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Mean and Variance of the Time to Recruitment in a Two Grade Manpower System Using the Threshold Has Kumaraswamy Generalized Gamma Distribution

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Abstract:
Recruitment is one of the dynamics of manpower system that can usually be effectively controlled, always assuming that there is at any time an adequate supply of recruits to a system. In this situation, number of exits from the organization reaches a certain threshold level; it could be viewed as a breakdown point. The time to attain breakdown point is an important characteristic for the management of the organization. In this paper an organization having two grades with independent threshold follows the special case of Kumaraswamy Generalized Gamma (Kum-GG) distribution is considered and the mean and variance of the time for recruitment in this organization are obtained using the cumulative damage process of reliability theory.

Key words*: Threshold, Kum-GG distribution, two grades*

1. Introduction

Wastage of personnel due to retirement, death and resignations is a common phenomenon in administrative as well as production oriented organizations. A number of models are discussed in Bartholomew [1], Bartholomew and Forbes [2]. Optimal appointment policies using Markov Chain model is discussed by Grinold [4]. Optimal time interval between recruitment is derived using shock model approach by Sathiamoorthy and Elangovan [7]. Frequent exists and recruitment of personnel is very common in such organization. Assuming the exists of personnel as shocks to the manpower availability (in terms of man hours) in an organization system, the expected duration to the breakdown of the organization due to the depletion of manpower is studied. A threshold level which in other words can be called as the breakdown point of the system. The breakdown point or level can also be interpreted as that point at which the immediate recruitment is necessitated to make up the manpower loss by recruitment. For a detailed study of shocks models and cumulative damage process one can refer Esary, Marshall and Proschan [3]. Parthasarathy, Ravichandran and Vinoth [6] have obtained the expected time to recruitment for a two grade system assuming exponentiated exponential distribution as threshold level. In this paper, an organization having two grades with independent The Kum-GG distribution which is introduced by Pascoa et al [5] as threshold for the loss of man hours is considered. The expected time to recruitment is obtained allowing for the mobility of manpower from one category to the other, where is more of depletion. The two grades are assumed to have Kum-GG distribution. Obviously the breakdown occurs only when the total depletion crosses the maximum of the two threshold levels.

The distribution function of Kum-GG is defined by

$F_k(x) = 1 - \left[1 - G(x)^{\lambda}\right]^{\varphi}$

The p.d.f of Kum-GG is

$f_k(x) = \lambda\varphi g(x) G(x)^{\lambda-1} \left[1 - G(x)^{\lambda}\right]^{\varphi-1}$

Where $g(x) = \frac{dG(x)}{dx}$

The corresponding survival function is

$S_k(x) = 1 - F_k(x)$

$S_k(x) = \left[1 - G(x)^{\lambda}\right]^{\varphi}$

Using Kum-GG distribution

$G(t; \alpha, \tau, \kappa) = \frac{\gamma(\kappa\left(\frac{t}{\alpha}\right)\hat{}\tau)}{\Gamma\kappa}$

where $\alpha, \tau, \kappa > 0$

$\gamma(\kappa, x) = \int_0^x \omega^{\kappa-1} e^{-\omega} d\omega$

Now, $S_k(x) = [1 - \frac{\int_0^x \omega^{\kappa-1} e^{-\omega} d\omega}{\Gamma\kappa}]^{\varphi}$

Where α is a scale parameter and the other parameter τ, κ and λ are shape parameter, when the shape parameter τ=λ= φ=1, then we get ,

$S_k(x) = 1 + e^{-t/\alpha} \sum_{n=1}^{\infty} \frac{1}{(k-n)!} (\frac{t}{\alpha})^{k-n}$

2. Assumptions

- The depletion of manpower is linear and cumulative.
- An organization takes decisions at random epochs in (0; 1) and at every decision making epoch a random number of persons quit the organization.
- There is an associated loss of man-hours to the organization if a person quits.
- The loss of manpower process is a sequence of independent and identically distribution continuous random variables.
- The depletion of manpower occurs to the two grades are independent of their threshold levels.

