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Optimal Control of Two-Phase $M^X/M/1$ Queueing System with Server Start-Up, N-Policy, Unreliable Server and Balking

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Abstract:
This paper deals with the optimal control policy of two-phase service$M^X/M/1$ queues with vacation, N-policy, server break downs and balking. Generating functions method is used to derive the system characteristics. The total expected cost function is developed to determine the optimal threshold of N at a minimum cost. Numerical experiment is performed to validate the analytical results. The sensitivity analysis has been carried out to examine the effect of different parameters in the system.

Key words: *Vacation, N-policy, Queueing System, Two-phase, Start-up, Breakdowns and Balking*

1. Introduction

We consider two-phase $M^x/M/1$ queuing system with N-policy, server breakdowns and balking. Customers arrive in batches according to a Poisson process and receive batch service in first phase and individual service in second phase. The server is turned off each time the system empties, as and when the queue length reaches or exceeds N (threshold) batch service starts. Before the batch service, the system requires a random startup time for pre-service. When the number of customers in the queue is less than or equal to N-1, the server is in vacation, when the number of customers in the queue is greater than or equal to N, it goes to a startup period for pre service. Arrivals during pre-service are also allowed to enter the batch. As soon as the startup period is over the server starts the batch service followed by individual service to all customers in the batch. During both batch as well as individual services, the server may breakdown at any time according to a Poisson process and if the server fails, it is immediately sent for repair. After repair the server resumes service. In the present chapter we have introduced the phenomenon of balking in a vacation as well as working queueing model. It can be commonly visualized that some customers may not enter the queue on finding the system down or the server is absent from the system while others; some may leave the queue system by seeing the long length of the queue,which is known as balking.

Two-phase queuing systems have been discussed in the past for their applications in various areas, such as computer, communication, manufacturing, and other stochastic systems. In many computer and communication service systems, the situation in which arriving packets receive batch mode service in the first phase followed by individual services in the second phase is common. As related literature we should mention some papers [3,8,11,17] arising from distributed system control where all customers receives batch mode service in the first phase followed by individual service in the second phase

Vacation queueing theory was developed as an extension of the classical queueing theory. The vacations may represent server working on some supplementary jobs, performing server maintenance inspection and repairs, or server's failures that interrupt the customer service. Furthermore, allowing servers to take vacations makes queueing models more flexible in finding optimal service policies. Therefore, queues with vacations or simply called *vacation models* attracted great attention of queueing researchers[9,13,16,19] and became an active research area. Miller was the first to study a queueing system in which the server becomes idle and is unavailable during some random length of time for the M/G/1 queueing system.

The subject of queueing systems wherein the server is subject to breakdowns from time to time is a popular subject which has received a lot of focus for the last five decades. In many real systems, the server may meet unpredictable breakdowns or any other interruptions. Understanding the behaviour of the unreliable server which includes the effect of machine breakdowns and repairs in these systems is important as this affects not only the system's efficiency but also the queue length and the customer's waiting time in the queue. Therefore, queueing models with server breakdowns are more realistic representation of the system. These are the most popular models which have attracted extensive researcher attention [17,18] over the past fifty years.

Research studies on queues with batch arrival and vacations have been increased tremendously and still many researchers [1,10] have been developing on the theory of different aspects of queuing.

The concept of customer impatience has been studied in 1950's. Haight (1957) has first studied about the concept of customer behaviour called balking, which deals the reluctance of a customer to join a queue upon arrival, since then a remarkable attention [5,7,12,14] has been given on many queueing models with customer impatience.
However, to the best of our knowledge, for two –phase queueing systems with N-Policy, server breakdowns, there is no literature which takes customers' impatience into consideration. This motivates us to study a two-phase queueing system with N-policy, server start-up, breakdowns and balking. Thus, in this present paper, we consider two-phase $M^X/M/1$ queueing system with server Start-up, N-Policy, unreliable server and Balking where customers become impatient when the server is unavailable.
The article is organized as follows. A full description of the model is given in Section. 2. The steady-state analysis of the system state probabilities is performed through the generating in Section. 3 while some, very useful for the analysis, results on the expected number of customers in different states are given in Section. 4. In Section. 5 the characteristic features of the system are investigated. Optimal control policy is explained in section.6, while, in Section. 7, numerical results are obtained and used to compare system performance under various changes of the parameters through sensitivity analysis. Finally, the conclusions are presented in section .8.
The main objectives of the analysis carried out in this paper for the optimal control policy are:

- to establish the steady state equations and obtain the steady state probability distribution of the number of customers in the system in each state.
- to derive expressions for the expected number of customers in the system when the server is in vacation, in startup, in batch service (working and broken conditions) and in individual service (working and broken conditions) respectively.
- to formulate the total expected cost functions for the system, and determine the optimal value of the control parameter N.
- to carry out sensitivity analysis on the optimal value of N ,Expected length of the system and the minimum expected cost for various system parameters through numerical experiments.

