **(-3,5,13)**		(10,12,14) **(4,14,24)**		(17,19,21)
D3		(9,11,13) **(2,3,4)**		(5,7,9) **(11,14,17)**	(15,17,19)
Demand	(10,11,12)	(2,3,4)	(12,14,16)	(14,16,18)	

Using zero point method, the optimal solution is $\tilde{x}_{11} = (-1,6,13)$; $\tilde{x}_{14} = (-3,2,7)$; $\tilde{x}_{21} = (-3,5,13)$; $\tilde{x}_{23} = (4,14,24)$; $\tilde{x}_{32} = (2,3,4)$; $\tilde{x}_{34} = (11,14,17)$ with total minimum transportation cost is (70,381,814).

Now, the active cost transportation problem of fuzzy BCTP for M = (67,68,69) is given below

	S1	**S2**	**S3**	**S4**	**Supply**
D1	(4,5,6)	(4,6,8)	-	(9,11,13)	(6,8,10)
D2	(4,6,8)	-	(10,12,14)	(13,14,15)	(17,19,21)
D3	-	(9,11,13)	(8,9,10)	(5,7,9)	(15,17,19)
Demand	(10,11,12)	(2,3,4)	(12,14,16)	(14,16,18)	

By zero point method, we get the allotment table

	S1	S2	S3	S4	Supply
D1	(4,5,6) **(-4,5,14)**	(4,6,8) **(-8,3,14)**			(6,8,10)
D2	(4,6,8) **(-2,6,14)**		(10,12,14) **(7,13,19)**		(17,19,21)
D3			(8,9,10) **(-3,1,5)**	(5,7,9) **(14,16,18)**	(15,17,19)
Demand	(10,11,12)	(2,3,4)	(12,14,16)	(14,16,18)	

Using zero point method, the optimal solution is $\tilde{x}_{11} = (-4,5,14)$; $\tilde{x}_{12} = (-8,3,14)$; $\tilde{x}_{21} = (-2,6,14)$; $\tilde{x}_{23} = (7,13,19)$; $\tilde{x}_{33} = (-3,1,5)$; $\tilde{x}_{34} = (14,16,18)$ with the minimum transportation cost is (60,356,786).
Now, the active cost transportation table of fuzzy BCTP for M = (71,73,75) is given below

	S1	S2	S3	S4	Supply
D1	(4,5,6)	(4,6,8)	(9,10,11)	(9,11,13)	(6,8,10)
D2	(4,6,8)	-	(10,12,14)	(13,14,15)	(17,19,21)
D3	-	(9,11,13)	(8,9,10)	(5,7,9)	(15,17,19)
Demand	(10,11,12)	(2,3,4)	(12,14,16)	(14,16,18)	

By zero point method, we get the allotment table

	S1	S2	S3	S4	Supply
D1		(4,6,8) **(-8,3,14)**	(9,10,11) **(-4,5,14)**		(6,8,10)
D2	(4,6,8) **(10,11,12)**		(10,12,14) **(5,8,11)**		(17,19,21)
D3			(8,9,10) **(-3,1,5)**	(5,7,9) **(14,16,18)**	(15,17,19)
Demand	(10,11,12)	(2,3,4)	(12,14,16)	(14,16,18)	

Using zero point method, the optimal solution is $\tilde{x}_{12} = (-8,3,14)$; $\tilde{x}_{13} = (-4,5,14)$; $\tilde{x}_{21} = (10,11,12)$; $\tilde{x}_{23} = (5,8,11)$; $\tilde{x}_{33} = (-3,1,5)$; $\tilde{x}_{34} = (14,16,18)$ with the minimum total transportation cost is (68,351,728).

5.6. The efficient solutions of the fully fuzzy BCTP

S. No.	Efficient solution of fully fuzzy BCTP	Objective value of fully fuzzy BCTP	Satisfaction Level α
1	$\tilde{x}_{11} = (-1,6,13)$; $\tilde{x}_{14} = (-3,2,7)$; $\tilde{x}_{21} = (-3,5,13)$; $\tilde{x}_{23} = (4,14,24)$; $\tilde{x}_{32} = (2,3,4)$; $\tilde{x}_{34} = (11,14,17)$ with time (65,66,67).	(70,381,814) & (65, 66,67)	(1,1,1)
2	$\tilde{x}_{11} = (-4,5,14)$; $\tilde{x}_{12} = (-8,3,14)$; $\tilde{x}_{21} = (-2,6,14)$; $\tilde{x}_{23} = (7,13,19)$; $\tilde{x}_{33} = (-3,1,5)$; $\tilde{x}_{34} = (14,16,18)$ with time (67,68,69).	(60,356,786) & (67,68,69)	(0.67,0.93,0.93)
3	$\tilde{x}_{12} = (-8,3,14)$; $\tilde{x}_{13} = (-4,5,14)$; $\tilde{x}_{21} = (10,11,12)$; $\tilde{x}_{23} = (5,8,11)$; $\tilde{x}_{33} = (-3,1,5)$; $\tilde{x}_{34} = (14,16,18)$ with time (71,73,75).	(68,351,728) & (71, 73,75)	(0.79,0.76,0.73)
4	$\tilde{x}_{13} = (6,8,10)$; $\tilde{x}_{21} = (10,11,12)$; $\tilde{x}_{22} = (2,3,4)$; $\tilde{x}_{23} = (1,5,9)$; $\tilde{x}_{33} = (-3,1,5)$ and $\tilde{x}_{34} = (14,16,18)$ with time (93,95,97).	(162,348,576) & (93,95,97)	(0,0,0)

6. Conclusion

The time of transport might be significant factor in several transportation problems. The proposed methods are quite simple from the computational point of view and also, easy to understand and apply. By blocking zero point method, we obtain a sequence of optimal solutions to a fully fuzzy bottleneck-cost transportation problem for a sequence of various times in a time interval. This method provides a set of transportation schedules to fully fuzzy bottleneck-cost transportation problems which helps the decision makers to select an appropriate transportation schedule, depending on his financial position and the extent of bottleneck that he can afford.

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