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Blind Image Restoration for Moving Objects Based on Normal Density and Variation Method

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Abstract:
The blind restoration of a scene is investigated, when multiple degraded (blurred and noisy) acquisitions are available. Distorted frames are modeled as deconvolution process; Finite normal density mixture modeling is performed for each noisy frame. Variation regularizer favors images of bounded variation without penalizing possible discontinuities. Variation approach is further used to piecewise smooth the frames. Variation based image deconvolution/deblurring, which is adaptive in the sense that it does not require the user to specify the value of the regularization parameter. Majaorization Minimization algorithm is used to update the current iteration of the image which satisfies upper bound condition; under mild condition stationary parameters converge monotonically. A quadratic bound function is derived further. The motivation is twofold: first minimizing quadratic functions is equivalent to solving linear systems; second, we do not need to solve exactly each linear system, but rather only to decrease the associated quadratic function, which can be achieved by running a few steps of conjugate gradient (CG).

Key words*: Single-Input Single-Output (SISO), Gauss Markov Random Fields (GMRF), Single-Input Multiple-Output (SIMO)*

1. Introduction

Image restoration is the process of reconstructing an approximation of an image from blurred and noisy measurements. In many applications such as medical imaging, remote sensing observed images are often degraded by distortion. The recovered original image from noisy image is shown in Fig1. Distortion may arise from much form, for example atmospheric turbulence, relative motion between object and camera.

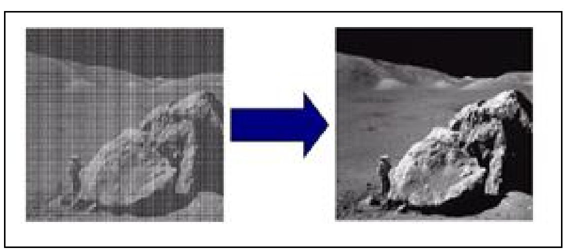

Figure 1: Noisy image (left) and Restored image

Image restoration remains an important research topic and one of the major applications driving the theory and practice of image processing since digital computers made processing large amounts of data possible. This section is not meant to provide a review of the extensive discussion on image restoration methods, but rather to provide some perspective on how modern multiframe blind image restoration algorithms grew out of the existing body of research and how these new multiframe methods define directions for future work. Image restoration techniques may be very broadly categorized into two classes based on the number of observed frames. Specifically, the categorization is into the classes of single-input and multi-input restoration methods.

1.1. Single Frame Restoration and Multiframe Restoration

The classical image restoration problem is concerned with restoration of a single output image from a single degraded observed image, also known as Single-Input Single-Output (SISO). The discussion on the restoration of a single input frame is extensive and spans several decades. A wide variety of techniques exist to tackle this problem and for a concise but representative review.

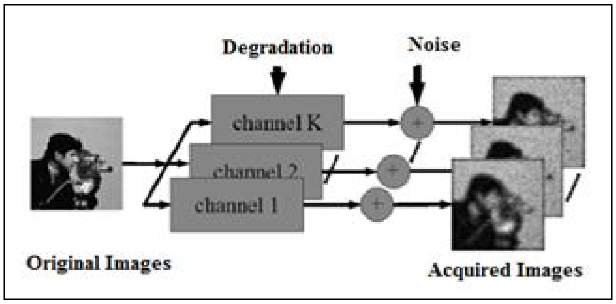

Figure 2: Single-input multiple-output model

The original scene is captured by K different channels which are subject to various degradations. Single input multiple output system is shown in Fig 2: More recently, with the growing interest in digital processing of image sequences, researchers have begun to address the problem of multiframe image restoration. The techniques developed for single frame restoration have often provided the theoretical basis for extending to the multiframe case. While the field of single frame image restoration appears to have matured, digital images have provided many new restoration problems for image processing researchers. The Multi-Input Single-Output (MISO) methods in turn spurred the development of more general Multi-Input Multi-Output (MIMO) approaches.

1.2. Blind Image Restoration

In many practical cases of interest point spread function (noise) is not known. For example when taking the photograph of a moving object, what is the shutter speed, and what is the speed of the object are unknowns. In this case a very difficult problem of "blind" image restoration is faced. In such cases prior knowledge of the process has to be utilized in order to recover the original image. Images are ubiquitous and indispensable in science and everyday life. It is natural to display observations of the world in graphical form. Images are obtained in areas ranging from everyday photography to astronomy, remote sensing, medical imaging, and microscopy. In each case, there is an underlying object or scene has to be observed; the original or true image is the ideal representation of the observed scene.

