THE INTERNATIONAL JOURNAL OF SCIENCE & TECHNOLEDGE

Fixed Point in Intuitionistic Fuzzy Metric Space for Weakly-Compatible Maps

M. S. Chauhan
Rajabhoj Govt. College, Mandideep District, Raisen, M.P., India
Dheeraj Aheere
Research Scholar, Vikram University, Ujjain, M.P., India
Bharat Singh
SOC & EIPS Academy Indore, M.P., India

Abstract:
The purpose of this paper is to obtain a common fixed point theorem for weakly compatible, using the property (IFE.A) and (IFH_E) non-weakly compatible intuitionistic fuzzy metric space. Also our result does not require the continuity of the maps. Our result generalizes the result of Yao Yao LAN. [22].

***Key words**: Fixed Point, Intuitionistic Fuzzy Symmetric Space, Non-compatible Maps and Weakly Compatible Maps*

1. Introduction

The concept of fuzzy sets was introduced by Zadeh [11]. Following the concept of fuzzy sets, fuzzy metric spaces have been introduced by Kramosil and Michalek [6], George and Veeramani [4] modified the notion of fuzzy metric space with the help of continuous t-norms.

As a generalization of fuzzy sets and introduced and studied the concept of intuitionistic fuzzy sets. Using the idea of intuitionistic fuzzy sets Park [15] defined the notion of intuitionistic fuzzy metric space with the help of continuous t norm and continuous t co-norm as a generalization of fuzzy metric space, George and Veeramani [4] had showed that every metric induces an intuitionistic fuzzy metric and found a necessary and sufficient condition for an intuitionistic fuzzy metric space to be complete. Choudhary [13] introduced mutually contractive sequence of self maps and proved a fixed point theorem. Kramosil and Michalek [6] introduced the notion of Cauchy sequences in an intuitionistic fuzzy metric space and proved the well known fixed point theorem of Banach spaces. Turkoglu et al. [12] gave the generalization of Jungek,s common fixed point theorem [19] to intuitionistic fuzzy metric space.Recently Aamri and Moutawakil [20] introduced the property (IFE.A) and thus generalized the concept of noncompatible maps. The results obtained in in the metric fixed point theory by using the notion of noncompatible maps or the properties (IFE.A) are very interesting. The question arises whether, by using the concept of incompatibility or its generalized notion, that is the property (IFE.A), we can find equally interesting results in intuitionistic fuzzy metric space also our answer in affirmative.

In this paper is to obtain a common fixed point theorem for weakly compatible self-maps satisfying the property (IFE.A) using implicit relation on a non complete Intuitionistic fuzzy metric space. Also our result does not require the continuity of the maps. Our result generalizes the result of Yao LAN. [22].

2. Preliminaries

Definition 2.1 [7] A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is called a continuous t - norm if ([0,1], $*$) is an abelian topological monoid with the unit 1 such that $a * b \le c * d$ whenever $a \le c$ and $b \le d$ for all $a, b, c, d \in [0,1]$.
Examples of t - norms are $a * b = ab$ and $a * b = \min\{a, b\}$.

Definition 2.2 [7] A binary operation $\Diamond: [0,1] \times [0,1] \to [0,1]$ is continuous t-conorm if $\Diamond$ is satisfying the following condition
(a) $\Diamond$ is commutative and associate,
(b) $\Diamond$ is continuous,
(c) $a \Diamond 0 = a$ for all $a \in [0,1]$,
(d) $a \Diamond b \le c \Diamond d$ Whenever $a \le c$ and $b \le d$ for all $a, b, c, d \in [0,1]$.

Definition 2.3 [4] The 3 - tuple (X , M , $*$) is called a fuzzy metric space (FM - space) if X is an arbitrary set $*$ is a continuous t-norm and M is a fuzzy set in $X^2 \times [0,\infty]$ satisfying the following conditions for all $x, y, z \in X$ and $t, s > 0$.
(FM-1) $M(x, y, 0) > 0$,
(FM-2) $M(x, y, t) = 1, \forall\ t > 0$ iff $x = y$,
(FM-3) $M(x, y, t) = M(y, x, t)$,
(FM-4) $M(x, y, t) * M(y, z, s) \le M(x, z, t + s)$,
(FM-5) $M(x, y, \ .): [0, \infty] \to [0,1]$ is continuous.
Remark 2.1 since $*$ is continuous, it follows from (FM-4)) that the limit of a sequence in FM-space is uniquely determined .