2 The System and Assumptions

We consider the $M^x/M/1$ queueing system with server startup, two phases of service, unreliable server and balking, where the unreliable server operates under N-policy with the following assumptions:

- The arrival process is a compound Poisson process (with rate λ) of independent and identically distributed random batches of customers, where each batch size X, has a probability density function $\{a_n: a_n = P(X=n), n\geq 1\}$. Batches are admitted to service on a first come first service basis.
- The service is in two phases. The first phase of service is batch service to all customers waiting in the queue. On completion of batch service, the server immediately proceeds to the second phase to serve all customers in the batch individually. Batch service time is assumed to follow exponential distribution with mean $1/\beta$ which is independent of batch size. Individual service times are assumed to be exponentially distributed with mean $1/\mu$. On completion of individual service, the server returns to the batch queue to serve the customers who have arrived. If the customers are waiting, the server starts the batch service followed by individual service to each customer in the batch. If no customer is waiting the server takes a vacation.
- Whenever the system becomes empty, the server is turned off. As soon as the total number of arrivals in the queue reaches or exceeds the pre-determined threshold N, the server is turned on and is temporarily unavailable for the waiting customers. The server needs a startup time which follows an exponential distribution with mean $1/\theta$. As soon as the server finishes startup, it starts serving the first phase of waiting customers.
- The customers who arrive during the batch service are also allowed to join the batch queue which is in service.
- The breakdowns are generated by an exogenous Poisson process with rates ξ_1 for the first phase of service and α_1 for the second phase of service. When the server fails it is immediately repaired at a repair rateξ_2 in first phase and α_2 in secondphase, where the repair times are exponentially distributed. After repair the server immediately resumes the concerned service.
- A customer may balk from the queue station with probability b_0 when the server is in vacation or may balk with a probability $b_1$when the server is in service mode due to impatience.

3 Steady-State Analyses

In steady – state the following notations are used.

- $p_{0,i,0}$ = The probability that there are i customers in the batch queue when the server is on vacation, where i = 0,1,2,3,…,N-1
- $p_{1,i,0}$ = The probability that there are i customers in the batch queue when the server is doing pre-service (startup work), where i = N, N+1, N+2,…
- $p_{2,i,0}$ = The probability that there are i customers in the batch queue when the server is in batch service where i = 1,2,3,…
- $p_{3,i,0}$= The probability that there are i customers in batch queue when the server is working but found to be broken down, where i = 1,2,3,…
- $p_{4,i,j}$ = The probability that there are i customers in the batch queue and j customers in individual queue when the server is in individual service,where i=0,1,2… and j=1,2,3,…

- $p_{5,i,j}$ = The probability that there are i customers in the batch queue and j customers in individual queue when the server is working but found to be broken down, where i = 0,1,2,… and j = 1,2,3, …

The steady-state equations governing the system size probabilities are as follows:
$\lambda b_0 p_{0,0,0} = \mu p_{4,0,1}$. (1) $\lambda b_0 p_{0,i,0} = \lambda b_0 \sum_{k=1}^{i} a_k p_{0,i-1,0}$; $1 \le i \le N-1$. (2) $(\lambda b_1 + \theta) p_{1,N,0} = \lambda b_0 \sum_{k=1}^{N} a_k p_{0,N-k,0}$. (3) $(\lambda b_1 + \theta) p_{1,i,0} = \lambda b_1 \sum_{k=1}^{i-N} a_k p_{1,i-k,0} + \lambda b_0 \sum_{k=i-N+1}^{i} a_k p_{0,i-k,0}$; $i > N$. (4) $(\lambda b_1 + \beta + \xi_1) p_{2,i,0} = \lambda b_1 \sum_{k=1}^{i} a_k p_{2,i-1,0} + \mu p_{4,i,1} + \xi_2 p_{3,i,0}$; $1 \le i \le N-1$. (5) $(\lambda b_1 + \beta + \xi_1) p_{2,i,0} = \lambda b_1 \sum_{k=1}^{i} a_k p_{2,i-1,0} + \mu p_{4,i,1} + \xi_2 p_{3,i,0} + \theta p_{1,i,0}$; $i \ge N$. (6) $(\lambda b_1 + \xi_2) p_{3,i,0} = \lambda b_1 \sum_{k=1}^{i} a_k p_{3,i-1,0} + \xi_1 p_{2,i,0}$; $i \ge 1$. (7) $(\lambda b_1 + \alpha_1 + \mu) p_{4,0,j} = \mu p_{4,0,j+1} + \beta p_{2,j,0} + \alpha_2 p_{5,0,j}$; $j \ge 1$. (8) $(\lambda b_1 + \alpha_1 + \mu) p_{4,i,j} = \mu p_{4,i,j+1} + \lambda b_1 \sum_{k=1}^{i} a_k p_{4,i-k,j} + \alpha_2 p_{5,i,j}$; $i, j \ge 1$. (9) $(\lambda b_1 + \alpha_2) p_{5,0,j} = \alpha_1 p_{4,0,j}$; $j \ge 1$. (10) $(\lambda b_1 + \alpha_2) p_{5,i,j} = \alpha_1 p_{4,i,j} + \lambda b_1 \sum_{k=1}^{i} a_k p_{5,i-k,j}$; $i, j \ge 1$. (11)

To obtain the analytical closed expression of $p_{0,0,0}$, the technique of probability generating function can be successfully applied as detailed below. Define probability generating functions associated with marginal queue size distributions as follows:
$G_0(z) = \sum_{i=0}^{N-1} p_{0,i,0} z^i$, $G_1(z) = \sum_{i=N}^{\infty} p_{1,i,0} z^i$,
$G_2(z) = \sum_{i=1}^{\infty} p_{2,i,0} z^i$, $G_3(z) = \sum_{i=1}^{\infty} p_{3,i,0} z^i$,
$G_4(z,y) = \sum_{i=0}^{\infty} \sum_{j=1}^{\infty} p_{4,i,j} z^i y^j$, $G_5(z,y) = \sum_{i=0}^{\infty} \sum_{j=1}^{\infty} p_{5,i,j} z^i y^j$
and $R_j(z) = \sum_{i=0}^{\infty} p_{4,i,j} z^i$.

