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An Observation on the Drawbacks and Corrections of Existing Rules of Generating Twin-Prime and Prime Numbers

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Abstract:

This article will provide correction on existing very easy and simple logic to generate twin prime numbers. By following the upcoming corrections and observations we can at least generate a prime number. There are many prime number generation logics available worldwide but many of them have drawbacks. Here a few major corrections were made with the example of drawback and correcting the drawbacks. The corrections will allow us to generate prime numbers effortlessly. The coding of these logics are very easy to implement in any high level language. Therefore a computer feed is also available.

Key words: Twin Prime, Prime Number Generation, Drawback of Euler Prime, Correction of generating prime numbers using squares

1. Introduction

There are many types of Prime numbers exist in the world. They mostly follow a pattern to generate prime number. Every day the internet users are growing very rapidly. The requirement of providing security to the storage of every one's information is very important. Here Twin Primes can be very useful because, - we can take one Prime as user name and another as password. One prime we take and assign and encrypt that as a user name or user Id and another assign and encrypt as password. While log in to some internet services the service provider simply decrypt that and allow us to log into our requested services. Thus, we can be safe from hackers. Very recently from a review over HTML and HTTPS on an article we have come to know that 27000 publicly-available keys (around 0.4%) of the total 7.1 million publicly available keys are insecure. As we know the whole security systems are based on mostly the encryption of prime numbers. There are infinite prime numbers existing in the world but, twin-primes are very few. Therefore this article will help the security providers to generate new prime numbers every moment to secure the web. There is a common logic for twin primes that they can be determine by using $6n \pm 1$ formulae. But, this not always provides twin prime. Sometimes it provides a single prime number some time both composite numbers. Years ago Leonhard Euler had provided logic to generate prime numbers. There is a drawback on the logic. Sometimes taking some random numbers we may not be able to generate prime numbers. These two logics are being used worldwide to generate prime numbers. There are some corrections and adjustments are made in the article. By using the corrections we can be able to generate prime numbers of any number of digits we want.

2. Important Definitions and Information's

2.1. Prime Number

A prime number is a natural number which can only divided by 1 or the number itself.

Example: 2, 3, 5, 7, 11, 103, 227, 29, 607, etc.

2.2. Twin Prime

If a pair of consecutive odd numbers is found prime numbers then the pair of number is said to be Twin prime. The Gap between two prime numbers here is always 2.

Example: (41, 43), (17, 19), (59, 61), (71, 73), (101, 103) etc.

2.3. Primes on Residue Class

If we get a prime number on a $a + d$ form, we can say that the prime is in Residue Class.

Example:

$2n+1$: 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53
 $4n+1$: 5, 13, 17, 29, 37, 41, 53, 61, 73, 89, 97, 101, 109, 113, 137
 $4n+3$: 3, 7, 11, 19, 23, 31, 43, 47, 59, 67, 71, 79, 83, 103, 107
 $6n+1$: 7, 13, 19, 31, 37, 43, 61, 67, 73, 79, 97, 103, 109, 127, 139
 $6n+5$: 5, 11, 17, 23, 29, 41, 47, 53, 59, 71, 83, 89, 101, 107, 113
 $8n+1$: 17, 41, 73, 89, 97, 113, 137, 193, 233, 241, 257, 281, 313, 337, 353
 $8n+3$: 3, 11, 19, 43, 59, 67, 83, 107, 131, 139, 163, 179, 211, 227, 251
 $8n+5$: 5, 13, 29, 37, 53, 61, 101, 109, 149, 157, 173, 181, 197, 229, 269
 $8n+7$: 7, 23, 31, 47, 71, 79, 103, 127, 151, 167, 191, 199, 223, 239, 263
 $10n+1$: 11, 31, 41, 61, 71, 101, 131, 151, 181, 191, 211, 241, 251, 271, 281
 $10n+3$: 3, 13, 23, 43, 53, 73, 83, 103, 113, 163, 173, 193, 223, 233, 263
 $10n+7$: 7, 17, 37, 47, 67, 97, 107, 127, 137, 157, 167, 197, 227, 257, 277
 $10n+9$: 19, 29, 59, 79, 89, 109, 139, 149, 179, 199, 229, 239, 269, 349, 359

In general we can say:

$10n+d$ ($d = 1, 3, 7, 9$) are primes ending in the decimal digit d .

2.4. Relatively Prime

Two numbers are said to be relatively prime, if they have no common factors other than 1

Example:

20 and 17 are relatively prime because,- they have no common factors other than 1.

But, 6 and 21 are not relatively prime because,- they have a common factor 3.

2.5. Euler Prime

According to Euler's Theorem, Every prime lies in $6n+1$ form some primes also come in the form of $3n+1$.

2.6. Euler Prime with Another View

According to Euler's theorem it is said that if a prime number lies on $6n+1$ those are also on the form of $3n+1$ form. That can be written as the sums of two squares can generate prime numbers according to Euler. Which can also represent as: $p = x^2 + y^2$, Where x and y are integers.

3. Existing Twin Prime Generation Rule

$6n \pm 1$ is the most available and easy way to generate prime numbers.