Definition 2.4 [16] A five –tuple (X, M, N, $*$, $\Diamond$) is said to be an intuitionistic fuzzy metric space if X is an arbitrary set $*$ is a continuous t – norm , $\Diamond$ is a continuous t-conorm and M , N are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions for all $x, y, z \in X, s, t > 0$
(IFM-1) $(x, y, t) + N(x, y, t) \le 1$,
(IFM – 2) $M(x, y, t) > 0$,
(IFM-3) $M(x, y, t) = 1$ if and only if x = y ,
(IFM-4) $(x, y, t) = M(y, x, t)$,
(IFM-5) $(x, y, t) * M(y, z, s) \le M(x, z, t + s)$,
(IFM – 6) $M(x, y, .): (0, \infty) \to (0,1)$ is continuous ,
(IFM-7) $N(x, y, t) > 0$,
(IFM-8) $N(x, y, t) = 1$ if and only if x = y ,
(IFM-9) $(x, y, t) = N(y, x, t)$,
(IFM-10) $(x, y, t) \Diamond N(y, z, s) \ge N(x, z, t + s)$,
(IFM – 11) $N(x, y, .): (0, \infty) \to (0,1]$ is continuous .
Then (M , N) is called an intuitionistic fuzzy metric on X , the function $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non- nearness between x and y with respect to t respectively .

Remark 2.1 In intuitionistic fuzzy metric spaces X, M(x,y,.) is non- decreasing and N(x , y,.) is non - increasing for all x, y $\in$ X .

Definition 2.5 A Sequence x_n in X convergent to x if and only if
$M(x_n, x, t) \to 1$ and $N(x_n, x, t) \to 0$.
Example 2.1 Let (X , d) be a metric space denote $a * b = ab$ and $a \Diamond b = \min\{1, a + b\}$ for all $a, b \in [0,1]$ and let M, and N be fuzzy sets on $X^2 \times (0,1)$ defined as follows

$$M(x, y, t) = \frac{t}{t+d(x,y)},$$

$$N(x, y, t) = \frac{d(x, y)}{t + d(x, y)}$$

Then $(X, M, N, *, \Diamond)$ is an intuitionistic fuzzy metric space. We call this intuitionistic fuzzy metric induced by a metric d the standard intuitionistic fuzzy metric space .

Remark 2.2 Note that the above example holds even with the t-norm $a * b = \min\{a, b\}$ and the t-conorm $a \Diamond b = \max\{a, b\}$.

Example 2.2 Let $X = N$ define $a * b = \max\{0, a + b - 1\}$ and $a \Diamond b = a + b - ab$ for all $a, b \in [0,1]$ and let M and N be fuzzy sets on $X^2 \times (0, \infty)$ as follows

$$M(x, y, t) = \begin{cases} \frac{x}{y} & \text{if } x \le y, \\ \frac{y}{x} & \text{if } y \le x. \end{cases}$$

$$N(x, y, t) = \begin{cases} \frac{y - x}{y} & \text{if } x \le y, \\ \frac{x - y}{x} & \text{if } y \le x. \end{cases}$$

for All $x, y \in X$ and $t > 0$ then $(X, M, N, *, \Diamond)$ is an intuitionistic fuzzy metric space.

Remark 2.3 Note that in above example, t-norm $*$ and t- conorm $\Diamond$ are not associated, and there exist no metric d on X satisfying
$$M(x, y, t) = \frac{t}{t+d(x,y)} ,$$

$N(x,y,t) = \frac{d(x,y)}{t+d(x,y)}$.
Where $M(x,y,t)$ and $N(x,y,t)$ are defined in above example, also note that the above functions
(M, N) is not an intuitionistic metric space with the t-norm and t-conorm defined as
$a * b = \min\{a,b\}$ $a \lozenge b = \max\{a,b\}$.