Let $A(z) = \sum_{i=i}^{\infty} a_i z^i$ be the probability generating function of the arrival batch size random variable X and $A'(z)$ and $A''(z)$ represent the first and second order derivatives of A(z) respectively.
Using equation (2), we get
$p_{0,i,0} = y_i p_{0,0,0}$,
where y_i 's are defined as $y_0 = 1$ and $y_i = \sum_{k=1}^{i} a_k y_{i-k}$,i = 1,2,3,…,N-1.
$G_0(z) = \sum_{i=0}^{N-1} p_{0,i,0} z^i = p_{0,0,0} \sum_{i=0}^{N-1} y_i z^i = p_{0,0,0} y_N(z)$ (12)
where $y_N(z) = \sum_{i=0}^{N-1} y_i z^i$ with $y_N(1) = \sum_{i=0}^{N-1} y_i$ and $y_N'(1) = \sum_{i=1}^{N-1} i y_i$.
Multiplication of equations (3) and (4) by z^i and adding over i($i \ge N$) gives $(\lambda b_1 (1 - A(z)) + \theta) G_1(Z) = \lambda b_0 G_0(z)(A(z) - 1) + \lambda b_0 p_{0,0,0}$. (13)
Multiplication of equations (5) and (6) by z^i and adding over i ($i \ge 1$) gives $(\lambda b_1 (1 - A(z)) + \beta + \xi_1)\, G_2(Z) = \xi_2 G_3(z) + \mu R_1(z) + \theta G_1(z) - \lambda b_0 p_{0,0,0}$. (14)
Multiplication of equation (7) by z^i and adding over i ($i \ge 1$) gives $(\lambda b_1 (1 - A(z)) + \xi_2)\, G_3(Z) = \xi_1 G_2(z)$. (15)
Multiplication of equations (8) and (9) by $z^i y^j$ and adding over Corresponding values of i and j gives

$$\left(\lambda b_1 y (1 - A(z)) + \alpha_1 y - \mu (1 - y)\right) G_4(z,y) = \left(\alpha_2 G_5(z,y) + \beta G_3(y) - \mu R_1(z)\right) y. \quad (16)$$

Multiplication of equations (10) and (11) by $z^i y^j$ and adding over Corresponding values of i and j gives $(\lambda b_1 (1 - A(z)) + \alpha_2) G_5(z,y) = \alpha_1 G_4(z,y)$. (17)
The total probability generating function G(z, y) is given by $G(z,y) = G_0(z) + G_1(z) + G_2(z) + G_3(z) + G_4(z,y) + G_5(z,y)$. (18)
The normalizing condition is

$$G(1,1) = G_0(1) + G_1(1) + G_2(1) + G_3(1) + G_4(1,1) + G_5(1,1) = 1. \quad (19)$$

From equations (12) to (19)

$$G_0(1) = y_N(1) p_{0,0,0}, \quad (20)$$

$$G_1(1) = \left(\frac{\lambda b_0}{\theta}\right) p_{0,0,0}, \quad (21) \qquad G_2(1) = \left(\frac{\mu}{\beta}\right) R_1(1), \quad (22)$$

$$G_3(1) = \left(\frac{\xi_1}{\xi_2}\right) G_2(1), \quad (23)$$

$$G_4(1,1) = \left(\frac{\alpha_2\left(\beta G_2'(1) - \mu R_1'(1)\right)}{\mu\alpha_2 - \lambda b_1 A'(1)(\alpha_1 + \alpha_2)}\right)$$

$$= \left(\frac{\lambda b_0 A'(1) p_{0,0,0}\left(\frac{y_N(1)\theta + \lambda b_1}{\theta}\right)\xi_2 + \lambda b_1 A'(1)(\xi_1 + \xi_2)\frac{\mu}{\beta} R_1(1)}{\mu\alpha_2 - \lambda b_1 A'(1)(\alpha_1 + \alpha_2)}\right)\frac{\alpha_2}{\xi_2}, \quad (24)$$

$$G_5(1,1) = \left(\frac{\alpha_1}{\alpha_2}\right) G_4(1,1). \quad (25)$$

The normalizing condition (19) gives,

$$R_1(1) = \frac{\left(\left(t_1\left(1-p_{0,0,0}\left(\frac{\lambda b_0}{\theta}+y_N(1)\right)\right)+(\alpha_1+\alpha_2)\frac{\lambda b_0(\lambda b_1+y_N(1)\theta)}{\theta}\right)\beta\xi_2\right)}{\mu^2\alpha_2(\xi_1+\xi_2)},$$

where $t_1 = \left(\mu\alpha_2 - \lambda b_1 A'(1)(\alpha_1+\alpha_2)\right)$.

Substituting the value of $R_1(1)$ from (22) to (25) gives $G_2(1)$, $G_3(1)$, $G_4(1,1)$and$G_5(1,1)$.

Probability that the server is neither in batch service nor in individual service is givenby

$$G_0(1) + G_1(1) = 1 - A'(1)\left(\frac{\lambda b_1}{\beta}\left(1+\frac{\xi_1}{\xi_2}\right) + \frac{\lambda b_1}{\mu}\left(1+\frac{\alpha_1}{\alpha_2}\right)\right).$$

This gives

$$p_{0,0,0} = (1-\rho)\frac{\theta}{(\lambda b_0 + y_N(1)\theta)}. \qquad (26)$$

Where $\rho = \left(\frac{\lambda b_1}{\beta}\left(1+\frac{\xi_1}{\xi_2}\right) + \frac{\lambda b_1}{\mu}\left(1+\frac{\alpha_1}{\alpha_2}\right)\right)$ is the utilizing factor of the system.