3. Notations

- Xi: a continuous random variable denoting the loss of man hours caused to the organization due to the i^{th} policy announcement i=1,2…
- Y_1,Y_2: a continuous random variable denoting the threshold level having KumGG distribution for the one grade and grade two respectively
- f(.): p.d.f of random variable denoting between successive policy announcement with the corresponding cdf F(.).
- $F_m(t)$: the m-fold convolution functions of F(.).
- S (.): The survivor functions of F (.).
- T_1: Time to breakdown of the system due to depletion in the _rst grade.
- T2: Time to breakdown of the system due to depletion in the first grade.
- T=max (T_1,T_2): Time to breakdown of the system or to recruitment.
- $F_m(.)$: The m-fold convolution functions of F(.)
- L(T): 1−S(t).
- $\boldsymbol{V_m}(.)$: Probability that there are exactly 'm' policies
- g (.): The probability density functions of X.
- gm(:): The m-fold convolution of g(.) i.e., p.d.f of ΣX_i
- g(.): Laplace transform of g(.).

4. Results

Let Y the random variable which has the cdf defined as

$H_g(x) = 1 - [1 - G(x)^{\lambda}]^{\varphi}$ $\lambda, \varphi > 0$

Therefore it has the density function

$H_g(x) = \lambda\varphi g(x) G(x)^{\lambda-1} [1 - G(x)^{\lambda}]^{\varphi-1}$

The corresponding survival function is

$S_m(x) = 1 + e^{-t/\alpha} \sum_{n=1}^{\infty} \frac{1}{(k-n)!} (\frac{t}{\alpha})^{k-n}$

Now, P($X_1 + X_2 + X_3 \ldots\ldots + X_m$<Y) = P[the system does not fail, after m epochs of exists].

$P(\sum_{i=1}^{m} X_i < Y) = \int_0^{\infty} g_m(t) \hat{H}(t) dt$

$P(\sum_{i=1}^{m} X_i < Y) = \int_0^{\infty} g_m(t) \left\{1 - \left[1 - \left(1 + e^{-\frac{t}{\alpha}} \sum_{n=1}^{\infty} \frac{1}{(k-n)!} (\frac{t}{\alpha})^{k-n}\right)\right]\right\} \left\{1 - \left[1 - \left(1 + e^{-\frac{t}{\alpha}} \sum_{n=1}^{\infty} \frac{1}{(k-n)!} (\frac{t}{\alpha})^{k-n}\right)\right]\right\} dt$

$$= \int_0^{\infty} g_m(t) \left\{1 - e^{-t\left(\frac{1}{\alpha_1} + \frac{1}{\alpha_2}\right)} \sum_{n=1}^{\infty} \frac{1}{(k-n)!} t^{k-n} \left[\left(\frac{1}{\alpha_1}\right)^{k-n} \left(\frac{1}{\alpha_2}\right)^{k-n}\right]\right\} dt$$

$$= \int_0^{\infty} g_m(t)(1) dt - \int_0^{\infty} g_m(t) \left\{1 - e^{-t\left(\frac{1}{\alpha_1} + \frac{1}{\alpha_2}\right)} \sum_{n=1}^{\infty} \frac{1}{(k-n)!} t^{k-n} \left[\left(\frac{1}{\alpha_1}\right)^{k-n} \left(\frac{1}{\alpha_2}\right)^{k-n}\right]\right\} dt$$

$$= -\left\{\sum_{n=1}^{\infty} \frac{1}{(k-n)!} t^{k-n} \left[\left(\frac{1}{\alpha_1}\right)^{k-n} \left(\frac{1}{\alpha_2}\right)^{k-n}\right]\right\}^m \left[\int_0^{\infty} g(t)\, e^{-t\left(\frac{1}{\alpha_1} + \frac{1}{\alpha_2}\right)} t^{k-n} \text{ dt}\right]^m$$