Definition 2.6 [1] Let $(X, M, N, *, \lozenge)$ be an intuitionistic fuzzy metric space then
(a) A sequence $\{x_n\}$ in X is said to be convergent x in X if for each ε > 0 and each $t > 0$ there exists $n_0 \in N$ such that
$M(x_n, x, t) > 1 - \varepsilon$,
and
$N(x_n, x, t) < \varepsilon$. For all $n \geq n_0$

(b) An intuitionistic fuzzy metric space in which every Cauchy sequence is convergent is said to be complete .
Definition 2.7 [19] Let A and B maps from a intuitionistic fuzzy metric space $(X, M, N, *, \lozenge)$into itself. The maps A and B are said to be compatible (or asymptotically commuting) if for all $t > 0$
$lim_{n\to\infty} M(ABx_n, BA_{x_n}, t) = 1$
and
$lim_{n\to\infty} N(AB_{x_n}, BA_{x_n}, t) = 0.$
Whenever $\{x_n\}$ is a sequence in X such that $lim_{n\to\infty} Ax_n = lim_{n\to\infty} Bx_n$ = z for some $z \in X$.

Definition 2.8 [21] Two mappings A and B of a intuitionistic fuzzy metric space $(X, M, N, *, \lozenge)$ into itself are R-weakly commuting provided there exists some real number R such that
M (ABx, BAx, t) ≥ M (Ax, Bx, $\frac{t}{R}$)
and
N (ABx, BAx, t) ≤ N (Ax, Bx, $\frac{t}{R}$) , for each $x \in X$ and $t > 0$.
Definition 2.9 [20] Let f and g be two self mappings of an intuitionistic fuzzy metric space
$(X, M, N, *, \lozenge)$. We say that A and S satisfy the property (IFE.A) if there exists a sequence $\{x_n\}$ such that
$lim_{n\to\infty} fx_n = lim_{n\to\infty} gx_n = t$. for some $t \in X$.
Lemma 2.4 In intuitionistic fuzzy metric space X , M$(x, y, .)$ is non decreasing and N $(x, y, .)$ is non increasing for all $x, y \in X$.
Lemma 2.5
Let $(X, M, N, *, \lozenge)$ be an intuitionistic fuzzy metric space if there exist $k \in (0,1)$ such that
$M(x, y, kt) \geq M(x, y, t)$
and
$N(x, y, kt) \leq N(x, y, t)$, for all $x, y \in X$
Then $x = y$.

IFW. 1 [22] Given $\{x_n\}$, x and y in X ,
$\lim_{n\to\infty} M(x_n, x, t) = 1$, $\lim_{n\to\infty} N(x_n, x, t) = 0$,
and
$\lim_{n\to\infty} M(x_n, y, t) = 1$, $\lim_{n\to\infty} N(x_n, y, t) = 0$,
$\Longrightarrow$ x = y .

IFW. 2[22] Given $\{x_n\}$, $\{y_n\}$and x in X ,
$\lim_{n\to\infty} M(x_n, x, t) = 1$, $\lim_{n\to\infty} N(x_n, x, t) = 0$,
and
$\lim_{n\to\infty} M(x_n, y_n, t) = 1$, $\lim_{n\to\infty} N(x_n, y_n, t) = 0$,
$\Longrightarrow \lim_{n\to\infty} M(y_n, x, t) = 1$, $\lim_{n\to\infty} N(y_n, x, t) = 0$.

Definition 2.10[22] Let f and g be self – mappings of an intuitionistic fuzzy metric space (X, M, N) f and g are called compatible if
$\lim_{n\to\infty} M(fgx_n, gfx_n, t) = 1$, $\lim_{n\to\infty} N(fgx_n, gfx_n, t) = 0$.
Whenever $\{x_n\}$ is a sequence in X such that
$\lim_{n\to\infty} M(fx_n, y, t) = 1$, $\lim_{n\to\infty} N(fx_n, y, t) = 0$,
and
$\lim_{n\to\infty} M(gx_n, y, t) = 1$, $\lim_{n\to\infty} N(gx_n, y, t) = 0$.
For some $y \in X$.

Definition 2.11 [22] Let f and g be self –mappings of an intuitionistic fuzzy metric space (X, M, N) f and g are called weakly compatible if they commute at their coincidence points i.e. if
$fu = gu$ for some $u \in X$, then $fgu = gfu$.