The observation process is never perfect, there is uncertainty in the measurements, occurring as blur, noise, and other degradations in the recorded images. Digital image restoration aims to recover an estimate of the original image from the degraded observations. The key to being able to solve this ill posed inverse problem is proper incorporation of prior knowledge about the original image into the restoration process. Classical image restoration seeks an estimate of the true image assuming the blur is known. In contrast, blind image restoration tackles the much more difficult, but realistic, problem where the degradation is unknown. In general, the degradation is nonlinear (including, for example, saturation and quantization) and spatially varying (non uniform motion, imperfect optics); however, for most of the work, it is assumed that the observed image is the output of a Linear Spatially Invariant (LSI) system to which noise is added. Therefore it becomes a blind deconvolution (BD) problem, with the unknown blur represented as a Point Spread Function (PSF).

Many of the blind deconvloution algorithms have their roots in estimation theory, linear algebra, and numerical analysis. An important question one may ask is why is blind deconvloution useful? Is there any better observation procedure in the first place? Perhaps, but there always exist physical limits, such as photonic noise, diffraction, or an observation channel outside of our control, and often images must be captured in suboptimal conditions. Also there are existing images of unique events that cannot be retaken, or that will be very difficult to recover (for instance with forensics or archive footage); furthermore in these cases it is often infeasible to measure properties of the imaging system directly. Another reason is that of cost. High-quality optics and sensing equipment are expensive. However, processing power is abundant today and opens the door to the application of increasingly sophisticated models. Thus blind deconvloution represents a valuable tool that can be used for improving image quality without requiring complicated calibrations of the real-time image acquisition and processing system (i.e., in medical imaging, video conferencing, space exploration, x-ray imaging, bio-imaging, and so on). The Blind deconvloution problem is encountered in many different technical areas, such as astronomical imaging [1,3], remote sensing [1,4], microscopy [1,5], medical imaging, optics [13], photography [12], super resolution applications [12], and motion tracking applications [13] and so on. For example, astronomical imaging is one of the primary applications of blind deconvloution algorithms [1, 3]. Ground-based imaging systems are subject to blurring due to the rapidly changing index of refractions of the atmosphere. Extraterrestrial observations of the Earth and the planets are degraded by motion blur as a result of slow camera shutter speeds relative to the rapid spacecraft motion. Blind deconvloution is used for improving the quality of the Poisson distributed film grain noise present in blurred x-rays, mammograms, and digital angiographic images. In many applications, most of the time the degradations are unavoidable because the medical imaging systems limit the intensity of the incident radiation in order to protect the patient's health. In optics, Blind deconvloution is used to restore the original image from the degradation introduced by a microscope or any other optical instrument . The Hubble Space Telescope (HST) main mirror imperfections have provided an inordinate amount of images for the digital image processing community.

2. Literature Survey

The amount of *a priori* information about the degradation, like the size or shape of blurring functions and the noise parameters, significantly influences the success of restoration. When the blur function is known, many conventional approaches have been developed to compensate for the distortion [1]. The problem is ill posed, and, to overcome this difficulty, it is common to use regularization. The modeling of blurring can be divided in two parts: blurring function (PSF) and noise modeling. Some ideal PSF models are Gaussian, out-of-focus and linear motion blur [3]. In astronomy, data extracted from clear stars in observed image is used to fit a synthetic PSF function by weighted nonlinear least squares method [2]. The PSF measurement techniques are also discussed in [1].

The diversity of algorithms [3-9] developed nowadays reflects different ways of recovering a "best" estimate of the "true image". Wiener and regularized filters are better for known PSF and additive noise [6]. Some iterative restoration techniques [6-7], i.e. Expectation Maximization (EM) algorithms, work better for known PSF and unknown additive noise. Blind image restoration algorithms [5] are more proper for unknown PSF and additive noise. Flow imaging experiments have a lot in common with astronomy observations. They are both low light level imaging. Both images are degraded by imaging optical system and suffer from signal related noise (Poisson noise), CCD camera read-out noise and quantization noise etc. These physical similarities suggest that a better starting point in applying image restoration techniques in flow scalar image restoration is to consider those successful ones in astronomy.

The process of restoring the original frame when the blur function is unknown is called as blind image restoration. A basic survey of different blind restoration techniques is given in [2]. Most of the methods are iterative or recursive. They involve regularization terms based on available prior information which assumes various statistical properties of the image and constrains the estimated image and/or restoration filter. As in the nonblind case, regularization is required to improve stability. For images with sharp changes of intensity, the appropriate regularization is based on variational integrals. A special case of the variational integral, total variation was first proposed. Minimization of the variational integrals preserves edges and fine details in the image and it was applied to image denoising and to blind restoration, as well. Since the blind case is strongly ill posed, all the methods suffer from convergence and stability problems. If the images are smooth and homogeneous, an autoregressive model can be used to describe the measuring process. The autoregressive model simplifies the blind problem by reducing the number of unknowns and several techniques were proposed for finding its solution [13].

