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Multivariate Analysis of Price Volatility in Petroleum Products Using the DCC and BEKK-GARCH Models

Benjamin Odoi
Assistant Lecturer, Department of Mathematical Sciences,
University of Mines and Technology (UMAT), Ghana
Dr. Sampson Twumasi-Ankrah
Lecturer, Department of Statistics,
Kwame Nkrumah University of Science and Technology, Ghana
Sulemana Al-Hassan
Assoc. Prof, Department of Mining Engineering,
University of Mines and Technology (UMAT), Ghana
Dr. Sampson Takyi Appiah
Senior Lecturer (Dr), Department of Mathematical Sciences,
University of Mines and Technology (UMAT), Ghana

Abstract:
The basic idea to extend univariate models to multivariate is its significance in predicting the dependence in the co-movements of asset returns in a portfolio. The main purpose of this paper was to model the dependence in the co-movement of petroleum products in Ghana using Dynamic Conditional Correlation (DCC) and Baba Engle Kraft and Kroner (BEKK) GARCH models and determine whether BEKK or DCC should be preferred in practical application. The results revealed that though the BEKK suffers from the archetypal curse of dimensionality whereas DCC does not. It was found that the models were adequate since the coefficient of both models were significant. Also, it was found that there was a co-movement in the petroleum products. The result also shows that BEKK was preferred to DCC in optimal model for the estimating conditional covariance regardless of whether targeting was used.

Keywords*: Co-Movements, conditional covariance, DCC, BEKK-GARCH*

1. Introduction

When the general level of prices is relatively stable, the uncertainties of time-related activities such as investment will have diminished. This helps to promote full employment and strong economic growth. Sobel et al. (2006) stipulated that when price stability is achieved and maintained, monetary policy makers have done a good job. The importance of price stability is also emphasized in the Maastricht agreement, which defined the framework for single European currency, Euro, and identified price stability as the main objective (Nortey et al. (2015))

Energy usage pattern determines the foundation of the whole global economy. Any kind of physical production and transportation are completely impossible without energy. Even non-physical production (services) is unlikely to be performed without energy. Remarkably, engineers, physicists and historians often consider energy to be the primary factor to industrial and economic development.

The importance of energy is obvious but our understanding of its being volatile is not good enough. Consequently, it is obvious that the prices of petroleum products have a huge influence on the world economy and, being highly uncertain and volatile, are a source of economic and political risks and instability. Petroleum products are also the world's most actively traded commodity, accounting for 10% of total world trade (Ruta and Venables, 2012).

Ghana suffers in decision making due to political crises and numerous challenges, consequently its budget runs at a deficit in the balance of trade account which tends to weaken its currency. Prices in petroleum products are an essential area in Ghana's economy.

It is well known that the volatility of financial data often varies over time and tends to cluster in periods. The GARCH model and its extensions have been proved to be able to capture the volatility clustering and predict volatilities in the future (Su and Huang, 2010).

When analyzing the co-movements of financial returns, it is always essential to estimate, construct, evaluate, and forecast the co-volatility dynamics of asset returns in a portfolio. This task can be fulfilled by multivariate GARCH (MGARCH) models. The development of MGARCH models could be thought as a great breakthrough against the curse of dimensionality in financial modeling. Many different approaches have been applied parsimoniously and still remain necessary flexibility.
Patnaik (2013) applied dynamic conditional correlation model in the foreign exchange rates of the Indian rupee and four other prominent foreign currencies to measure volatility spillover across these exchange rates.

Padhi and Lagesh (2012) studied volatility transmission between five Asia equity markets, India and USA. Malik and Ewing (2009) studied volatility transmission between oil prices and five different US equity sector indexes.

Chevallier (2012) studied dynamic nature of correlation among oil, gas and CO_2 of European climate exchange, Bloomberg and Reuter's dataset using Baba-Engle-Kraft-Kroner, Constant Conditional Correlation (CCC) and Dynamic Conditional Correlation (DCC) models.

Lin and Li (2015) studied price and volatility spillover effect of monthly data of natural gas of US, Europe and Japan in a VEC-MGARCH model framework. Modern time series methods such as co-integration reflects the price transmission mechanism between futures and spot market.

Kanchan et al. (2017), studied transmission of price signals and volatility spillover effects between the spot and futures market of black pepper, using Johansen co-integration test, VEC-BEKK and Dynamic Conditional Correlation (DCC) model.

In this paper, MGARCH models (DCC and BEKK) are estimated for the dependence in the co-movement volatility and co-volatility of the prices of petroleum products.

2. Data and Methodology

Monthly historical data of prices of petroleum products (kerosene, gas oil, premium gasoline, liquefied petroleum gas (LPG) were acquired from National Petroleum Authority website. The data was span from 1st August, 2007 to 16th November, 2016. The prices were transformed into returns by taking the log difference of the previous price p_{t-1} from the current price p_t.

The data were analysed using multivariate GARCH, DCC and BEKK models. The procedure applied most often in the model estimation involves the maximization of a likelihood function constructed on the assumption of independently and identically distributed standardized residuals.

Eagle (2001), said in analysing and understanding how univariate GARCH works is fundamental for the study of Dynamic Conditional correlation multivariate GARCH model. The DCC model is a non-linear combination of univariate GARCH and its matrix.

2.1. Generalized Arch (GARCH) Model

This model is a Generalized Autoregressive Conditional Heteroscedasticity (GARCH) (Bollerslev, 1986) model when considering the conditional variance. In that case the GARCH (p, q) model where q is the order of the GARCH terms σ^2 and p is the order of the ARCH terms $\in^2$ is given as in Equation (1)

$$\sigma_{\in}^2 = a_0 + a_1 \in_{t-1}^2 + \cdots + a_p \in_{t-p}^2 + \beta_1\sigma_{t-1}^2 + \cdots + \beta_q\sigma_{t-q}^2 \quad (1)$$

$$\sigma_{\in}^2 = a_0 + \sum\nolimits_{i=1}^{p} a_i \in_{t-i}^2 + \sum\nolimits_{j=1}^{q} \beta_j \sigma^2{}_{t-j} \quad (2)$$

where a_i -measures the extents to which volatility shocks today feeds through into next period volatility a_i + β_j - measures the rate at which this effect dies over , a_0 -represent ambient volatility (the weighted long run variance), $\sum_{i=1}^{p} a_i \in_{t-i}^2$ -is the moving average term, which is the sum of the p previous lags of the squared-innovations multiplied by the assigned weight α_i for each lagged square innovation. $\sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$ is the autoregressive term, which is the sum of the q previous lagged variances multiplied by the assigned β_j for each lagged variance. $\in_{t-i}^2$ - $\sigma^2{}_{t-j}$ is now the volatility shock.