Definition 2.12[22] Let f f and gg be self –mappings of an intuitionistic fuzzy metric space
(X, M, N) We say that f and g satisfy the property (IFE.A) if there exists a sequence
$\{x_n\}$ such that

$\lim_{n\to\infty} M(fx_n, y, t) = 1$, $\lim_{n\to\infty} N(fx_n, y, t) = 0$,
and
$\lim_{n\to\infty} M(gx_n, y, t) = 1$, $\lim_{n\to\infty} N(gx_n, y, t) = 0$.
For some $y \in X$.

Definition 2.13 [22] Let (X , M, N) be an intuitionistic fuzzy metric space .
We say that (X, M, N) satisfy the property (IFH_E) if there exists sequences $\{x_n\}$, $\{y_n\}$ such that
$\lim_{n\to\infty} M(x_n, x, t) = 1$, $\lim_{n\to\infty} N(x_n, x, t) = 0$,
and
$\lim_{n\to\infty} M(y_n, x, t) = 1$, $\lim_{n\to\infty} N(y_n, x, t) = 0$,
Implies that $\lim_{n\to\infty} M(x_n, y_n, t) = 1$, $\lim_{n\to\infty} N(x_n, y_n, t) = 0$.

Remark 2.6 Let Φ be the sets of increasing and continuous and Ψ be the sets of decreasing and continuous function $\varphi, \Psi : [0,1] \to [0,1]$ such that $(\varphi) > t$ and $(\Psi)\varphi > t$ for all $\in (0,1)$.

3. Main Results

Theorem 3.1 Let $(X, M, N, *, \lozenge)$ be an intuitionistic fuzzy symmetric space, which satisfies (IFW.1) and (IFH_E). Let f and g be two weak compatible self mappings of an IFM-space $(X, M, N,)$ such that
(3.1.1) $fX \subset gX$,
(3.1.2) f and g satisfy the property (IFEA),

(3.1.3) $M(fx, fy, kt) \geq M(gx, gy, t)$ and $N(fx, fy, kt) \leq N(gx, gy, t)$, $k \geq 0$,

(3.1.4) $M(fx, ffx, t) > \varphi\left(min\begin{Bmatrix} M(ggx, gfx, t), \sup_{t_1+t_2=\frac{2}{k}t} \min\{M(fx, gx, t_1), M(gx, f^2x, t_2)\}, \\ \sup_{t_3+t_4=\frac{2}{k}t} \max\{M(f^2x, gfx, t_3), M(fx, gfx, t_4)\} \end{Bmatrix}\right.$ and

$$N(fx, ffx, t) < \Psi\left(max\begin{Bmatrix} N(ggx, gfx, t), sup_{t_1+t_2=\frac{2}{k}t} min\{N(fx, gx, t_1), N(gx, f^2x, t_2)\}, \\ sup_{t_3+t_4=\frac{2}{k}t} max\{N(f^2x, gfx, t_3), N(fx, gfx, t_4)\} \end{Bmatrix}\right.$$

Whenever $fx \neq f^2x$, for all $x, y \in X, t > 0$. If the range of f or g is a complete subspace of X. then f and g have a common fixed point .

Proof: Since f and g satisfy the property (IFE.A) there exists a sequence $\{x_n\}$ in X such that
$\lim_{n\to\infty} M(fx_n, z, t) = 1$, $\lim_{n\to\infty} N(fx_n, z, t) = 0$.
and
$\lim_{n\to\infty} M(gx_n, z, t) = 1$, $\lim_{n\to\infty} N(gx_n, z, t) = 0$,
and for some $z \in X$. By (IFH_E), it follows
$\lim_{n\to\infty} M(fx_n, gx_n, t) = 1$, $\lim_{n\to\infty} N(fx_n, gx_n, t) = 0$.
Suppose that gX is a complete subspace of X. Then $z = gu$ for some $u \in X$.
If $fu \neq gu$. Then, Using (3.1.3) putting $x = x_n$ and $y = u$,
$M(fx_n, fu, kt) \geq M(gx_n, gu, t)$ and $N(fx_n, fu, kt) \leq N(gx_n, gu, t)$,
taking limit as $n \to \infty$, we get
$M(gu, fu, Kt) \geq M(gu, gu, t)$ and $N(gu, fu, Kt) \leq N(gu, gu, t)$
Therefore by lemma 2.5 $fu = gu$.
Since f and g are weakly compatible so $fgu = gfu$ and therefore $fgu = ffu = gfu = ggu$
If $ffu \neq fu$ then by contractive condition we have, Using (3.1.4) putting x = u and $t = t_0$ then