Bayesian maximum a posterior (MAP) estimation has been shown to be effective in blind restoration. In particular, A F M Smith [4] has recently demonstrated the effectiveness of generalized Gauss Markov random fields (GGMRF) in blind restoration of extended objects. Here both the source and blur were modeled as GGMRF's which have a parametric form allowing a great variety of image representations, including hard edged fields typical of real images and smooth fields typical of blurring point spread functions. However, the GGMRF model is not well suited to point-like sparse images. A Markov random field model which favors sparse solutions is essential if high resolution restorations and accurate point localizations are to be achieved, particularly in the blind case.

The ability to exploit known structure in the problem and impose a sparse form on the solution is essential in overcoming convolutional ambiguity in the blind problem. Blind restoration is a highly ill-posed inverse problem, and algorithms which incorporate known image structure in solutions will invariably perform better.

F Sroubek & J Flusser, have presented an MRF model that exploits the sparse nature of point source input images in the context of MEG-based imaging. Their model involves a dual field representation: first, a binary activity process determines which pixels have non-zero amplitudes, and then a Gaussian amplitude process represents active point intensity levels.

There are many applications, where different blurred versions of the same original image are observed (MC) models: the single-input multiple-output (SIMO) model and the multiple-input multiple-output (MIMO) model. The SIMO model is typical for one-sensor imaging under varying environment conditions, where individual channels represent the conditions at time of acquisition. The MIMO model refers, for example, to multi sensor imaging, where the channels represent different spectral bands or resolution levels. Color images are the special case of the MIMO model. An advantage of MIMO is the ability to model cross-channel degradations which occur in the form of channel crosstalks, leakages in detectors, and spectral blurs. Many techniques for solving the MIMO problem were proposed and could be found in [9]–[11].

Multichannel measuring processes are common, e.g., in remote sensing and astronomy, where the same scene is observed at different time instants through a time-varying inhomogeneous medium such as the atmosphere; in confocal microscopy, where images of the same sample are acquired at different focusing lengths; or in broadband imaging through a physically stable medium, but which has a different transfer function at different frequencies. Nonblind MC restoration is potentially free of the problems arising from the zeros of blurs. The lack of information from one blur in one frequency can be supplemented by the information at the same frequency from the others. Intuitively, one may expect that the blind restoration problem is also simplified by the availability of different channels. Two classes of MC blind image restoration algorithms exist. Extensions of single-channel blind restoration approaches form the first class, but since they suffer from similar drawbacks as their single-channel counterparts, they are of not much interest. The other class consists of intrinsic MC approaches and will be considered here. One of the earliest intrinsic multichannel blind deconvolution (MBD) methods [12] was designed particularly for images blurred by atmospheric turbulence. Harikumar*et al.* [6] proposed an indirect algorithm, which first estimates the blur functions and then recovers the original image by standard nonblind methods. The blur functions are equal to the minimum eigenvector of a special matrix constructed by the blurred images. Necessary assumptions for perfect recovery of the blur functions are noise-free environment and channel coprimeness, i.e., a scalar constant is the only common factor of the blurs. Harikumar*et al.* [6] developed another indirect algorithm based on identity of coprime polynomials which finds restoration filters and by convolving the filters with the observed images recovers the original image.

3. Mathematical Modeling

3.1. Image Restoration Mathematical Model

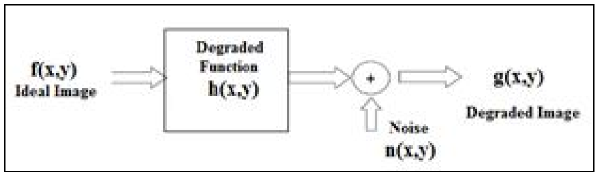

Figure 3: Image restoration mathematical model

The image degradation process shown in Fig 3 can be modeled by the equation given by:

$$g = H.f + w$$

Where, H represents a convolution matrix that models the blurring that many imaging systems introduce. For example, camera defocus, motion blur, imperfections of the lenses all can be modeled by H. The vectors g, f, and w represent the observed, the original and the noise images. More specifically, w is a random vector that models the random errors in the observed data. These errors can be due to the electronics device used (thermal and shot noise) for the recording medium (film grain) or the imaging process (photon noise).

3.2. Noise Model

The principal sources of noise in digital images arise during image acquisition (digitization) and/or transmission of digital images. For example, imaging sensors can be affected by ambient conditions; interference can be added to an image during transmission. Two common assumptions in Spatial and Frequency Properties of Noise: (1) Noise is independent of spatial coordinates and it is uncorrelated with respect to the image itself (that is, there is no correlation between pixel values and the values of noise components) and (2) the Fourier spectrum of noise is constant (The noise usually is called white noise).