$$= -\left\{\sum_{n=1}^{\infty} \frac{1}{(k-n)!} t^{k-n} \left[\left(\frac{1}{\alpha_1}\right)^{k-n} \left(\frac{1}{\alpha_2}\right)^{k-n}\right]\right\}^m \left[g^* \left\{ (t)^{k-n} \left(\frac{1}{\alpha_1} + \frac{1}{\alpha_2}\right)\right\}\right]^m$$

$$= -\left\{\sum_{n=1}^{\infty}\frac{1}{(k-n)!}\ t^{k-n}\ \left[\left(\frac{1}{\alpha_1}\right)^{k-n}\ \left(\frac{1}{\alpha_2}\right)^{k-n}\right]\right\}^{m}\left[L\left\{\ (t)^{k-n} g\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)\right\}\right]^{m}$$

$$= -\left\{\sum_{n=1}^{\infty}\frac{1}{(k-n)!}\ t^{k-n}\ \left[\left(\frac{1}{\alpha_1}\right)^{k-n}\ \left(\frac{1}{\alpha_2}\right)^{k-n}\right]\right\}^{m}\left\{(-1)^{k-n}\ \frac{d^{k-n}}{d\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)^{k-n}}\ L(\mu e^{-\mu t})\right\}^{m}$$

$$= -\left\{\sum_{n=1}^{\infty}\frac{1}{(k-n)!}\ t^{k-n}\ \left[\left(\frac{1}{\alpha_1}\right)^{k-n}\ \left(\frac{1}{\alpha_2}\right)^{k-n}\right]\right\}^{m}\quad\left\{(-1)^{k-n}\ \frac{d^{k-n}}{d\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)^{k-n}}\ \left[\frac{\mu}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{k-n}\right\}^{m}$$

$$= -\left\{\sum_{n=1}^{\infty}\frac{1}{(k-n)!}\ t^{k-n}\ \left[\left(\frac{1}{\alpha_1}\right)^{k-n}\ \left(\frac{1}{\alpha_2}\right)^{k-n}\right]\right\}^{m}\quad\left\{\mu\,\Gamma(k-n)\ \left[\frac{1}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{k-n+1}\right\}^{m}$$

$$= -\left[\frac{\mu}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{m}\ \sum_{n=1}^{\infty}\left\{\left[\frac{1}{(k-n)}\ \left(\frac{1}{\alpha_1\alpha_2}\right)^{k-n}\right]\ \left[\frac{1}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{k-n}\right\}^{m}$$

The survival function S(t) which is the probability that an individual survives for a time t.
S(t) = P(T>t) = Probability that the system survives beyond t.
P(exactly m policy decisions in (0,t)) = $F_m(t) - F_{m+1}(t)$ with $F_0(t) = 1$.

S(t) = $\sum_{m=0}^{\infty}[V_m(t)\mathrm{P}(\sum_{i=1}^{m} X_i < Y)]$

$$S(t) = -\sum_{m=0}^{\infty}[F_m(t) - F_{m+1}(t)]\ \left[\frac{\mu}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{m}\ \sum_{n=1}^{\infty}\left\{\left[\frac{1}{(k-n)}\ \left(\frac{1}{\alpha_1\alpha_2}\right)^{k-n}\right]\ \left[\frac{1}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{k-n}\right\}^{m}$$

$$S(t) = P(T>t) = -1+\left\{1 - \left[\frac{\mu}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{m}\ \sum_{n=1}^{\infty}\left\{\left[\frac{1}{(k-n)}\ \left(\frac{1}{\alpha_1\alpha_2}\right)^{k-n}\right]\ \left[\frac{1}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{k-n}\right\}\right\}$$

$$\sum_{m=1}^{\infty} F_m\ (t)\ \left[\frac{\mu}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{m}\ \sum_{n=1}^{\infty}\left\{\left[\frac{1}{(k-n)}\ \left(\frac{1}{\alpha_1\alpha_2}\right)^{k-n}\right]\ \left[\frac{1}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{k-n}\right\}^{m-1}$$