From Equation (26) we have $\rho < 1$, which is the necessary and sufficient condition under which steady state solution exits.

Under steady state conditions, let $p_0, p_1, p_2, p_3, p_4,$ *and* $p_5$be the probabilities that the server is in vacation, startup, in batch service, in batch service with break down, in individual service and in individual service with breakdown states respectively. Then,

$p_0 = G_0(1)$, (27)

$p_1 = G_1(1)$,(28)

$p_2 = G_2(1)$,(29)

$p_3 = G_3(1)$,(30)

$p_4 = G_4(1,1)$,(31)

$p_5 = G_5(1,1)$. (32)

4. Expected Number of Customers at Different States of the Server

Using the probability generating functions expected number of customers in the system at different states are presented below. Let L_0, L_1, L_2, L_3, L_4 and L_5 be the expected number of customers in the system when the server is in idle, startup, batch service, break down in batch service, individual service and break down in individual states respectively.

Then

$$L_0 = \sum_{i=0}^{N-1} i\, p_{0,i,0} = G_0'(1) = y_N'(1)p_{0,0,0}, \qquad (33)$$

$$L_1 = \sum_{i=N}^{\infty} i\, p_{1,i,0} = G_1'(1) = \frac{\lambda b_0 A'(1)(\lambda b_1 + y_N(1)\theta)}{\lambda b_1 \theta^2} p_{0,0,0}, (34)$$

$$L_2 = \sum_{i=1}^{\infty} i\, p_{2,i,0} = G_2'(1) = \left(\frac{\lambda b_1 A'(1)(\xi_1+\xi_2)G_2(1) + \theta\xi_2 G_1'(1)}{t_1\beta\xi_2}\right)\mu\alpha_2, \qquad (35)$$

$$L_3 = \sum_{i=1}^{\infty} i\, p_{3,i,0} = G_3'(1) = \frac{\xi_1(G_2'(1)\xi_2 + \lambda b_1 A'(1)G_2(1))}{\xi_2^2}, (36)$$

$$L_4 = \sum_{i=0}^{\infty}\sum_{j=1}^{\infty}(i+j)\, p_{4,i,j} = G_4'(1,1) = \frac{\left[\alpha_2(\beta G_2''(1) - \mu R_1''(1)) + 2(\alpha_2 - \lambda b_1 A'(1))\left(\beta G_2'(1) - \mu R_1'(1)\right)\right]}{2t_1} - \frac{\left[G_4(1,1)(2\left(\lambda b_1 A'(1)\right)^2 - A''(1)\lambda b_1(\alpha_1+\alpha_2) - 2\lambda b_1 A'(1)(\alpha_1+\alpha_2+\mu))\right]}{2t_1}, (37)$$

$$L_5 = \sum_{i=0}^{\infty}\sum_{j=1}^{\infty}(i+j)\, p_{5,i,j} = G_5'(1,1)$$

$$= \frac{\alpha_1}{\alpha_2} L_4 + \frac{\lambda b_1}{\alpha_2} A'(1) G_5(1,1) \quad (38)$$

The expected number of customers in the system is given by

$$L(N) = L_0 + L_1 + L_2 + L_3 + L_4 + L_5 . (39)$$

5 Characteristic features of the system

In this section, we obtain the expected system length when the server is in different states. Let E_0, E_1, E_2, E_3, E_4 *and* E_5denote the expected length of vacation period, startup period, batch service period , batch service breakdown period, individual service period, and breakdown period during individual service respectively. Then the expected length of a busy cycle is given by

$$E_c = E_0 + E_1 + E_2 + E_3 + E_4 + E_5$$

The long run fractions of time the server is in different states are as follows:

$\frac{E_0}{E_c} = p_0$, (40)

$\frac{E_1}{E_c} = p_1$, (41)

$\frac{E_2}{E_c} = p_2$, (42)

$\frac{E_3}{E_c} = p_3$, (43)

$\frac{E_4}{E_c} = p_4$, (44)

$\frac{E_5}{E_c} = p_5$. (45)

Expected length of vacation period is given by

$$E_v = \frac{y_N(1)}{\lambda b_0}. \quad (46)$$

Hence,

$$E_c = \frac{1}{(\lambda b_0 p_{0,0,0})}. \quad (47)$$

6. Optimal Control Policies

In this section, we determine the optimal value of N that minimizes the long run average cost of two- phase $M^X/M/1$, N-policy queue with server break downs with balking. To determine the optimal value of N we consider the following linear cost structure.

$$T(N) = C_h L(N) + C_o\left(\frac{E_2}{E_c} + \frac{E_4}{E_c}\right) + C_m\left(\frac{E_5}{E_c}\right) + C_{b1}\left(\frac{E_3}{E_c}\right) + C_{b2}\left(\frac{E_5}{E_c}\right) + C_s\left(\frac{1}{E_c}\right)$$

$$+C_b\big(\lambda(1-b_0)p_0 + \lambda(1-b_1)(p_1+p_2+p_3+p_4+p_5)\big) - C_r\left(\frac{E_0}{E_c}\right) \quad (48)$$

Where

C_h = Holding cost per unit time for each customer present in the system,

C_o = Cost per unit time for keeping the server on and in operation,

C_m = Startup cost per unit time,

C_s = Setup cost per cycle,

C_{b1} = Break down cost per unit time for the unavailable server in batch service mode,

C_{b2} = Break down cost per unit time for the unavailable server in individual service mode,

C_b = Cost per unit time when a customer balks,

C_r = Reward per unit time as the server is doing secondary work in vacation.