3.3. Formulation of the Multiframe Deconvolution Process

Let us consider y_1, $y_{2\ldots}y_M$ be the set of M degraded images (observed frames) of a scene, obtained in a noisy multi-frame environment. Each frame, of (M *X* N) pixels in dimension, is considered to be subject to different version of blurring and different kind of noise. In other words, these set of images consists of different distorted versions of either the same original scene, or slightly different scenes, whose information, however, can be efficiently exploited as follows.

A linear space invariant (LSI) model has been assumed for modeling the blurring process given by

$$y_m = h_m * x_m + n_m \quad, m = 1 \ldots M \qquad (1)$$

where y_m is the degraded image corresponding to particular sensor m, x_m is the original noiseless or undistorted image, h_m is the blurring function also known as the point-spread function (PSF) and n_m is additive zero mean white noise. It should be noted that it has been assumed that $x_1=x_2=\ldots x_M$ such image model approximations, as discussed earlier, are frequently encountered in medical imaging applications, e.g., in phased-array beam forming for Ultrasound imaging or brain/cardiac MRI and CT scans. The ultimate goal of the proposed algorithm is to reconstruct a single an enhanced representation of the original scene, based upon the set of distorted observations

The multiframe blind image restoration problem has recently considerable attractable attentions. The first blind deconvolution attempts were based on single-channel formulations. The problem is extremely ill-posed in the single-channel framework and cannot be resolved in the fully blind form. These kinds of methods do not exploit the potential of the multichannel framework, because in the single-channel case missing information about the original image in one channel is not supplemented by information in the other channels.

3.4. Image Modeling Using Finite Normal Density Mixtures

In statistics and probability, a mixture distribution is the probability distribution of a random variable whose values can be interpreted as being derived in a simple way from an underlying set of other random variables. In particular, the final outcome value of the process is selected at random from among the underlying values, with a certain probability of selection of the process being associated with each. Here the underlying random variables, or may be random vectors, each having the same dimension, in which case the mixture distribution is a multivariate distribution.

In cases where each of the underlying random variables of the process is continuous, then outcome variable will also be continuous and its probability density function is sometimes referred to as a mixture density. The can be expressed as a convex combination (i.e. a weighted sum, with non-negative weights that sum to 1) of other distribution functions and density functions. The individual distributions that are combined to form the mixture distribution are called the mixture components, and the probabilities associated with each component are called the mixture weights of the process. The number of components in mixture distribution is often restricted to being finite, although in some cases the components may approach to infinite. More general cases (i.e. an uncountable set of component distributions), as well as the countable case, are treated under the title of compound distributions.

A clear cut distinction needs to be made between a random variable whose distribution function is the sum of a set of components (i.e. a mixture distribution) and a random variable whose value is the sum of the values of two or more underlying random variables, in which case the distribution is given by the convolution operator. As an example, the sum of two Gaussian distributed random variables, each with different means, will still be a Gaussian distribution. On the other hand, a mixture density created as a mixture of two Gaussian distributions with different means will have two peaks provided that the two means are far enough apart, showing that this distribution is radically different from a Gaussian distribution.

In probability theory, the normal (or Gaussian) distribution is a continuous probability distribution that has a bell-shaped probability density function, known as the *Gaussian function* or informally the *bell curve*:
Gaussian density function is described by equation given by

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2}} \qquad (2)$$

The parameter σ^2 is the variance. σ is known as the standard deviation (indicates how it disperse from peak value) and μ is the *mean* or *expectation* (indicates location of the peak) . The distribution with $\mu = 0$ and $\sigma^2 = 1$ is called the standard normal distribution or the unit normal distribution. To describe real-valued random variables that cluster around a single mean value a normal distribution is often used as a first approximation.

The most prominent probability distribution in statistics is normal distribution. There are so many reasons for this: first, the normal distribution derived from the central limit theorem, which states that under the mild conditions the mean of a large number of random variables of the process drawn from the same distribution is approximately normally distributed, *irrespective* of the form of the original distribution. And this gives it exceptionally wide application in, for example, sampling. Secondly, the normal distribution is very tractable analytically, that is, a large number of results involving this distribution can be derived in explicit form.

Mixture distributions arise in many contexts in the literature and arise naturally where a statistical population contains two or more sub-populations. They are also sometimes used as a means of representing non-normal distributions. Data analysis concerning statistical models involving mixture distributions is discussed under the title of mixture models, while the present article concentrates on simple probabilistic and statistical properties of mixture distributions and how these relate to properties of the underlying distributions.

Finite mixture models have been broadly developed and widely applied to classification, clustering, density estimation and pattern recognition problems. A finite mixture of normal densities is employed to model the distribution of each distorted images y_m, m=1..M. Finite mixture distributions have been widely used as statistical models in problems where measurements are available from experimental units that belong to one of a set of classes, but whose individual class-memberships are unavailable.