L(t) = 1−S(t)

$$L(t) = 1+1-\left\{1 - \left[\frac{\mu}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{m}\ \sum_{n=1}^{\infty}\left\{\left[\frac{1}{(k-n)}\ \left(\frac{1}{\alpha_1\alpha_2}\right)^{k-n}\right]\ \left[\frac{1}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{k-n}\right\}\right\}\sum_{m=1}^{\infty} F_m\ (t)$$

$$\left[\frac{\mu}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{m}\ \sum_{n=1}^{\infty}\left\{\left[\frac{1}{(k-n)}\ \left(\frac{1}{\alpha_1\alpha_2}\right)^{k-n}\right]\ \left[\frac{1}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{k-n}\right\}^{m-1}$$

$$L(t) = 2-\left\{1 - \left[\frac{\mu}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{m}\ \sum_{n=1}^{\infty}\left\{\left[\frac{1}{(k-n)}\ \left(\frac{1}{\alpha_1\alpha_2}\right)^{k-n}\right]\ \left[\frac{1}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{k-n}\right\}\right\}\sum_{m=1}^{\infty} F_m\ (t)$$

$$\left[\frac{\mu}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{m}\ \sum_{n=1}^{\infty}\left\{\left[\frac{1}{(k-n)}\ \left(\frac{1}{\alpha_1\alpha_2}\right)^{k-n}\right]\ \left[\frac{1}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{k-n}\right\}^{m-1}$$

$$L^{*}(s) = 2+\left\{\left[\frac{\mu}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{m}\ \sum_{n=1}^{\infty}\left\{\left[\frac{1}{(k-n)}\ \left(\frac{1}{\alpha_1\alpha_2}\right)^{k-n}\right]\ \left[\frac{1}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{k-n}\right\}-1\right\}\sum_{m=1}^{\infty} f_m^{*}(s)$$

$$\left[\frac{\mu}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{m}\ \sum_{n=1}^{\infty}\left\{\left[\frac{1}{(k-n)}\ \left(\frac{1}{\alpha_1\alpha_2}\right)^{k-n}\right]\ \left[\frac{1}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{k-n}\right\}^{m-1}$$

$$= 2+\left\{\left[\frac{\mu}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{m}\ \sum_{n=1}^{\infty}\left\{\left[\frac{1}{(k-n)}\ \left(\frac{1}{\alpha_1\alpha_2}\right)^{k-n}\right]\ \left[\frac{1}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{k-n}\right\}-1\right\}$$

$$f^{*}(s)\sum_{m=1}^{\infty} f^{*}(s)\ \left[\frac{\mu}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{m}\ \sum_{n=1}^{\infty}\left\{\left[\frac{1}{(k-n)}\ \left(\frac{1}{\alpha_1\alpha_2}\right)^{k-n}\right]\ \left[\frac{1}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{k-n}\right\}^{m-1}$$

$$=2+\frac{\left\{\left[\frac{\mu}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{m}\ \sum_{n=1}^{\infty}\left\{\left[\frac{1}{(k-n)}\left(\frac{1}{\alpha_1\alpha_2}\right)^{k-n}\right]\left[\frac{1}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{k-n}\right\}-1\right\}f^{*}(s)}{\left[\frac{\mu}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{m}\ \sum_{n=1}^{\infty}\left[\frac{1}{(k-n)}\left(\frac{1}{\alpha_1\alpha_2}\right)^{k-n}\right]\left[\frac{1}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{k-n}-f^{*}(s)}$$