For the determination of the optimal operating N-policy, minimize T (N) in equation48.

An approximate value of the optimal threshold N* can be found by solving the equation

$$\left.\frac{dT_1(N)}{dN}\right|_{N=N^*} = 0 \quad (49)$$

MATLAB software is used to develop the computational program.

We can consider different batch size distributions like deterministic, Positive Poisson, Geometric etc.

Where

Here the Geometric distribution is assumed.

For the Geometric batch size distribution, the generating function is

$A(z) = p(z^{-1} - (1-p))^{-1}$.

This gives $A'(1) = \frac{1}{p}$ and $A''(1) = \frac{2(1-p)}{p^2}$.

7. Sensitivity Analysis

In order to verify the efficiency of our analytical results, we perform numerical experiment by using MATLAB. The variations of different parameters (both monetary and non-monetary) on the optimal threshold N*, mean number of jobs in the system and minimum expected cost are shown. By fixing

The non-monetary parameters as

$\lambda=0.5, \mu=8, \alpha_1=0.2, \alpha_2=3.0, \xi_1=0.2, \xi_2=0.3, \theta=6, \beta=12, b_0=0.4$ and $b_1=0.2, p=0.2$

and monetary parameters as

$C_r=15, C_{b1}=50, C_{b2}=75, C_b=15, C_m=200, C_h=5$ and $C_s=1000$,We perform the sensitivity analysis.

7.1. Effect of variation in the non-monetary parameters

(i)Variation in λ

For specified range of values of λ the optimal threshold N*, the mean number of customers in the system L (N*) and minimum expected cost T(N*) are presented in figure 1 .

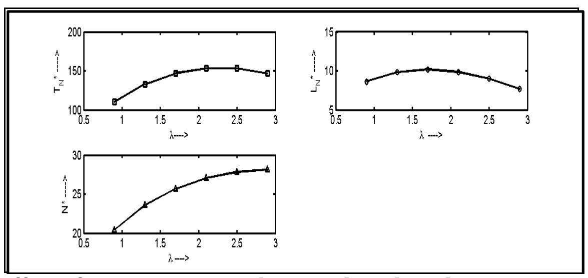

Figure 1: Effect of ***λ*** *on N*, expected system length and minimum expected cost*

It is observed from figure 1 thatwith increase in the values of **λ**,

- N* is increasing.
- Mean number of customers in the system is convex function.
- Minimum expected costs convex function.

(ii)Variation in μ

For specified range of values of **μ** the optimal threshold N*, the mean number of customers in the system L (N*) and minimum expected cost T (N*) are presented in figure 2.

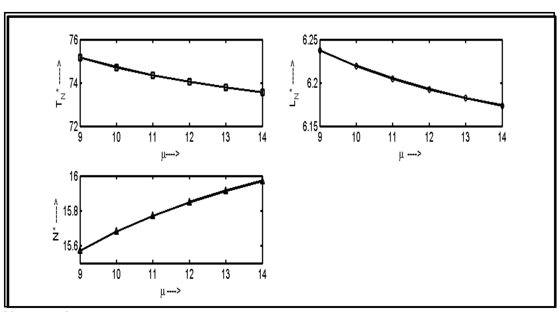

Figure 2: Effect of ***μ*** *on N*, expected system length and minimum expected cost*

It is observed from figure 2 thatwith increase in the values of **μ**,

- N* is increasing.
- Mean number of customers in the system is decreasing.
- Minimum expected cost isdecreasing.

(iii) Variation in α_1

For specified range of values of $\boldsymbol{\alpha_1}$ the optimal threshold N*, the mean number of customers in the system L (N*) and minimum expected cost T (N*) are presented in figure 3.

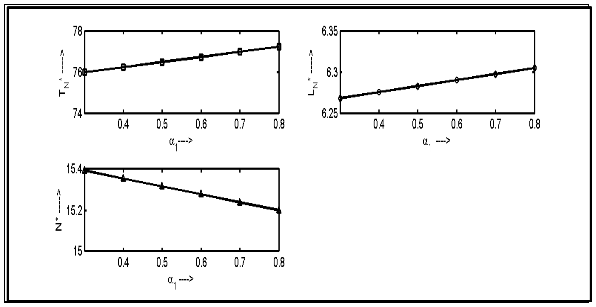

Figure 3: Effect of $\boldsymbol{\alpha_1}$ on N, expected system length and minimum expected cost*

It is observed from figure3 that with increase in the values of $\boldsymbol{\alpha_1}$,

- N* is decreasing.
- Mean number of customers in the system is slightly increasing.
- Minimum expected cost is also slightly increasing.

(iv)Variation in α_2

For specified range of values of $\boldsymbol{\alpha_2}$ the optimal threshold N*, the mean number of customers in the system L (N*) and minimum expected cost T (N*) are presented in figure 4.

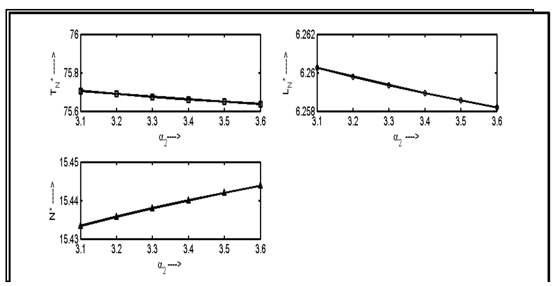

Figure 4: Effect of $\boldsymbol{\alpha_2}$ on N, expected system length and minimum expected cost*

It is observed from figure 4 that with increase in the values of $\boldsymbol{\alpha_2}$,

- N* is increasing.
- Mean number of customers in the system is decreasing.
- Minimum expected cost is decreasing.