For each distorted image (given any pixel y_i), the FNM model given by

$$P_m^{FNM}(y_i|\varphi_m) = \sum_{k=1}^{K} \pi_{m,k} P_{m,k}(y_i|\Phi_{m,k}) \ , m = 1,\dots,M, n = 1\dots N \qquad (3)$$

where K denotes the number of Gaussian components in the model and $N=N_1N_2$ is the total number of pixels. Furthermore, the vector denotes the distinct unknown mean and variance values of the K Gaussian distributions and

$$\Phi_m = \left(\mu_{m,1}, \mu_{m,2}, \mu_{m,k}, \ \sigma_{m1}^2, \sigma_{m2}^2, \sigma_{m,k}^2\right)$$

$\varphi_m=(\Phi_m^T, \pi_{m,1}, \pi_{m,2},\dots \pi_{m,k})$ denotes all unknown parameters. The vector $(\pi_{m,1}, \pi_{m,2},\dots \pi_{m,k})$ represents the mixture proportions, where $\sum_K \pi_{m,k}=1$, for m=1…M,k=1…K

The K th component density of the mixture, can be described by the Gaussian kernel (2.2.3) (for m=1…M and i=1….N)

$$P_{m,k}(y_i|\Phi_{m,k}) = \frac{1}{\sqrt{2\pi\sigma_{m,k}^2}} exp\left\{-\frac{(y-\mu_{m,k})^2}{2\sigma_{m,k}^2}\right\} \qquad (4)$$

the problem becomes that of estimating the model parameters ($\mu_{m,K}$, $\sigma^2_{m,k}$, $\pi_{m,k}$) , based upon the observations (y_1,y_2……y_m), for all P_m^{FNM} ,m=1…..M .

3.5. Variation Method

Total variation (TV) regularization was introduced by Rudin, Osher, and Fatemi, and has become popular in recent years. Recently, the range of application of TV based methods has been successfully extended to inpainting, blind deconvolution, and processing of vector-valued images (e.g., color). The successes of Variation based regularization lies on a good balance between the ability to model piecewise smooth images and the difficulty of the resulting optimization problems. In fact, the Variation regularizer favors images of bounded variation, without penalizing possible discontinuities. Furthermore, the Variation regularizer is convex, though not differentiable, and has stimulated a good amount of research on efficient algorithms for computing optimal or nearly optimal solutions.

Let x and y denote vectors containing the true and the observed image gray levels, respectively, arranged in column lexicographic order. Herein, we consider the linear observation model y = Hx+n , where H is the observation matrix and n is a sample of a zero-mean white Gaussian noise vector with covariance s 2I (where I denotesthe identity matrix)

As in many recent publications [15], [21-24], Total Variational model is adopted to handle the ill-posed nature of the problem of inferring x. This amounts to computing the herein termed variation estimate, which is given by

$$X = \arg\max L(x) \qquad (5)$$

with

$$L(x) = \left\|y - Hx\right\|^2 + \lambda\, VA(x) \qquad (6)$$

where λ controls the relative weights of the data compliance and regularization terms in (3.3.2).

Here the assumptions are that image is defined on discrete domains, the definition of total variance is used and is given by

$$VA(x) = \sum_i \sqrt{(\Delta_i^h x)^2 + (\Delta_i^v x)^2} \qquad (7)$$

where Δ_i^h and Δ_i^v are linear operators corresponding to horizontal and vertical first order differences, at pixel *i*, respectively; i.e., $(\Delta_i^h x) = x_i - x_j$, (where j_i is the first order neighbor to the left of *i*) and $(\Delta_i^h x) = x_i - x_k$ (where k_i is the first order neighbor above *i*). At this point, It would be mentioned that quite often the l_1 regularizer, $l_1(x) = \sum_i |(\Delta_i^h x) + (\Delta_i^v x)|$ has been used to

approximate Va(x), or even wrongly considered itself as the variation regularizer. However, the distinction between these two regularizers should be kept in mind, since, as least in deconvolution problems, Va(x) leads to significantly better results.
The variation estimate given by (7) favors images with bounded variation without penalizing possible discontinuities. Since both smooth and sharp edges have the same Va(x), this does not mean that total variation favors sharp edges relatively to smooth ones, but rather that, for a given value of Va(x), the estimated edge is decided by the observed image y.
The objective function *L*(x) is convex, although not strictly so. Nevertheless, its minimization represents a significant numerical optimization challenge, owing to the non-differentiability of Va(x). In the next section, a new optimization algorithm is introduced which is fully developed on the discrete domain, which is simple and yet computationally efficient. To obtain meaningful image estimates, some form of regularization (prior knowledge, from a Bayesian viewpoint) has to be enforced to penalize "undesirable" solutions. Accordingly, typical criteria given by

$$X = \arg\min \left\{ \frac{1}{2\sigma^2} \|y - Hx\|^2 + \lambda\, Va(x) \right\} \quad (8)$$