2.2. BEKK-GARCH Models

To ensure positive definiteness, a new parameterization of the conditional variance matrix H_t was defined by Baba et al. (1990) and became known as BEKK model, which is viewed as another restricted version of the VEC model. It achieves the positive definiteness of the conditional covariance by formulating the model in a way that is a property implied by the model structure.

The form of the BEKK model is expressed as Equation (3)

$$H_t = CC' + \sum_{k=1}^{k}\sum_{i=1}^{q} A_{ki}'\varepsilon_{t-i}\varepsilon_{t-i}'A_{ki} + \sum_{k=1}^{k}\sum_{j=1}^{p} B_{kj}'H_{t-j}B_{kj} \qquad (3)$$

where A_{ki} , B_{kj} are $N \times N$ parameter matrices, and C is a lower triangular matrix. The purpose of decomposing the constant term into a product of two triangular matrices is to guarantee the positive semi-definiteness of H_t. Where K>1 an identification problem would be generated for the reason that there is not only a single parameterization that can obtain the same representation of the model.

The first –order BEKK model is given as Equation (4)

$$H_t = CC' + A'\varepsilon_{t-1}\varepsilon_{t-1}'A + B'H_{t-1}B. \qquad (4)$$

The BEKK model also has its diagonal form by assuming A_{ki} , B_{kj} matrices are diagonal. It is a restricted version of the DVEC model. The most restricted version of the diagonal BEKK model is the scalar BEKK one with $A = aI$, $B = bI$ where *a* and *b* are scalars.

Estimation of the BEKK model still bears large computation due to several matrix transpositions. The numbers of parameters of the complete BEKK model is $(p+q)KN^2 + N(N+1)/2$. even in the diagonal one, the number of parameters soon reduces to $(p+q)K \times N + N(N+1)/2$ but it is still large. The BEKK form is not linear in parameters, which makes the convergence of the model difficult; however, the strong point lies such that the model structure automatically guarantees the positive definiteness of H_t. Under the overall consideration, it is typically assumed that $p = q = k = 1$ in BEKK form applications.

One condition has to be fulfilled in order to ensure covariance stationarity, which the absolute eigenvalues of the expression $\sum_{i=1}^{p}\sum_{k=1}^{K} A_{ik} \otimes A_{ik} + \sum_{j=1}^{q}\sum_{k=1}^{K} B_{jk} \otimes B_{jk}$ have to be less than one. (Silvennoinen and Terasvirta, 2009).

2.3. The Dynamic Conditional Correlation (DCC) Model

To extend the assumptions in the univariate GARCH to multivariate case, suppose that we have *n* assets in a portfolio and the return vector $x_t = (x_{1t}, x_{2t}, x_{3t}, ..., x_{nt})$. Furthermore, assume that the conditional returns are normally distributed with zero mean and conditional covariance matrix $H_t = E\{x_t x_t / \lambda_{t-1}\}$.This implies that $x_t = H_t^{1/2} y_t$ and $x_t / \lambda_{t-1} \sim N(0, H_t)$ $z_t = (z_{1t}, z_{2t}, z_{3t}, \ldots, z_{nt})' \sim N(0, I_n)$ and I_n is identity matrix of order *n*. $H_t^{1/2}$ may be obtained by cholesky decomposition of H_t. In DCC- model, the covariance matrix is decomposed into $H_t \equiv D_t X_t D_t$ where D_t is the diagonal matrix of time varying standard variation from univariate GARCH process as shown in Equation (5).

$$D_t = \begin{bmatrix} \sqrt{h_{1t}} & 0 & \ldots & 0 \\ 0 & \sqrt{h_{2t}} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \sqrt{h_{nt}} \end{bmatrix} \qquad (5)$$

X_t is the conditional correlation matrix of the standardized disturbances ε_t;

$$X_t = \begin{bmatrix} 1 & Q_{12,t} & \ldots & Q_{1n,t} \\ Q_{21,t} & 1 & \ldots & Q_{2n,t} \\ \vdots & \vdots & \ddots & \vdots \\ Q_{n1,t} & Q_{n2,t} & \ldots & 1 \end{bmatrix} \qquad (6)$$

and

$$\varepsilon_t = D_t^{-1} x_t \sim N(0, X_t) \qquad (7)$$

The conditional correlation is the conditional covariance between the standardized disturbances. By the definition of the covariance matrix, H_t has to be positive definite. Since H_t is a quadratic form based on X_{ts}, it follows from linear algebra that X_t has to be positive definite to ensure that H_t is positive definite. By the definition of the conditional correlation matrix all the elements have to be equal to or less than one. To ensure that all these requirements are met, X_t is decomposed into

$X_t = Q_t^{*-1} Q_t Q_t^{*-1}$ where Q_t is a positive definite matrix defining the structure of the dynamics and Q_t^{*-1} rescales the elements in Q_t to ensure that $|q_{ij}| \leq 1.$ this implies that, Q_t^{*-1} is simply the inverted diagonal matrix with the squared root diagonal elements of Q_t .

$$Q_t^{*-1} = \begin{bmatrix} 1/\sqrt{Q_{11,t}} & 0 & \dots & 0 \\ 0 & 1/\sqrt{Q_{11,t}} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1/\sqrt{Q_{11,t}} \end{bmatrix} \quad (8)$$

Suppose that Q_t has the following dynamics:

$$Q_t = (1 - \alpha - \beta)\bar{Q} + \alpha \varepsilon_{t-1}^{'} + \beta Q_{t-1} \quad (9)$$

where $\bar{Q}$ is the unconditional covariance of the standardized disturbances $\bar{Q} = \text{cov}(\varepsilon_t \varepsilon_t^{'}) = E\{\varepsilon_t \varepsilon_t^{'}\}$ $and \beta$ are scalars.