(v)Variation in ξ_1

For specified range of values of $\boldsymbol{\xi_1}$ the optimal threshold N*, the mean number of customers in the system L(N*) and minimum expected cost T(N*) are presented in figure 5 .

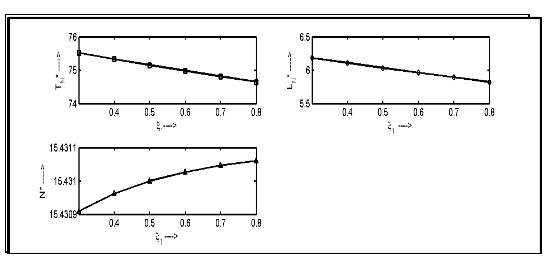

Figure5: Effect of $\boldsymbol{\xi_1}$ on N, expected system length and minimum expected cost*

It is observed from figure 5 that with increase in the values of ξ_1

- N* is slightly increasing.
- Mean number of customers in the system is decreasing.
- Minimum expected cost is decreasing.

(vi)Variation in ξ_2

For specified range of values of ξ_2 the optimal threshold N*, the mean number of customers in the system L (N*) and minimum expected cost T(N*) are presented in figure 6 .

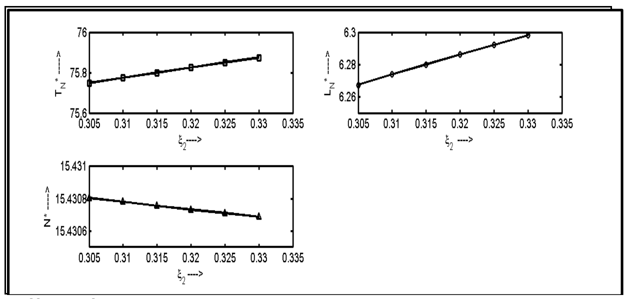

Figure 6: Effect of ξ_2 on N, expected system length and minimum expected cost*

It is observed from figure 6that with increase in the values of ξ_2,

- N* is decreasing.
- Mean number of customers in the system is slightly increasing
- Minimum expected cost is increasing.

(vii) Variation in θ

For specified range of values of **θ** the optimal threshold N*, the mean number of customers in the system L (N*) and minimum expected cost T(N*) are presented in figure 7.

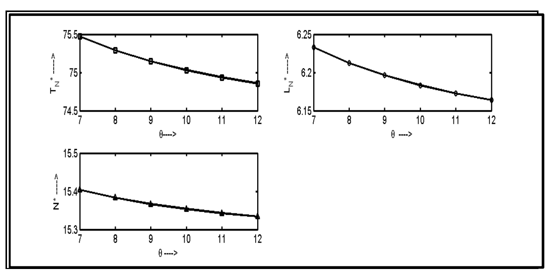

*Figure 7: Effect of **θ** on N*, expected system length and minimum expected cost*

It is observed from figure 7that with increase in the values of θ,

- N* is slightly decreasing.
- Mean number of customers in the system is decreasing.
- Minimum expected cost is decreasing.

viii) Variation in β

For specified range of values of **β** the optimal threshold N*, the mean number of customers in the system L (N*) and minimum expected cost T(N*) are presented in figure8 .

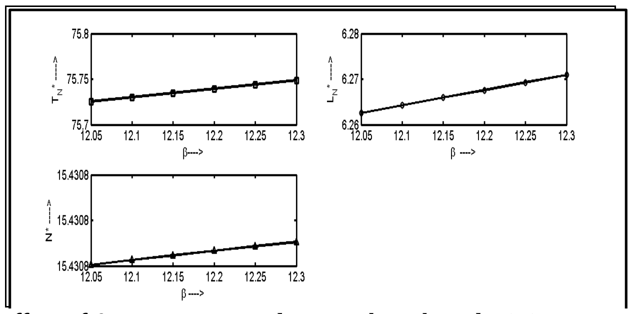

*Figure 8: Effect of **β** on N*, expected system length and minimum expected cost*

It is observed from figure 8that with increase in the values of β,

- N* is increasing.
- Mean number of customers in the system isincreasing.
- Minimum expected cost isincreasing.

ix)Variation in b_0

For specified range of values of $\mathbf{b_0}$ the optimal threshold N*, the mean number of customers in the system L (N*) and minimum expected cost T(N*) are presented in figure 9 .

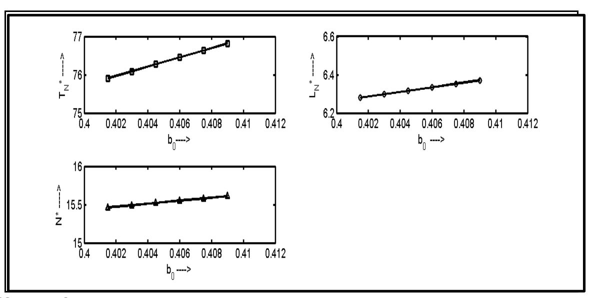

Figure 9: Effect of $\mathbf{b_0}$on N, expected system length and minimum expected cost*

It is observed from figure 9 that with increase in the values of $\mathbf{b_0}$,

- N* is increasing.
- Mean number of customers in the system is increasing.
- Minimum expected costis increasing.

x) Variation in b_1

For specified range of values of $\mathbf{b_1}$ the optimal threshold N*, the mean number of customers in the system L (N*) and minimum expected cost T(N*) are presented in figure 10 .