$$M(fu, ffu, t_0) > \varphi\left(min\begin{Bmatrix} M(ggu, gfu, t_0), sup_{t_1+t_2=\frac{2}{k}t_0} min\{M(fu, gu, t_1), M(gu, f^2u, t_2)\}, \\ sup_{t_3+t_4=\frac{2}{k}t_0} max\{M(f^2u, gfu, t_3), M(fu, gfu, t_4)\} \end{Bmatrix}\right.$$

$$= \varphi(min\begin{Bmatrix} M(ffu, ffu, t_0),\ min\left\{M(fu, fu, \varepsilon), M\left(fu, f^2u, \frac{2}{k}\ t_0 - \varepsilon\right)\right\}, \\ max\left\{M(f^2u, gfu, \varepsilon), M\left(fu, ffu, \frac{2}{k}\ t_0 - \varepsilon\right)\right\} \end{Bmatrix}$$

and

$$N(fu, ffu, t_0) < \Psi(max\begin{Bmatrix} N(ggu, gfu, t_0), sup_{t_1+t_2=\frac{2}{k}t_0}\ min\{N(fu, gu, t_1), N(gu, f^2u, t_2)\}, \\ sup_{t_3+t_4=\frac{2}{k}t_0}\ max\{N(f^2u, gfu, t_3), N(fu, gfu, t_4)\} \end{Bmatrix}$$

$$= \Psi(max\begin{Bmatrix} N(ffu, ffu, t_0),\ min\left\{N(fu, fu, \varepsilon), N\left(fu, f^2u, \frac{2}{k}\ t_0 - \varepsilon\right)\right\}, \\ max\left\{N(f^2u, gfu, \varepsilon), N\left(fu, ffu, \frac{2}{k}\ t_0 - \varepsilon\right)\right\} \end{Bmatrix}$$

$\forall\ \varepsilon \in (0, \frac{2}{k}\ t_0)$, as $\varepsilon \to 0$ it follows

$M(fu, ffu, t_0) > \varphi\{M\left(fu, ffu, \frac{2}{k}\ t_0 - \varepsilon\right)\} > M\left(fu, ffu, \frac{2}{k}\ t_0\right)$

and

$N(fu, ffu, t_0) < \Psi\{N\left(fu, ffu, \frac{2}{k}\ t_0 - \varepsilon\right)\} < N\left(fu, ffu, \frac{2}{k}\ t_0\right)$.

Which is a contradiction .Therefore $fu = ffu$ and $= ffu = fgu = gfu = ggu$. Hence fu is a common fixed point of f and g. The case when fX is a complete subspace of X is similar to the above since $fX \subset gX$, this completes the proof of the theorem.

Theorem 3.2 Let $(X, M, N, *, \Diamond)$ be an intuitionistic fuzzy symmetric space, which satisfies (IFW.1) , (IFW.2) , (IFE.A) and (IFH_E). Let f, g, h and ℓ be self mappings of (X, M, N) such that

(3.2.1) $fX \subset hX$, $gX \subset \ell X$,

(3.2.2) $$M(lx, gy, t) > \varphi(min\begin{Bmatrix} M(\ell x, hy, t), sup_{t_1+t_2=\frac{2}{k}t}\ min\{M(fx, gy, t_1), M(hy, \ell x, t_2)\}, \\ sup_{t_3+t_4=\frac{2}{k}t}\ max\{M(hy, gy, t_3), M(\ell x, gx, t_4)\} \end{Bmatrix}$$

And

$$N(lx, gy, t) < \Psi(max\begin{Bmatrix} N(\ell x, hy, t), sup_{t_1+t_2=\frac{2}{k}t}\ min\{N(fx, gy, t_1), N(hy, \ell x, t_2)\}, \\ sup_{t_3+t_4=\frac{2}{k}t}\ max\{N(hy, gy, t_3), N(\ell x, gx, t_4)\} \end{Bmatrix}$$

for all $x, y \in X, t > 0$. If the range of the one of the mappingsgf, $g, h\ or\ \ell$ is a complete subspace of X, then f, $g, h\ and\ \ell$ have a unique common fixed point and the fixed point is discontinuity.