The dynamic structure defined above is the simplest multivariate GARCH called the scalar GARCH. A major cavet of this structure is the all correlations obey the same structure.
The structure can be extended to the general DCC (P, Q)

$$Q_t = \left(1 - \sum_{i-1}^{P} \alpha_i - \sum_{j=1}^{Q} \beta_j\right)\bar{Q} + \sum_{i=1}^{P} \alpha_i \varepsilon_{t-1} \varepsilon_{t-1}^{'} + \sum_{j=1}^{Q} \beta_j Q_{t-1} \quad (10)$$

3. Diagnostic Checking

3.1. Testing Conditional Heteroscedasticity

There are many different test for testing Conditional Heteroscedasticity, in this paper the portmanteau test and ranked- based test were used.

3.1.1. Portmanteau Test

The test statistics is given as:

$$Q_k(m) = T^2 \sum_{i=1}^{m} \frac{1}{T-i} b_i^{'} (\hat{\rho}_0^{(a)-1} \otimes \hat{\rho}_0^{(a)-1}) b_i \quad (11)$$

3.1.2 Rank –Based Test

The test statistics is given as:

$$Q_R(m) = \sum_{i=1}^{m} \frac{[\tilde{\rho}_i - E(\tilde{\rho}_i)]^2}{\text{var}(\tilde{\rho}_i)} \quad (12)$$

4. Results and Discussion

4.1. Tests for Multivariate ARCH Effect

The test for the presence of ARCH effect confirms the presence of ARCH in all the petroleum products. This is because the p-values associated to the LM test are small, therefore the null hypothesis of no ARCH effect is rejected. The confirmation of the presence of ARCH effects in each case indicates that volatility in the prices of these petroleum products is time varying as shown in Table 1.

Petroleum Products	LM Test	P –Values
Gas Oil	3.3534	0.0000
LPG	2.7534	0.0000
Kerosene	4.0213	0.0000
Premium Gasoline	1.8724	0.0200

Table 1: ARCH-LM Test Results

4.2. The BEKK Model Parameters

The BEKK model (1, 1) applied to the returns on all the petroleum products are in three pairs, that is (rkero, rgas, and rlpg), (rkero, rgas, and rpga), (rkero, rlpg, rpga), and (rgas, rlpg, rpga). Parameters estimations of the BEKK form are provided in Table 2.

It was observed that BEKK model for (**rgas, rlpg, and rkero)** was the best model since its parameters has the greatest number to be highly significant.

The BEKK model takes the form;

$$H_t = CC' + \sum_{j=1}^{q}\sum_{k=1}^{k} A'_{kj}\varepsilon_{t-j}\varepsilon'_{t-j}A_{kj} + \sum_{j=1}^{p}\sum_{k=1}^{k} B'_{kj}H_{t-j}B_{kj} \quad (13)$$

Where A_{kj} , B_{kj} are $N \times N$ parameter matrices, and C is a lower triangular matrix.

Using the first order of the BEKK model

$$H_t = CC' + A'\varepsilon_{t-1}\varepsilon'_{t-1} + A + B'H_{t-1}B \quad (14)$$

Coefficient(s):	Estimate	Std. Error	T Value	Pr(>|T|)
mu1.rkero	0.0112	0.0040	1.6300	0.1000
mu2.rgas	0.0102	0.0040	3.3000	0.0000
mu3.rlpg	0.0223	0.0050	3.3000	0.0000
C(1,1)	0.0412	0.0050	7.3900	0.0000
C(2,1)	0.0411	0.0070	5.4700	0.0000
C(3,1)	0.0310	0.0090	3.7200	0.0000
C(2,2)	0.0212	0.0060	2.8000	0.0000
C(3,2)	0.0300	0.0080	3.5700	0.0000
C(3,3)	0.0200	0.0030	9.0500	0.0000
A(1,1)	0.4300	0.2570	1.6900	0.0340
A(2,1)	-0.5000	0.2590	-1.9300	0.0400
A(3,1)	-0.4100	0.2520	-1.6200	0.1000
A(1,2)	-0.5000	0.2070	-2.4100	0.0200
A(2,2)	0.8000	0.2030	3.9400	0.0000
A(3,2)	-0.1100	0.2100	-0.5200	0.6100
A(1,3)	-0.5000	0.3480	-1.4300	0.1500
A(2,3)	-0.5000	0.2950	-1.7000	0.0900
A(3,3)	0.0600	0.3160	0.1900	0.8500
B(1,1)	0.7000	0.1420	4.9400	0.0000
B(2,1)	0.1800	0.1200	1.5200	0.1300
B(3,1)	0.2500	0.1360	1.8600	0.0450
B(1,2)	-0.1814	0.1930	-0.9500	0.0000
B(2,2)	0.3612	0.1600	2.2500	0.0200
B(3,2)	-0.1623	0.1600	-1.0200	0.3100
B(1,3)	-0.2040	0.2630	-0.7400	0.4600
B(2,3)	-0.0424	0.2000	-0.2100	0.0400
B(3,3)	0.4340	0.2140	2.0100	0.0400

Table 2: The Estimation BEKK-GARCH Model Parameters (rkero, rgas, and rlpg)

We can see from Table 2 most of the variables estimated are statistically significant.
The estimated BEEK- GARCH model can be obtained by substituting the following matrices into Equation (4).

$$A = \begin{pmatrix} 0.43 & -0.50 & -0.50 \\ -0.50 & 0.80 & -0.50 \\ -0.50 & -0.11 & 0.06 \end{pmatrix} \qquad B = \begin{pmatrix} 0.70 & -0.18 & -0.20 \\ 0.18 & 0.36 & -0.04 \\ 0.25 & -0.16 & 0.43 \end{pmatrix}$$

$$C = \begin{pmatrix} 0.04 & 0 & 0 \\ 0.04 & 0.02 & 0 \\ 0.03 & 0.03 & 0.02 \end{pmatrix}$$

When evaluating Table 2 it can be observed that the non-diagonal elements are non-significant which shows the theoretical fact that these parameters often are redundant. Due to this, the estimation is instead performed with diagonal BEKK model, trying to achieve significant parameters.