$$=2+\frac{\left\{\left[\frac{\mu}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{m}\ \sum_{n=1}^{\infty}\left\{\left[\frac{1}{(k-n)}\left(\frac{1}{\alpha_1\alpha_2}\right)^{k-n}\right]\left[\frac{1}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{k-n}\right\}-1\right\}\frac{a}{a+s}}{\left[\frac{\mu}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{m}\ \sum_{n=1}^{\infty}\left[\frac{1}{(k-n)}\left(\frac{1}{\alpha_1\alpha_2}\right)^{k-n}\right]\left[\frac{1}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{k-n}-\frac{a}{a+s}}$$

$$L^*(s)=2+\frac{\left\{\left[\frac{\mu}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^m \sum_{n=1}^{\infty}\left\{\left[\frac{1}{(k-n)}\left(\frac{1}{\alpha_1\alpha_2}\right)^{k-n}\right]\left[\frac{1}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{k-n}\right\}-1\right\}a}{\left[\frac{\mu}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^m \sum_{n=1}^{\infty}\left[\frac{1}{(k-n)}\left(\frac{1}{\alpha_1\alpha_2}\right)^{k-n}\right]\left[\frac{1}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{k-n}(a+s)-a}$$

$$E(T)=-\frac{d}{ds}L^*(s)$$

$$=-\frac{d}{ds}\left\{2+\left[a\left[\frac{\mu}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^m \sum_{n=1}^{\infty}\left[\frac{1}{(k-n)}\left(\frac{1}{\alpha_1\alpha_2}\right)^{k-n}\right]\left[\frac{1}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{k-n}-a\right]\left\{\left[a\left[\frac{\mu}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^m \sum_{n=1}^{\infty}\left[\frac{1}{(k-n)}\left(\frac{1}{\alpha_1\alpha_2}\right)^{k-n}\right]\left[\frac{1}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{k-n}+s\left[\frac{\mu}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^m \sum_{n=1}^{\infty}\left[\frac{1}{(k-n)}\left(\frac{1}{\alpha_1\alpha_2}\right)^{k-n}\right]\left[\frac{1}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{k-n}-a\right]\right\}^{-1}\right\}$$

$$=\left[a\left[\frac{\mu}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^m \sum_{n=1}^{\infty}\left[\frac{1}{(k-n)}\left(\frac{1}{\alpha_1\alpha_2}\right)^{k-n}\right]\left[\frac{1}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{k-n}-a\right]/\left\{\left[a\left[\frac{\mu}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^m \sum_{n=1}^{\infty}\left[\frac{1}{(k-n)}\left(\frac{1}{\alpha_1\alpha_2}\right)^{k-n}\right]\left[\frac{1}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{k-n}+s\left[\frac{\mu}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^m \sum_{n=1}^{\infty}\left[\frac{1}{(k-n)}\left(\frac{1}{\alpha_1\alpha_2}\right)^{k-n}\right]\left[\frac{1}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{k-n}-a\right]\right\}^2$$

Where s=0

$$E(T)=\frac{1}{a\left[\frac{\mu}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^m \sum_{n=1}^{\infty}\left[\frac{1}{(k-n)}\left(\frac{1}{\alpha_1\alpha_2}\right)^{k-n}\right]\left[\frac{1}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{k-n}-a}$$

$$E(T^2)=\frac{d^2}{ds^2}L^*(s)$$

$$E(T^2)=\frac{d}{ds}\left\{\frac{d}{ds}L^*(s)\right\}$$

$$E(T^2)=\frac{d}{ds}\{-E(T)\}$$

$$E(T^2)=-\frac{d}{ds}\left[a\left[\frac{\mu}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^m \sum_{n=1}^{\infty}\left[\frac{1}{(k-n)}\left(\frac{1}{\alpha_1\alpha_2}\right)^{k-n}\right]\left[\frac{1}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{k-n}-a\right]/\left\{\left[a\left[\frac{\mu}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^m \sum_{n=1}^{\infty}\left[\frac{1}{(k-n)}\left(\frac{1}{\alpha_1\alpha_2}\right)^{k-n}\right]\left[\frac{1}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{k-n}+s\left[\frac{\mu}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^m \sum_{n=1}^{\infty}\left[\frac{1}{(k-n)}\left(\frac{1}{\alpha_1\alpha_2}\right)^{k-n}\right]\left[\frac{1}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{k-n}-a\right]\right\}^2$$