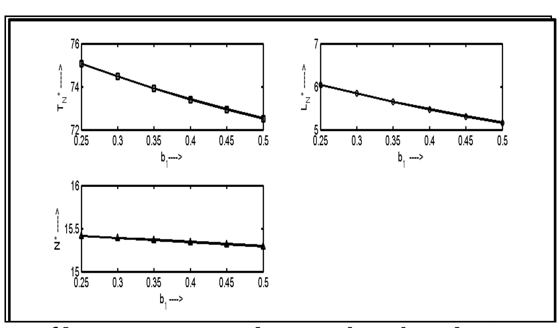

Figure 10: Effect of $\mathbf{b_1}$ on N, expected system length and minimum expected cost*

It is observed from figure 10 that with increase in values of $\mathbf{b_1}$,

- N* is slightly decreasing.
- Mean number of customers in the system is decreasing.
- Minimum expected cost is decreasing.

xi) Variation in p

For specified range of values of **p** the optimal threshold N*, the mean number of customers in the system L (N*) and minimum expected cost T(N*) are presented in figure 11 .

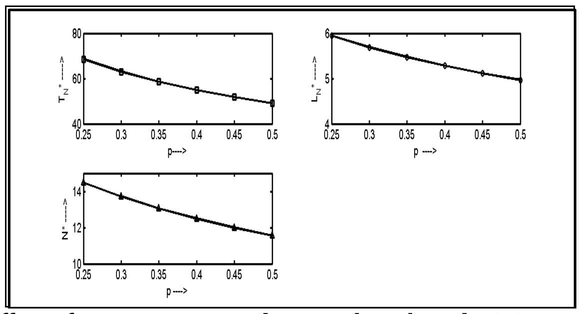

*Figure 11: Effect of **p** on N*, expected system length and minimum expected cost*

It is observed from figure 11 that with increase in the values of **p,**

- N* is decreasing.
- Mean number of customers in the system is decreasing.
- Minimum expected cost is decreasing.

7.2. Effect of variation in the monetary parameters

xii)Variation in C_r

For specified range of values of $\mathbf{C_r}$ the optimal threshold N*, the mean number of customers in the system L (N*) and minimum expected cost T(N*) are presented in figure 12 .

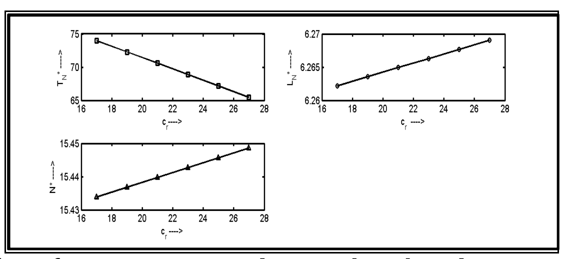

Figure 12: Effect of C_r on N, expected system length and minimum expected cost*

It is observed from figure 12that with increase in the values of $\mathbf{C_r}$,

- N* is slightly increasing.
- Mean number of customers in the system is slightly increasing.
- Minimum expected cost is decreasing.

xiii)Variation in C_{b1}

For specified range of values of $\mathbf{C_{b1}}$ the optimal threshold N*, the mean number of customers in the system L (N*) and minimum expected cost T(N*) are presented in figure 13 .

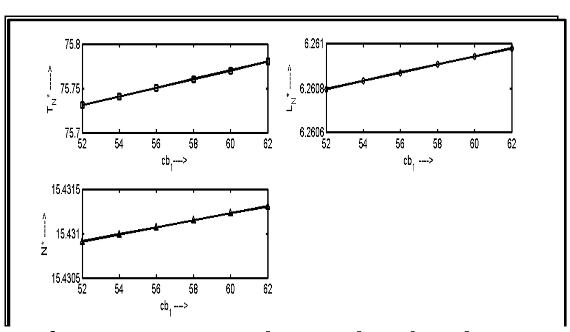

Figure 13: Effect of C_{b1} on N, expected system length and minimum expected cost*

It is observed from figure 13 that with increase in the values of $\mathbf{C_{b1}}$

- N* is almost insensitive.
- Mean number of customers in the system is increasing.
- Minimum expected cost is slightly increasing.

xiv) Variation in C_{b2}

For specified range of values of $\mathbf{C_{b2}}$ the optimal threshold N*, the mean number of customers in the system L (N*) and minimum expected cost T(N*) are presented in figure 14.

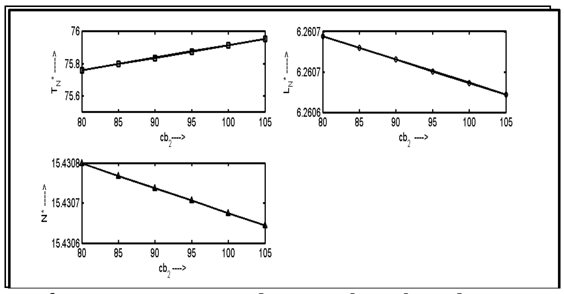

Figure 14: Effect of $\boldsymbol{C_{b2}}$on N, expected system length and minimum expected cost*

It is observed from figure 14 that with increase in the values of $\mathbf{C_{b2}}$,

- N* is insensitive.
- Mean number of customers in the system is almost insensitive.
- Minimum expected cost is slightly increasing**.**

xv) Variation in $\mathbf{C_b}$

For specified range of values of $\mathbf{C_b}$ the optimal threshold N*, the mean number of customers in the system L (N*) and minimum expected cost T(N*) are presented in figure 15.