It can be observed that eighteen parameters are highly significant. Additionally, the remaining parameters are very close to the parameter estimations of the BEKK model. Despite the restrictions of the model construction, the model seems to suit the selected sample of the data.

4.3. DCC-GARCH Model Parameters (rkero, rgas, and rlpg)

The DCC model (1, 1) applied to the returns on all the petroleum products are in three pairs, that is (rkero, rgas, and rlpg), (rkero, rgas, and rpga), (rkero, rlpg, rpga), and (rgas, rlpg, rpga). Parameters estimations of the DCC form is provided in Table 3.

It seems that DCC model for (rgas, rlpg, and rkero) was the best model since its parameters has the greatest number to be highly significant.

Coefficient(s):	Estimate	Std. Error	T Value	Pr(>|T|)
mu1.rkero	0.0112	0.004012	1.43300	0.1000
mu2.rgas	0.0102	0.0042341	3.145000	0.0000
mu3.rlpg	0.0223	0.0050322	3.673000	0.0000
C(1)	0.003112	0.0050342	7.316900	0.0000
C(2)	0.05211	0.0070331	5.584700	0.0000
C(3)	0.0610	0.0090244	2.72003	0.0000
A(1)	0.5600	0.257012	7.69005	0.0340
A(2)	0.3624	0.259012	-2.73000	0.0400
A(3)	-0.13179	0.2520513	-2.13200	0.1000
B(1)	0.18123	0.1520233	5.45400	0.0000
B(2)	0.2350	0.1600301	2.4300	0.1300
B(3)	0.38012	0.1360343	6.8600	0.0450
DCC(1)	0.0845	0.007013	3.6000	0.0000
DCC(2)	0.0234	0.00850	2.7700	0.0000

Table 3: The Estimation of the Parameters of DCC Model

The estimated DCC model's unconditional covariance matrix is given as

$$
\begin{aligned}
h_{11t} &= 3.112\times10^{-3} + 0.5600\varepsilon_{1,t-1}^{2} + 0.18123h_{11,t-1} \\
h_{22t} &= 5.2\times10^{-2} + 0.3624\varepsilon_{2,t-1}^{2} + 0.2350h_{22,t-1} \\
h_{33t} &= 6.1\times10^{-2} - 0.1379\varepsilon_{3,t-1}^{2} + 0.38012h_{33,t-1} \\
Q_t &= (1-0.0845-0.0234)\overline{Q} + 0.0845u_{t-1}u_{t-1}^{'} + 0.0234Q_{t-1} \\
R_t &= diag(q_{11t}^{1/2}, q_{22t}^{1/2}, q_{33t}^{1/2})Q_t diag(q_{11t}^{1/2}, q_{22t}^{1/2}, q_{33t}^{1/2})
\end{aligned}
\tag{15}
$$

$$
u_{it} = \varepsilon_{it} / \sqrt{h_{it}} \tag{16}
$$

5. Diagnostic Test for the Multivariate-BEKK and DCC Model

In checking whether the BEKK and DCC models estimated are adequate to predicts volatility in the petroleum products, two tests were conducted that is the ranked test and the portmanteaus (Qk (m)) test.

The portmanteaus test is to check the Conditional Heteroscedasticity that is the cross serial correlation on the returns of the estimated BEKK and DCC models and the Ranked –Based Test checks the autocorrelation of the residuals on the returns of the petroleum products. The significant level was 5 percent. It can be observed that the p values for both tests are greater than the significant level.

We can therefore conclude that the BEKK and DCC models estimated shows evidence that volatility model is adequate; hence the model is adequate to predict the volatility in the co-movement of the returns of the petroleum products as shown in Table 4. The graphs shown in Figure 1 and 2 reveals features, of volatility clustering and the relation between maturity and volatility, that is longer maturity corresponds to higher volatility for the BEKK model. In the same situation, the Figure 2 shows the fitting performance of volatility clustering for the DCC models between the various petroleum products.

	BEKK-MODEL		DCC-MODEL	
Returns	Ranked Test	Portmanteaus Test	Ranked test	Portmanteus Test
Kerosene, Gas Oil, LPG	16.9790 *0.0748	13.5726 *0.9920	18.9012 *0.0765	19.9440 *0.9210
Kerosene, LPG, Premium Gasoline	65.4493 *0.0564	31.1633 *0.9911	60.4246 *0.0902	32.4550 *0.8953
Gas Oil, LPG, Premium Gasoline	27.8772 *0.0643	82.4582 *0.7013	30.2346 *0.0791	80.3452 *0.8923
Kerosene, Gas Oil, Premium Gasoline	23.2010 *0.0791	52.4201 *0.9911	25.5350 *0.0891	53.2345 *0.9010

Table 4: Test for Cross Serial Correlation and Autocorrelation on BEKK and DCC Models