where $\|z\|^2$ is the sum of all the squared elements of some z, i.e., squared Euclidean norm, if z is a discrete image or vector,

$$\|z\|^2 = \int z(t)^2\, dt \quad (9)$$

Defining z on the continuous domain Ω, and Va(x) denotes a regularizer or a penalty function (or minus log-prior) which is designed to have small values for "desirable" estimates. The regularization parameter controls the weight that we are going to assign to the regularizer, relatively to the data misfit term.
The Variation regularizer (introduced in [13], see also [12] for a review of recent advances and pointers to the literature) appears in the context of bounded variation (BV) functions. In a continuous domain formulation, the estimation criterion takes the form of a variational problem (10)

$$X = \arg\min \left\{ \frac{1}{2\sigma^2} \int \left(y(t) - Hx(t)\right)^2 dt + \lambda\, VA(x) \right\}, \quad (11)$$

where Va(x) measures the variation of x and is given by

$$VA(x) = \int |\partial x(t)|\, dt, \quad (12)$$

and the minimization is to be carried out in x belongs to $L^2(\Omega)$ (the set of square integrable functions defined on). The Variation regularizer is very well suited for piecewise smooth images, as it avoids oscillatory solutions while it preserves edges/discontinuities. These characteristics have fostered the use of TV regularization in denoising and deconvolution of real world images with very good results.
Given that the Variation regularizer is not differentiable (due to the presence of the absolute value function), solving is a challenging task, which has been the focus of a considerable amount of work over the last decade. Most of the approaches adopted fall into one of three classes: (i) solving the associated Euler-Lagrange equation, which is a nonlinear partial differential equation; (ii) using methods based on duality, still formulated in the continuous domain, which avoid some of the difficulties at the cost of replacing it by a constrained variational problem; (iii) optimization methods applied to a discrete version. Almost all the literature on denoising/debluring follows one of the first two approaches; however, in practice, since computer implementations can only handle images on discrete lattices, the solution methods derived on the continuous domain have to be replaced, at some point, by discrete formulations. The choice to be made is between: (a) deriving a solution method on the continuous domain and then discretizing it; (b) discretizing the problem and then using a finite normal density algorithm.

3.6 Majaroization and Minimization Algorithm

Consider the objective function with l fixed and, for notational simplicity, let $\sigma^2 = 1/2$*:* Let $x^{(t)}$ denote the current image iterate and $Q(x|x^{(t)})$ a function that satisfies the following two conditions:

$$L(x^{(t)}) = Q(x^{(t)}|x^{(t)}) \quad (13)$$

$$L(x^{(t)}) \le Q(x|x^{(t)}) \quad, x \ne x^{(t)} \quad (14)$$

i.e, $Q(x|x^{(t)})$, as a function of x, majorizes (*i.e.*, upper bounds) *L*(x). Suppose now that $x^{(t+1)}$ is obtained by

$$x^{(t+1)} = \arg\min Q(x|x^{(t)}). \quad (15)$$

then,

$$\begin{aligned} L(x^{(t+1)}) &= L(X^{(t)}) - Q(X^{(t+1)}, X^{(t)}) + Q(X^{(t+1)}, X^{(t)}) \\ &\le Q(X^{(t+1)}, X^{(t)}) \\ &\le Q(X^{(t)}, X^{(t)}) \\ &= L(x^{(t)}) \end{aligned} \quad (16)$$

where the left hand inequality follows from the definition of *Q* and the right hand inequality from the definition of $x^{(t+1)}$. The sequence $L(x^{(t)})$, for $t = 1,2,\ldots\ldots$ is, therefore, non increasing. Under mild conditions, namely assuming that $Q(x\,|X^{\,i})$ is continuous in both x and x^i, all limit points of the MM sequence $L(x^{(t)})$ are stationary points of *L*, and $L(x^{(t)})$ converges monotonically to $L^* = L(x^*)$, for some stationary point x^*. If, in addition, *L* is strictly convex, then $x^{(t)}$ converges to the global minimum of *L*.
Observe that in order to have $L(x^{(t)}) \le Q(x|x^{(t)})$, it is not necessary to minimize $Q(x|x^{(t)})$ w.r.t x, but only to assure that $Q(x^{(t+1)}|x^{(t)}) \le Q(x^{(t)}|x^{(t)})$. This property has a relevant impact, namely when the minimum of *Q* cannot be found exactly or it is hard to compute.
First step of the project is acquiring the original image. Simulate blurry image and blurred noisy image. Formulate these images as multiframe images for blind deconvolution. Finite normal density modeling is performed and then variation method is applied. Finally, Majorization Minimization algorithm is applied to minimize the maximizer thus amount of solving the linear equations of the system. Complete design of the project is shown in Fig 4 and its implementation steps are as follows.