Proof: Suppose that (g, h)satisfies the property (IFE.A) then there exists a sequence $\{x_n\}$ in X such that

$\lim_{n\to\infty} M(gx_n , y, t) = 1$, $\lim_{n\to\infty} N(gx_n , y, t) = 0$.

and

$\lim_{n\to\infty} M(hx_n , y, t) = 1$, $\lim_{n\to\infty} N(hx_n , y, t) = 0$.

for some $y \in X$. Since $gX \subset \ell X$ there exists $\{y_n\}$ such that $gx_n = \ell y_n$.Therefore,

$\lim_{n\to\infty} M(\ell y_n , y, t) = 1$, $\lim_{n\to\infty} N(\ell y_n , y, t) = 0$.

We claim that

$\lim_{n\to\infty} M(fy_n , y, t) = 1$, $\lim_{n\to\infty} N(fy_n , y, t) = 0$.

Using (3.2.2) putting x= y_n, y = x_n, and t = t_0 then ,

$$M(\ell y_n, gx_n, t_0) > \varphi(min\begin{Bmatrix} M(\ell y_n, hx_n, t_0), sup_{t_1+t_2=\frac{2}{k}t_0}\ min\{M(fy_n, gx_n, t_1), M(hx_n, \ell y_n, t_2)\}, \\ sup_{t_3+t_4=\frac{2}{k}t_0}\ max\{M(hx_n, gx_n, t_3), M(\ell y_n, gy_n, t_4)\} \end{Bmatrix}$$

$$= \varphi(min\begin{Bmatrix} M(\ell y_n, hx_n, t_0),\ min\left\{M(fy_n, gx_n, \varepsilon), M\left(hx_n, \ell y_n, \frac{2}{k}\ t_0 - \varepsilon\right)\right\}, \\ max\left\{M(hx_n, gx_n, \varepsilon), M\left(\ell y_n, gy_n, \frac{2}{k}\ t_0 - \varepsilon\right)\right\} \end{Bmatrix}$$

And

$$N(\ell y_n, gx_n, t_0) < \Psi(max\begin{Bmatrix} N(\ell y_n, hx_n, t_0), sup_{t_1+t_2=\frac{2}{k}t_0}\ min\{N(fy_n, gx_n, t_1), N(hx_n, \ell y_n, t_2)\}, \\ sup_{t_3+t_4=\frac{2}{k}t_0}\ max\{N(hx_n, gx_n, t_3), N(\ell y_n, gy_n, t_4)\} \end{Bmatrix}$$

$$= \Psi(max \begin{Bmatrix} N(\ell y_n, hx_n, t_0\ ),\ min\left\{N(fy_n, gx_n, \varepsilon), N\left(hx_n, \ell y_n, \frac{2}{k}\ t_0 - \varepsilon\right)\right\}, \\ max\left\{N(hx_n, gx_n, \varepsilon), N\left(\ell y_n, gy_n, \frac{2}{k}\ t_0 - \varepsilon\right)\right\} \end{Bmatrix}$$

for all $\varepsilon \in (0, \frac{2}{k}\ t_0)$, as $\varepsilon \to 0$ it follows

$$\left.\begin{matrix} M(\ell y_n, gx_n, t_0) > \varphi\left\{M\left(\ell y_n, gy_n, \frac{2}{k}\ t_0\right)\right\} > M\left(\ell y_n, gy_n, \frac{2}{k}\ t_0\right) \\ and \\ N(\ell y_n, gx_n, t_0\ ) < \Psi\left\{ N\left(\ell y_n, gy_n, \frac{2}{k}\ t_0\right)\right\} < N\left(\ell y_n, gy_n, \frac{2}{k}\ t_0\right) \end{matrix}\right\}$$