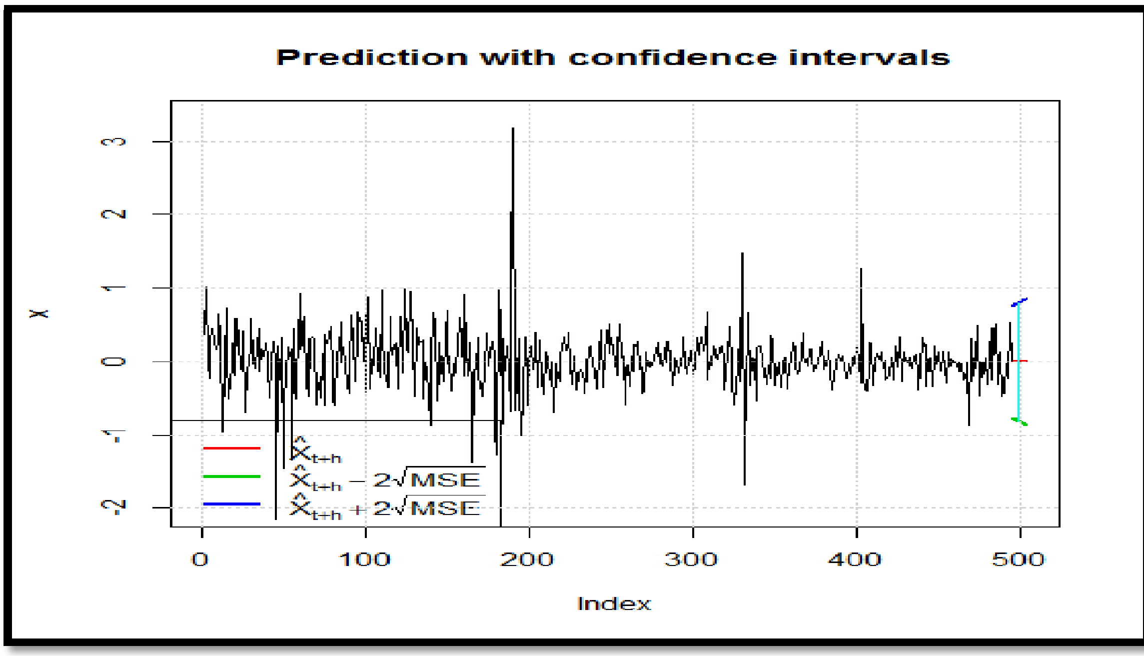

Figure 1: Time Varying Volatility Clustering Using BEKK Model

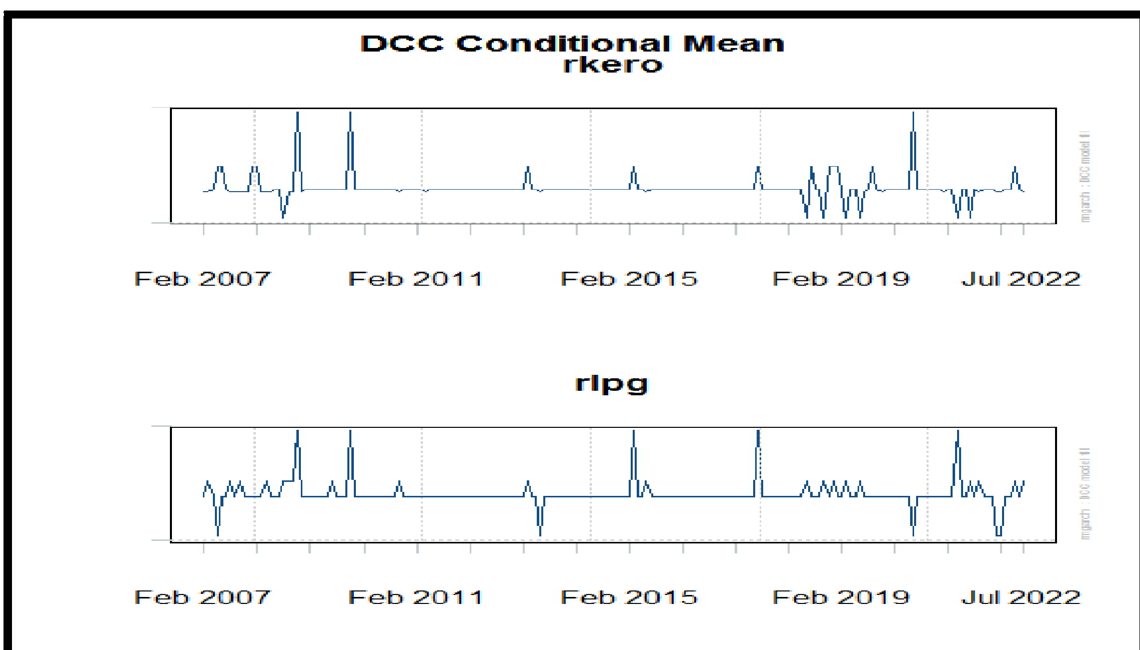

Figure 2: Time Varying Volatility Clusters from DCC Model

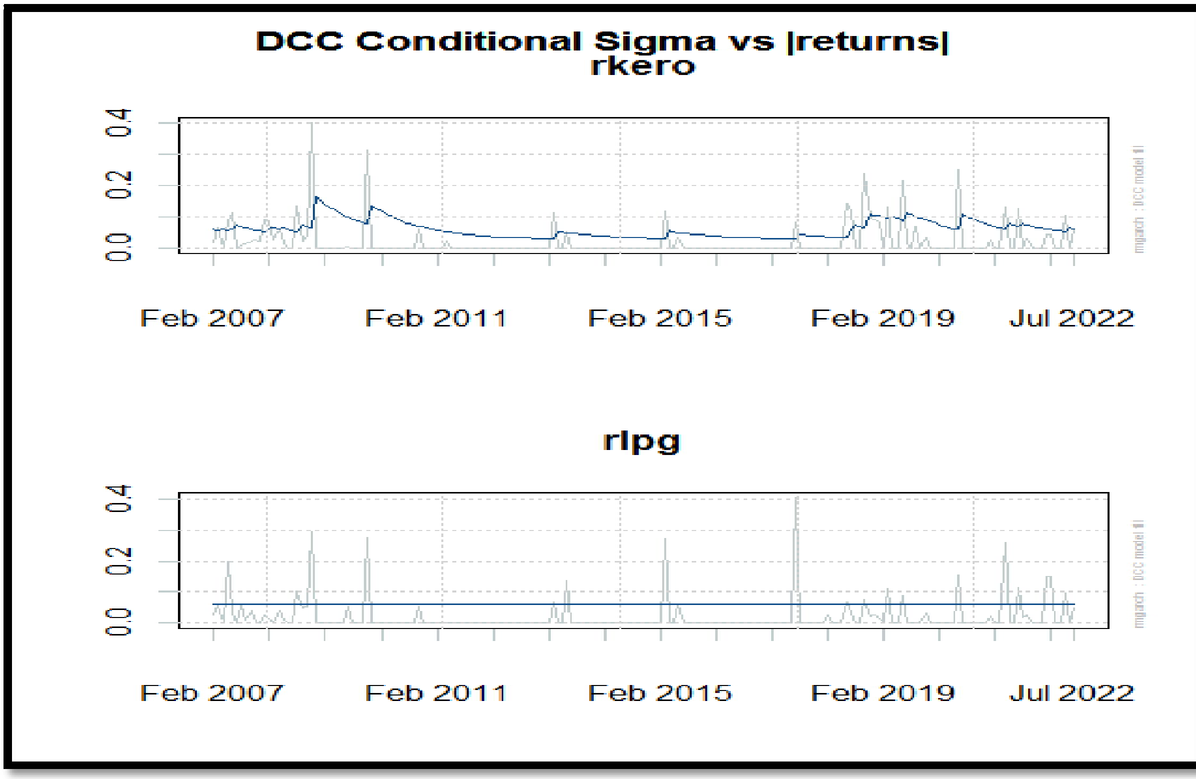

Figure 3

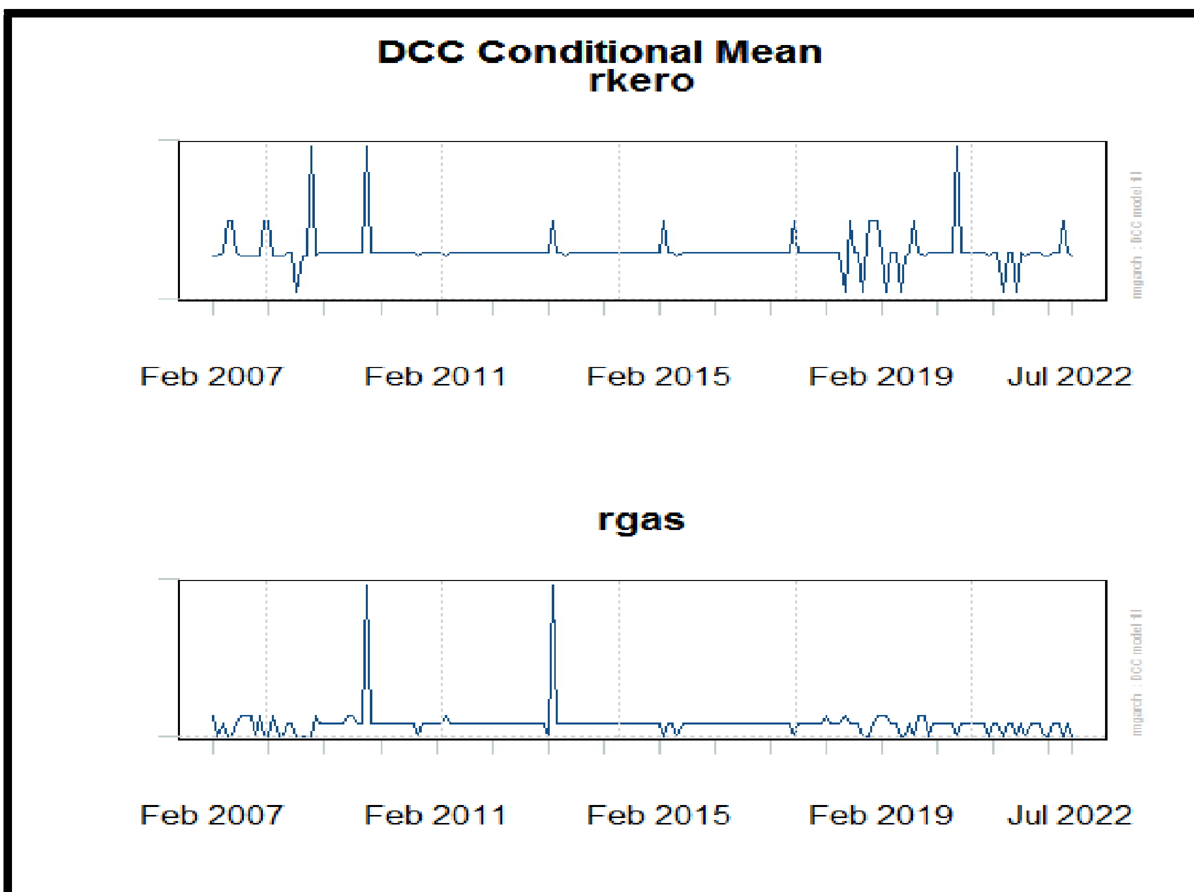

Figure 4: Time Varying Volatility Clusters from DCC Model

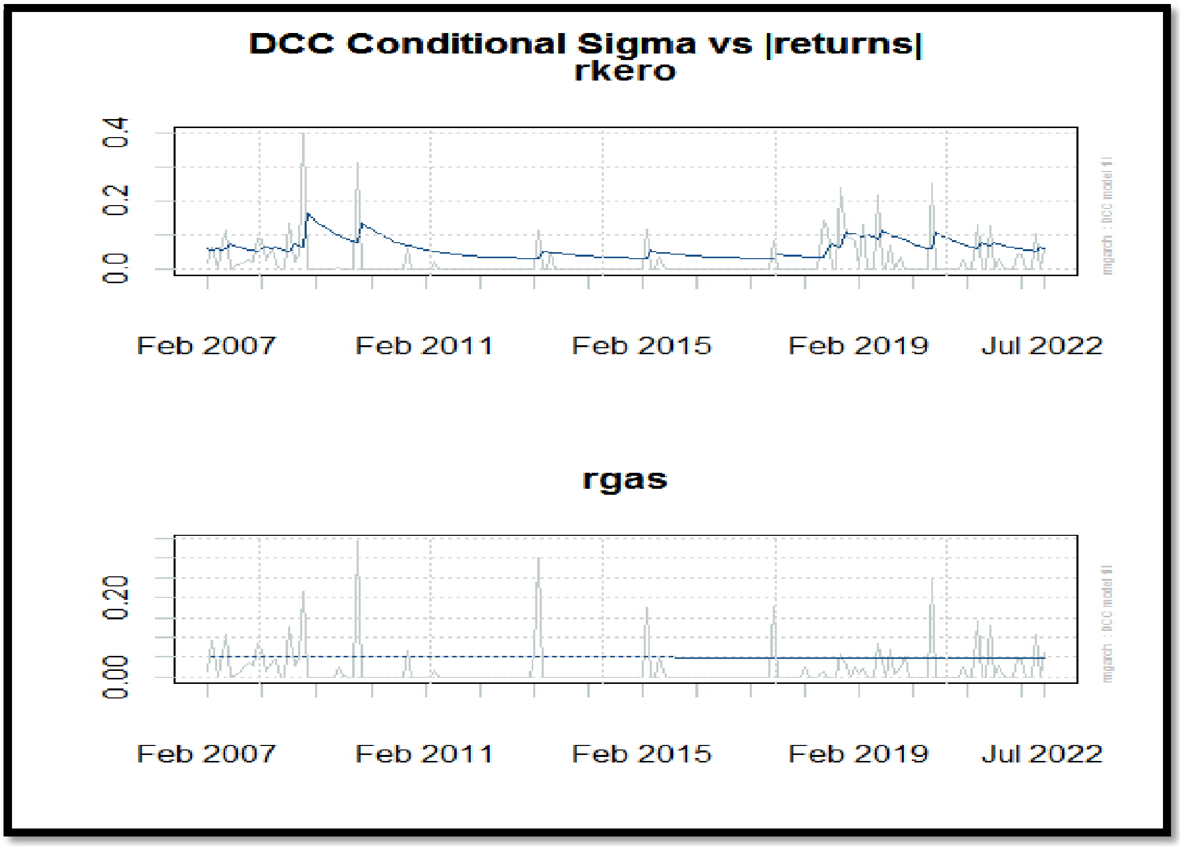

Figure 5

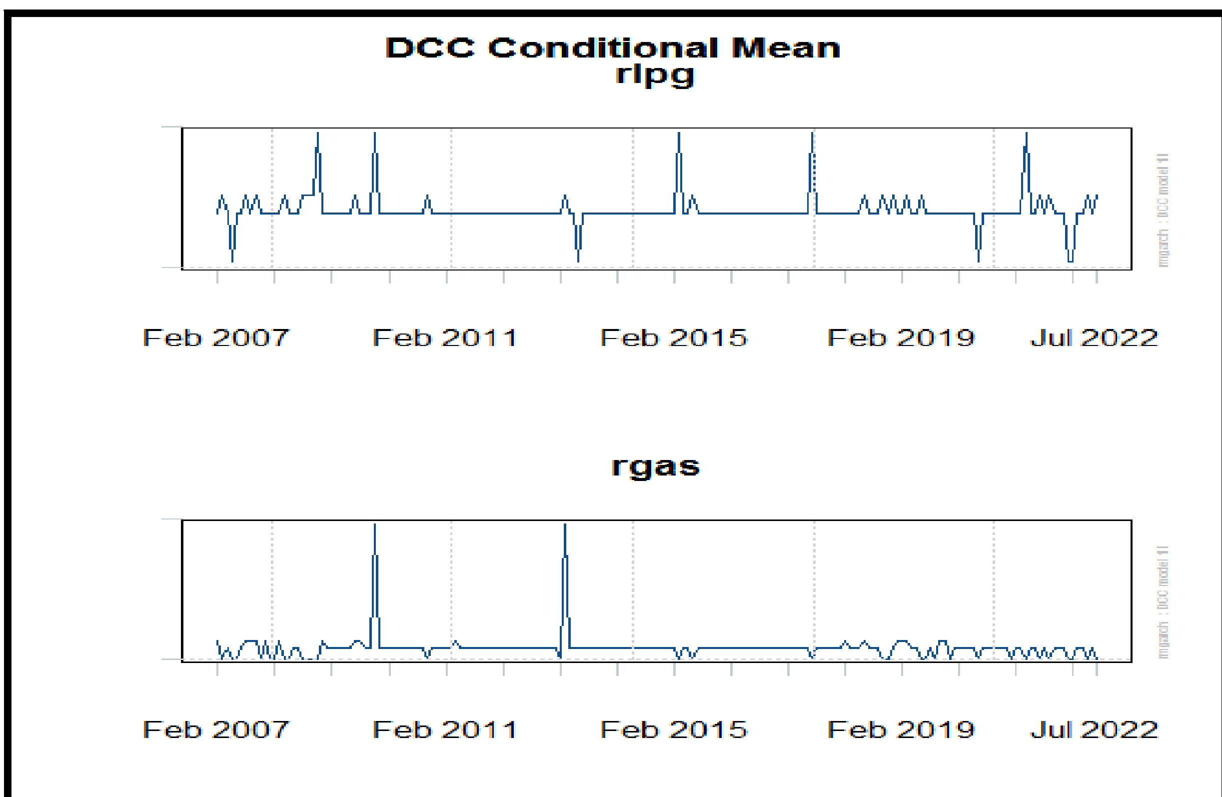