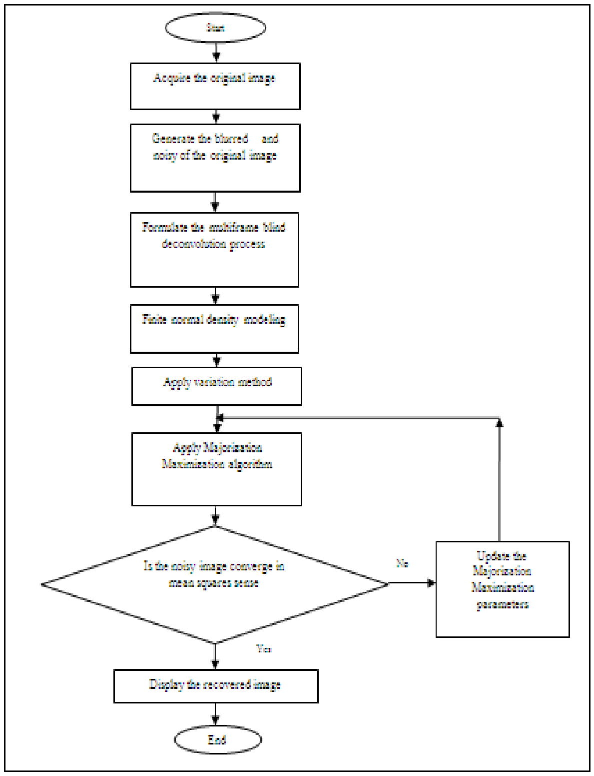

Figure 4: Flowchart of the project

Image can be read by the *imread* function which stores the input image intensity values in I. Image is cropped to resize it. Fig 5: shows the board image read by *imread* function.

Figure 5: Input the original image for blind deconvolution process

Simulate a real-life image that could be blurred (e.g., due to camera motion or lack of focus) and noisy (e.g., due to random disturbances). The example simulates the blur by convolving a Gaussian filter with the true image (using *imfilter*).

```
PSF = fspecial('gaussian',5,5);
Blurred = imfilter(I,PSF,'symmetric','conv');
figure;imshow(Blurred);title('Blurred');
```

Figure 6: Input imaged blurred by Gaussian noise of mean 0 and variance 0.02

The example simulates the noise by adding a Gaussian noise of variance V to the blurred image (using *imnoise*). The noise variance V is used later to define a damping parameter of the algorithm. The blurred image due to Gaussian noise is as shown in Fig 7 and blurred and noisy image is shown in Fig 8.

```
V = .002;
BlurredNoisy = imnoise(Blurred,'gaussian',0,V);
figure;imshow(BlurredNoisy);title('Blurred & Noisy');
```

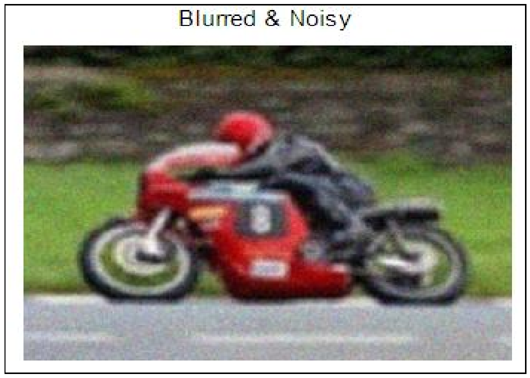

Figure 7: Adding noise to blurred image to get noisy image

Create a gmdistribution object defining a two-component mixture of bivariate Gaussian distributions:

```
MU = [1 2;-3 -5];
SIGMA = cat(3,[2 0;0 .5],[1 0;0 1]);
p = ones(1,2)/2;
obj = gmdistribution(MU,SIGMA,p);
```

y = pdf(obj,X) returns a vector y of length *n* containing the values of the probability density function (pdf) for the gmdistribution object obj, evaluated at the *n*-by-*d* data matrix X, where *n* is the number of observations and *d* is the dimension of the data. obj is an object created by gmdistribution or fit.

```
mu = .05/max(sigma,1.e-12);
MU = [mu/5 mu mu*5];
gamma   = beta/mu;
Denom   = Denom1 + gamma*Denom2;
vartol1 = tol*(beta_max/beta)^2;
varmaxit = maxit/2;
```

Majorization Minimization is performed after initializing parameters so that parameter of image is updated at each iterations. Majorization Minimization algorithm steps are as follows

1. Start with Initial estimate $x^{(0)}$
2. Compute $y' = H^T y$; with t:= 0.
3. Till “Majorization Maximization stopping criterion” is not satisfied do 3 to 6
4. Compute $W^{(t)}$
5. $x^{(t+1)} := x^{(t)}$
6. Till $x^{(t+1)}$ does not satisfy “Conjugate Gradient stopping criterion” do 6
7. $x^{(t+1)}$:= Conjugate Gradient iteration for system A(t)x = y′, initialized at $x^{(t+1)}$
8. End