Therefore $\ell y_n = gx_n$.
Letting n $\to \infty$, by (IFH_E), it follows
$\lim_{n\to\infty} M(fy_n , gx_n, t) = 1, \lim_{n\to\infty} N(fy_n , gx_n , t) = 0$.
Hence by (IFW.2), one has
$\lim_{n\to\infty} M(fy_n , y, t) = 1, \lim_{n\to\infty} N(fy_n , y, t) = 0$.
Assume that ℓX is a complete subspace of X. Then y = ℓu for some u ∈ X. Consequently.
$\lim_{n\to\infty} M(gx_n ,\ell u, t) = \lim_{n\to\infty} M(hx_n ,\ell u, t) = \lim_{n\to\infty} M(\ell y_n ,\ell u ,t) = \lim_{n\to\infty} M(fy_n, \ell u , t) = 1$.
and
$\lim_{n\to\infty} N(hx_n , \ell u, t) = \lim_{n\to\infty} N(\ell y_n , \ell u, t) = \lim_{n\to\infty} N(fy_n , \ell u,t) = \lim_{n\to\infty} N(gx_n , \ell u, t) = 0$.
Now Using (3.2.2), it follows putting y_n = u. Then

$$M(\ell u, gx_n, t_0) > \varphi(min \begin{Bmatrix} M(\ell u, hx_n, t_0\ ), sup_{t_1+t_2=\frac{2}{k}t_0}\ \ min\{M(fu, gx_n, t_1), M(hx_n, \ell u, t_2)\}, \\ sup_{t_3+t_4=\frac{2}{k}t_0}\ \ max\{M(hx_n, gx_n, t_3), M(\ell u, gu, t_4)\} \end{Bmatrix}$$

$$= \varphi(min \begin{Bmatrix} M(\ell u, hx_n, t_0\ ),\ min\left\{M(fu, gx_n, \varepsilon), M\left(hx_n, \ell u, \frac{2}{k}\ t_0 - \varepsilon\right)\right\}, \\ max\left\{M(hx_n, gx_n, \varepsilon), M\left(\ell u, gu, \frac{2}{k}\ t_0 - \varepsilon\right)\right\} \end{Bmatrix}$$

I and

$$N(\ell u, gx_n, t_0\ ) < \Psi(max \begin{Bmatrix} N(\ell u, hx_n, t_0\ ), sup_{t_1+t_2=\frac{2}{k}t_0}\ \ min\{N(fu, gx_n, t_1), N(hx_n, \ell u, t_2)\}, \\ sup_{t_3+t_4=\frac{2}{k}t_0}\ \ max\{N(hx_n, gx_n, t_3), N(\ell u, gu, t_4)\} \end{Bmatrix}$$

$$= \Psi(max \begin{Bmatrix} N(\ell u, hx_n, t_0\ ),\ min\left\{N(fu, gx_n, \varepsilon), N\left(hx_n, \ell u, \frac{2}{k}\ t_0 - \varepsilon\right)\right\}, \\ max\left\{N(hx_n, gx_n, \varepsilon), N\left(\ell u, gu, \frac{2}{k}\ t_0 - \varepsilon\right)\right\} \end{Bmatrix}$$

for all $\varepsilon \in (0, \frac{2}{k}\ t_0)$, as $\varepsilon \to 0$ it follows

$$M(\ell u, gx_n, t_0) > \varphi\left\{M\left(\ell u, gy_n, \frac{2}{k}\ t_0\right)\right\} > M\left(\ell u, gy_n, \frac{2}{k}\ t_0\right)$$

and

$$N(\ell u, gx_n, t_0\ ) < \Psi\left\{N\left(\ell u, gy_n, \frac{2}{k}\ t_0\right)\right\} < N\left(\ell u, gy_n, \frac{2}{k}\ t_0\right),$$

Implies that $y_n = gx_n$.
Letting n $\to \infty$, by (IFH_E), it follows
$\lim_{n\to\infty} M(fu , gx_n, t) = 1, \lim_{n\to\infty} N(fu , gx_n , t) = 0$.
By (IFW.1), one has $fu = \ell u$. The weak compatibility of f and ℓ implies that $f\ell u = \ell fu$ and then ffu u = $f\ell u = \ell fu = \ell\ell u$.
Similarly $fX \subset hX$ is also satisfies for the condition (3.2.2) .