Figure 6: Time Varying Volatility Clusters from DCC Model

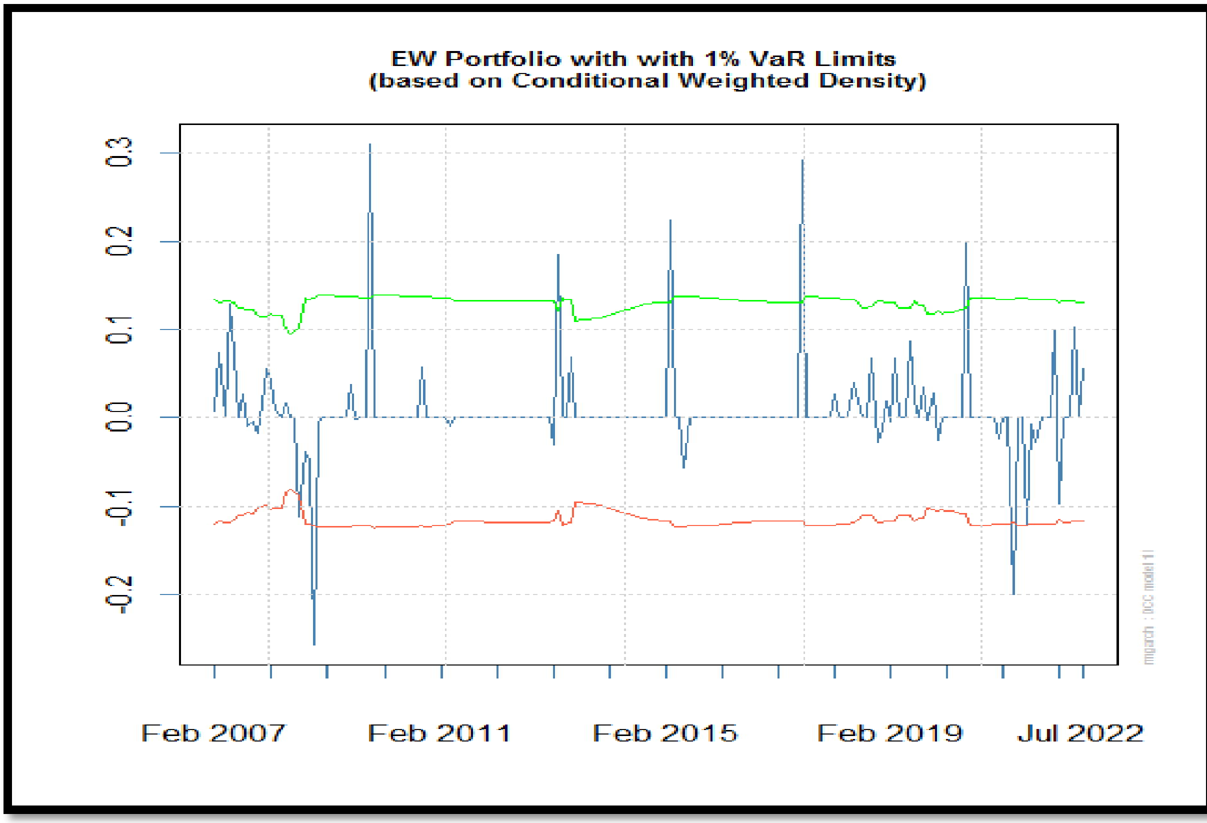

Figure 7: Conditional Covariance from DCC MODEL

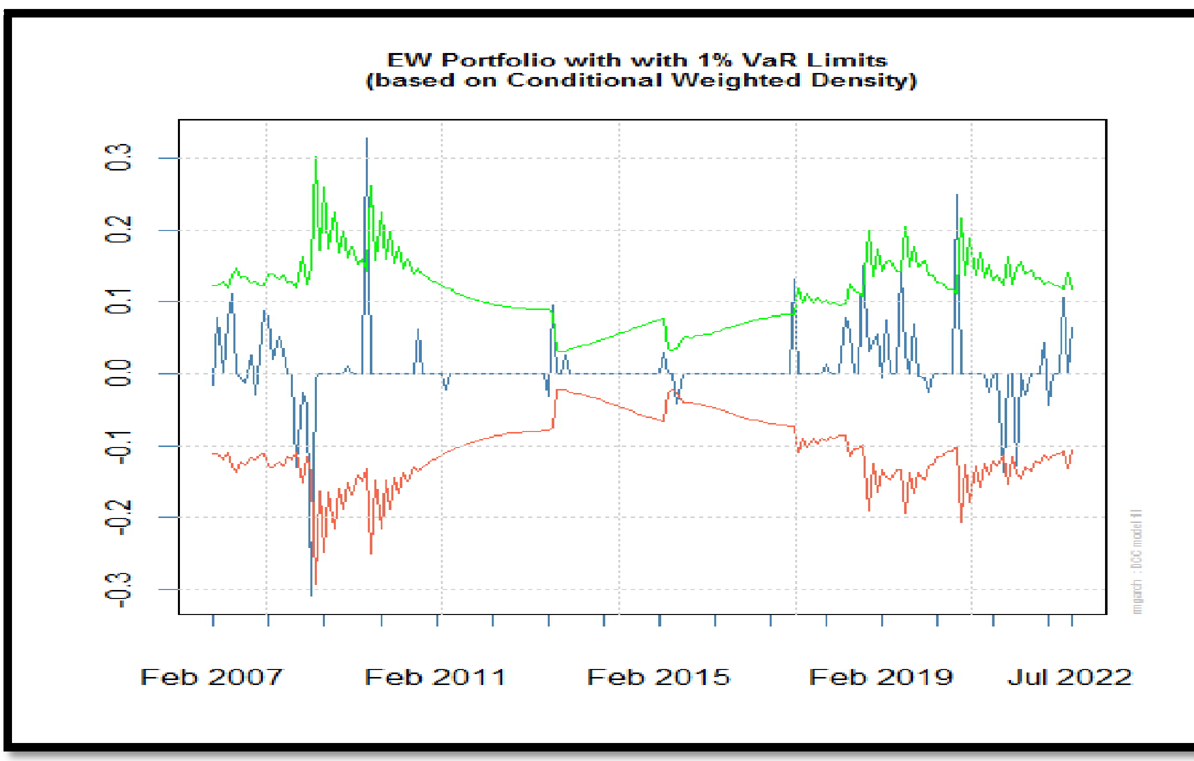

Figure 8: Conditional Covariance for BEKK Model

6. Conclusion

The results revealed that though the BEKK suffers from the archetypal curse of dimensionality whereas DCC does not, the models were adequate since the coefficient of both models were significant and showed there was a co-movement in the petroleum products.

BEKK was preferred to DCC in optimal model for the estimating conditional covariance regardless of whether targeting was used.

That is, the forecasting performance of the BEKK models looks better than that of the DCC model, the forecast volatility generally follows the dynamics of realized volatility.

Though the number of estimated parameters in the DCC model was smaller than that of the BEKK model, which may suggest that the errors accumulated by each parameter of the BEKK models tends to be larger than that of DCC model yet, the BEKK Model prediction performance was better based on the cross validation analysis of the data.

6.1. Recommendation

It was recommended that, the best model obtained in this study are used for policy decisions.

Also, citizens should be educated on factors causing increase in price volatility

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