3.7 Calculate SNR and PSNR

```
mse = mean((ref(:)-sig(:)).^2);
dv = var(ref(:),1);
x = 10*log10(dv/mse);
```

PSNR is computed as follows

```
error_diff = ref - sig;
decibels = 20*log10(255/(sqrt(mean(mean(error_diff.^2)))));
```

Figure 8: Recovered original image from blurred and noisy image

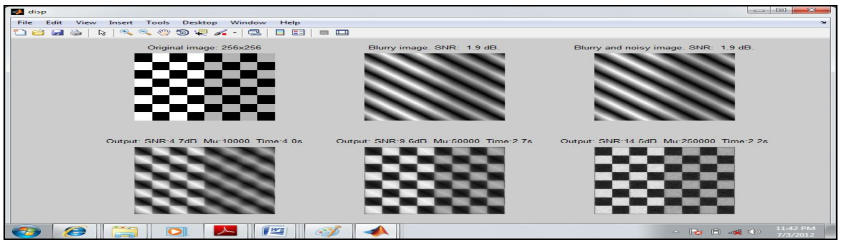

Figure 9: Displaying the original, blurred, noisy and restored images (second row) with SNR values

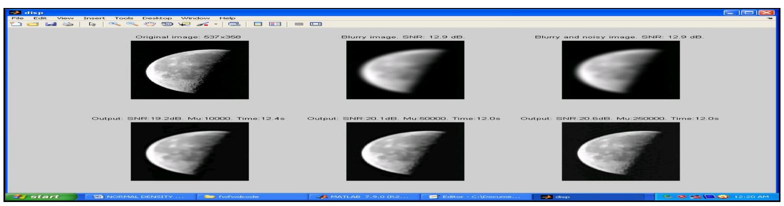

Figure 10: Displaying the original, blurred, noisy and restored images (second row) with SNR values

- Tables of comparision

	Blurry image	**Blurry and noisy Image**	**Restored Image at (mu=10000)**	**Restored Image (mu=5000)**	**Restored Image at (mu=250000)**
SNR	+12.9dB	+12.9dB	+19.2dB	+20.1db	+20.7dB
PSNR	+71.36dB	+ 71.36dB	+77.57dB	+78.51dB	+79.13dB

4. Conclusion

Two classes of MC blind image restoration algorithms exist. Extensions of single-channel blind restoration approaches form the first class, but since they suffer from similar drawbacks as their single-channel counterparts, they are of not much interest. The other class consists of intrinsic MC approaches and will be considered here.

	Blurry image	**Blurry and noisy Image**	**Restored Image at (mu=1000)**	**Restored Image (mu=5000)**	**Restored Image at (mu=250000)**
SNR	+1.9dB	+1.9dB	+4.7dB	+9.7Db	+14.5dB
PSNR	+57.2dB	+ 57.2dB	+60dB	+65dB	+69.8dB

One of the earliest intrinsic multichannel blind deconvolution (MBD) methods [8] was designed particularly for images blurred by atmospheric turbulence.Harikumar*et al.* [6] proposed an indirect algorithm, which first estimates the blur functions and then recovers the original image by standard nonblind methods. The blur functions are equal to the minimum eigenvector of a special matrix constructed by the blurred images. Necessary assumptions for perfect recovery of the blur functions are noise- free environment and channel coprimeness, i.e., a scalar constant is the only common factor of the blurs. Harikumar*et al.* [6] developed another indirect algorithm based on identity of coprime polynomials which finds restoration filters and by convolving the filters with the observed images recovers the original image. Both algorithms are vulnerable to noise and even for a moderate noise level restoration may break down. In the latter case, noise amplification can be attenuated to a certain extent by increasing the restoration filter order, which comes at the expense of deblurring. Pai *et al.* [10] suggested two MC restoration algorithms that, contrary to the previous two indirect algorithms, estimate directly the original image from the null space or from the range of a special matrix.

The blind restoration of an image is investigated, when multiple degraded (blurred and noisy) acquisitions are available. Distorted frames are modeled as deconvolution process. Finite normal density mixture model is employed to find the intensity variation within its neighbor. Variation method is applied to find the first order intensity variation. Majorization Maximization is applied to minimize upper bound.

Observed images recover the original image. Both algorithms are vulnerable to noise and even for a moderate noise level restoration may break down. In the latter case, noise amplification can be attenuated to a certain extent by increasing the restoration filter order, which comes at the expense of deblurring. Pai *et al.* [10] suggested two MC restoration algorithms that, contrary to the previous two indirect algorithms, estimate directly the original image from the null space or from the range of a special matrix.

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