Many have said that the minimum set of natural numbers to contain all primes is:

$\{2, 3, (6n \pm 1)\} n \in \mathbb{N}$

4. Drawbacks of the Existing Rules**4.1. There is a drawback on the generating rule of $6n \pm 1$.**

When we use $n = 20, 79$ etc. numbers we cannot get any prime using the generalized formulae.

$$6 \times 20 = 120$$

$$120-1 = 119 \text{ and } 120+1 = 121$$

Both of them are composite numbers.

$$6 \times 79 = 474$$

$$474-1 = 473 \text{ and } 474+1 = 475$$

These are also not prime numbers.

4.2. Using any Integers we cannot get a pair of prime every time on $x^2 + 3y^2$ rule

Example:

$$12^2 + 3 \times 15^2 = 819 \text{ this is not a prime number.}$$

$$11^2 + 3 \times 12^2 = 553 \text{ is not a prime number.}$$

$$11^2 + 3 \times 17^2 = 988 \text{ is not a prime number.}$$

$$11^2 + 3 \times 13^2 = 628 \text{ it is not a prime number.}$$

5. Corrections**5.1. For the correction of $6n \pm 1$ rule**

5.1.1. For the corrections we can apply the logic $6n \times 2 \pm 1$ to get prime or twin prime number.

Example: $6 \times 20 \pm 1 = 119, 121$

Now, $6 \times 2 \times 20 \pm 1 = 239, 241$. This pair is a twin prime. $6 \times 2 \times 79 \pm 1 = 947, 949$. Here 947 is a prime number.

In the above I have shown we cannot get prime number using the rule $6n \pm 1$, where $n = 20$ and 79 .

Now, if the corrected logic also failed to generate prime numbers then we need to multiply 2 with the result $6n \times 2 \pm 1$.

Therefore, $6n \times 4 \pm 1$ will generate a prime number and then again it fails to generate prime numbers then we need to apply the next logic.

$$6n \times 2^{2m} \pm 1.$$

For Example:

If we take $n = 5749$.

$$6 \times 5749 = 34494$$

$34494 \pm 1 = 34493, 34495$ both are composite number.

$34494 \times 2 \pm 1 = 68987, 68989$ both are composite number.

$68988 \times 2 \pm 1 = 137975, 137977$ both are composite number.

$137976 \times 2 \pm 1 = 275951, 275953$ both are composite number.

$275952 \times 2 \pm 1 = 551903551905$ both are composite number.

$551904 \times 2 \pm 1 = 1103807, 1103809$ both are composite number.

$1103808 \times 2 \pm 1 = 2207615, 2207617$ here 2207617 is a prime number.

From the above list we can say $6 \times 5749 \times 2^6 \pm 1 = 2207615, 2207617$ here 2207617 is a Prime Number.

Here, $m = 6$ we get.

5.2. For the Correction of $x^2 + 3y^2 = p$ ($p = \text{prime}$)

5.2.1. Whenever we cannot generate the prime number using the $x^2 + 3y^2$ rule, then we can apply the $6n \times 2^{2m} \pm 1$ logic.

For example:

$12^2 + 3 \times 15^2 = 819$ this is not a prime number. Here $x = 12$ and $y = 15$

Now, if we can apply the $6n \times 2^{2m} \pm 1$

We can get, $6 \times 819 \times 2^1 \pm 1 = 9827, 9829$. Here $m = 1$.

Here, 9829 is a prime number.

$11^2 + 3 \times 12^2 = 553$ is not a prime number.

Now, again applying the $6n \times 2^{2m} \pm 1$ we can get 3317 and 3319 . 3319 is a prime number.

Here, $x = 11$ and $y = 12$ and $m = 1$.

$11^2 + 3 \times 17^2 = 988$ is not a prime number.

Now, again applying the $6n \times 2^{2m} \pm 1$ we get 5927 and 5929 . 5927 is a prime number.

Here, $x = 11$ and $y = 17$ and $m = 1$.

$11^2 + 3 \times 13^2 = 628$ it is not a prime number.

Now, again applying the $6n \times 2^{2m} \pm 1$ we get 3769 and 3767 here we get a twin prime.

Here $x = 11$ and $y = 13$ and $m = 1$

5.2.2. A correction of Correction

It was said that if we use two relatively prime numbers and put them on $x^2 + 3y^2$ we can get prime number.

But, 6 and 7 are relatively prime.

Now, putting them in the $x^2 + 3y^2$ we get 183 .

183 is not a prime number.

Therefore a correction is also required here.

Again applying $6n \times 2^{2m} \pm 1$ we get, 1097 and 1099 . 1097 is a prime number.

6. Conclusion

The main purpose of this paper is to generate prime numbers to provide security for internet users and security providers to secure the web. Another purpose of this article is for the correction of the generating functions of prime numbers which are being used widely. Applying the corrections and methods we can generate prime number from any natural number. Although on the previous corrections made by others it is said that $x^2 + 3y^2 = \text{prime}$, this can be true if x and y are relatively prime numbers.

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