4. Acknowledgements

We are grateful to the Professor Bijendra Singh, Professor & Head V. H. Badshah and Reader, S. K. Tiwari, School of Studies in Mathematics Vikram University, Ujjain (M.P.) India, for their cooperation and valuable suggestions in the preparation of this paper.

5. References

1. Alaca , C. Turkoglu , D. and C. Yildiz, fixed points in IFM-space, Chaos Solutions and Fractals 29 (2006), 1073-1078.
2. Fang , J. X. on fixed point theorems in fuzzy metric space Fuzzy Set and System.
3. Park **,** J. H. , intuitionistic fuzzy metric space, Chaos, Solutions and Fractals 22(2004), 1039-1046.
4. George , A. and Veeramani , P. on some results in fuzzy metric spaces, fuzzy sets and systems 64,(1994), 395-399.
5. Jain , S. and Wagle , P. 'common fixed point theorem for six mappings in fuzzy metric space' Varahmihir, Journal of Mathematical Sciences, (2001), 195-201.
6. Kramosil , I and Michalek, J. fuzzy metric and statistical metric space, kybernetica, 11(1975), 336-344.
7. Rathore, M. S. , Singh , N. and Rathore , S. common fixed points of four mapping of a fuzzy metric spaces, Fuzzy Sets and Systems, 115(2000).
8. Singh , B. and Jain , S. semi-compatibility and fixed point theorem in fuzzy metric space, Journal of the Chuncheong Mathematical Society, (2005), 1-22.
9. B. Singh , B. and Chauhan , M. S. common fixed points of compatible maps in fuzzy metric spaces, Fuzzy Sets and Systems, 115(2000), 471-475.
10. Vasuki, R. common fixed points for R-weakly commuting maps in fuzzy metric space, Indian J. pure Appl. Maths, 30(1999), 419-423.
11. Zadeh , L. A. Fuzzy Sets, Inform and Control, 189(1965), 338-353.
12. Turkoglu , D. Altun , I.and Cho. Y. J. Common fixed points of compatible mappings of type and type (II), in fuzzy metric space, J. fuzzy math. 15(2007), 435-448.
13. Choudhary, B. S. a unique common fixed point theorem for sequence of self maps in menger spaces, Bull. Korean Math. Soci.37(2000) no.3, 569-575 .
14. Abhay S. Ranadive, Anuja P. Chouhan Absorbing maps and fixed point theorem in fuzzy metric spaces, Inter. Math. Forum, 5, 2010, no. 10, 493-502 **.**
15. Park , J. H. , intuitionistic fuzzy metric spaces, Chaos, Solutions & Fractals 22(2004), 1039- 1046.
16. Ishak , Cihangir, Altun, and Turkoglu , D. On compatible mappings of type (I) and (II) in IFM-spaces, Commun. Korean. Math. Soci.23 (2008), No.3 pp. 427-446.
17. Sedghi , Shaban , Nabi Shobe , Abdelkrian Aliouche, A common fixed point theorem for weakly compatible mappings in fuzzy metric spaces, General Mathematics vol.18, no, 3 (2010), 3-12.
18. Singh , Virendra , V.H. Badshah and M.S. Chauhan, fixed points in fuzzy metric spaces for weakly compatible maps. Int. J. Contemp. Math. Sciences, Vol. 5, 2010, no 3, 145-151.
19. Turkoglu D. Alca , C. and Yildiz. C Compatible maps and compatible maps of type (ά) and (β) in IFM-spaces, Demonstratio Math. 39:671:684, 2006.
20. Ansari, M. , Moutawakil , D. El some new common fixed point theorems under strict contractive conditions , J. Math. Anal., Appl. 270, 181-188 (2002).
21. Turkoglu , D. , Alaca, C. Cho , Y. J. and Yildiz, C. common fixed point theorems in IFM-spaces, J. Appl. Math. & Computing, 22 (2006) , 411-424.
22. Yaoyao Lan , Common Fixed point Theorems in Intuitionistic Fuzzy Symmetric Spaces Under Nonlinear Contractive Conditions, Contemporary Mathematics and Statistics (2013) 1: 1-9, doi:107726/cms.2013.1001