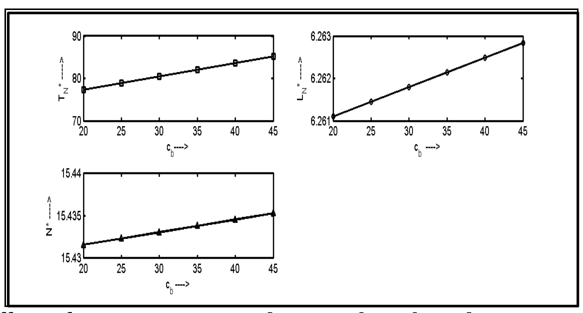

Figure 15: Effect of $\boldsymbol{C_b}$on N, expected system length and minimum expected cost*

It is observed from figure15that with increase in the values of $\mathbf{C_b}$,

- N* is increasing.
- Mean number of customers in the system is increasing.
- Minimum expected cost is increasing.

xvi) Variation in $\mathbf{C_m}$

For specified range of values of $\mathbf{C_m}$ the optimal threshold N*, the mean number of customers in the system L (N*) and minimum expected cost T(N*) are presented in figure 16 .

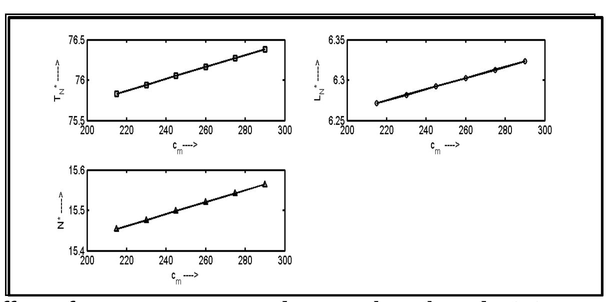

Figure 16: Effect of $\boldsymbol{C_m}$ on N, expected system length and minimum expected cost*

It is observed from figure 16that with increase in the values of $\mathbf{C_m}$,

- N* is increasing.
- Mean number of customers in the system is increasing.
- Minimum expected cost is increasing**.**

xvii)Variation in $\mathbf{C_o}$For specified range of values of $\mathbf{C_o}$ the optimal threshold N*, the mean number of customers in the system L (N*) and minimum expected cost T (N*) are presented in figure 17.

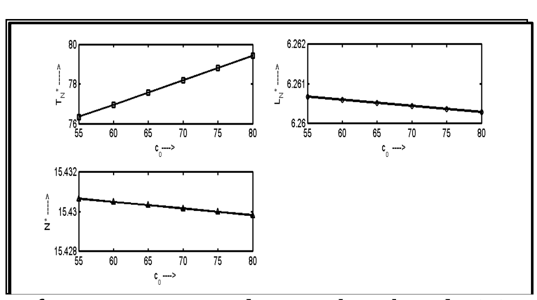

Figure 17: Effect of $\boldsymbol{C_o}$ on N, expected system length and minimum expected cost*

It is observed from figure 17that with increase in the values of $\mathbf{C_o}$,

- N* is almost insensitive.
- Mean number of customers in the system is decreasing
- Minimum expected cost is increasing.

xviii)Variation in c_h

For specified range of values of $\mathbf{C_h}$ the optimal threshold N*, the mean number of customers in the system L (N*) and minimum expected cost T(N*) are presented in figure 18.

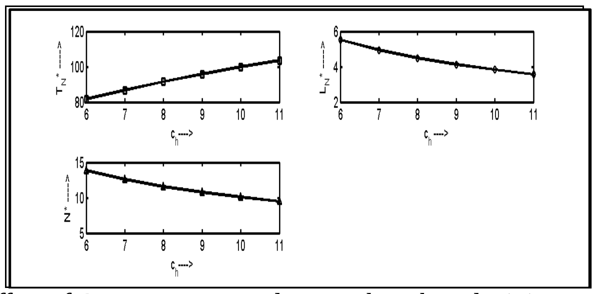

Figure 18: Effect of $\mathbf{C_h}$ on N, expected system length and minimum expected cost*

It is observed from figure 18that with increase in the values of $\mathbf{C_h}$

- N* is decreasing.
- Mean number of customers in the system is decreasing.
- Minimum expected cost is increasing.

xix) Variation in C_s

For specified range of values of $\mathbf{C_s}$ the optimal threshold N*, the mean number of customers in the system L (N*) and minimum expected cost T(N*) are presented in figure 19.

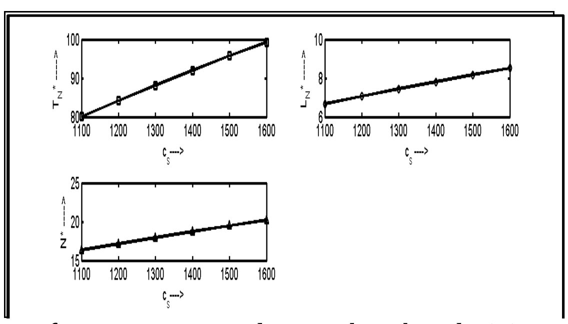

Figure 19: Effect of $\mathbf{C_s}$ on N, expected system length and minimum expected cost*

It is observed from figure 19that with increase in the values of Cs,

- N* is increasing.
- Mean number of customers in the system is increasing.
- Minimum expected cost isincreasing.

8. Conclusion

- Two–phase N-policy $M^X/M/1$ queueing system with server startup times, breakdowns and balking is studied. The closed expressions for the steady state distribution of the number of customers in the system when the server is at different states are obtained and hence the expected system length is derived.
- Total expected cost function for the system is formulated and determined the optimal value of the control parameter N that minimizes the expected cost.
- Sensitivity analysis is performed to discuss how the system performance measures can be affected by the changes of the both non-monetary and monetary input parameters.

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