$$E(T^2)=\left[a\left[\frac{\mu}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^m \sum_{n=1}^{\infty}\left[\frac{1}{(k-n)}\left(\frac{1}{\alpha_1\alpha_2}\right)^{k-n}\right]\left[\frac{1}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{k-n}-a\right]/\left\{\left[a\left[\frac{\mu}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^m \sum_{n=1}^{\infty}\left[\frac{1}{(k-n)}\left(\frac{1}{\alpha_1\alpha_2}\right)^{k-n}\right]\left[\frac{1}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{k-n}+s\left[\frac{\mu}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^m \sum_{n=1}^{\infty}\left[\frac{1}{(k-n)}\left(\frac{1}{\alpha_1\alpha_2}\right)^{k-n}\right]\left[\frac{1}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{k-n}-a\right]\right\}^3$$

Where s=0

$$= \frac{2}{\left\{a\left[\frac{\mu}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^m \sum_{n=1}^{\infty}\left[\frac{1}{(k-n)}\left(\frac{1}{\alpha_1\alpha_2}\right)^{k-n}\right]\left[\frac{1}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{k-n} - a\right\}^2}.$$

$$V(T) = E(T^2) - (E(T))^2$$

$$V(T) = \frac{2}{\left\{a\left[\frac{\mu}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^m \sum_{n=1}^{\infty}\left[\frac{1}{(k-n)}\left(\frac{1}{\alpha_1\alpha_2}\right)^{k-n}\right]\left[\frac{1}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{k-n} - a\right\}^2} - \left\{\frac{1}{a\left[\frac{\mu}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^m \sum_{n=1}^{\infty}\left[\frac{1}{(k-n)}\left(\frac{1}{\alpha_1\alpha_2}\right)^{k-n}\right]\left[\frac{1}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{k-n} - a}\right\}^2$$

$$= \frac{1}{\left[a\left[\frac{\mu}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^m \sum_{n=1}^{\infty}\left[\frac{1}{(k-n)}\left(\frac{1}{\alpha_1\alpha_2}\right)^{k-n}\right]\left[\frac{1}{\mu+\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}\right]^{k-n} - a\right]^2}$$

5. Numerical Data

The expected time to recruitment in an organization when the amount of loss / breakdown of manpower crosses the maximum of the two grades, where manpower from one grade to other is permitted, the overall behavior of the system follows Kum-GG distribution has threshold level observed which shown in the given table. When μ is fixed parameter increases and the amount of damage is fixed parameter 'a' then the expected time to recruitment decreases. When the interval between policy changing times increases with fixed parameter 'a' there trend to be decreases in expected time to recruitment which is natural. Since Kum-GG distribution consider here and also variation of α_1, α_2 we get the expected time to recruitment decreases when the inter arrival increases whereas when the expected time to recruitment increases when the inter arrival time is fixed.

a	1	2	3	4	5	6	7	8	9	10
E(T)	.0534	.0267	.0178	.0133	.0106	.0089	.0076	.0067	.0059	.0053
V(T)	.0029	.0007	.0003	.0002	.0001	.0001	.0001	.0001	.0000	.0000

TABLE 1

For a=1,2,3.....10 μ=1, α_1 = .07, α_2 = .08, k=6, n=1,2,3,4,5 and k>n.

a	1	2	3	4	5	6	7	8	9	10
E(T)	.0262	.0131	.0081	.0065	.0053	.0044	.0038	.0033	.0029	.0026
V(T)	.0001	.0001	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000

TABLE 2

For a=1,2,3,.....10 μ=2, α_1 = .09, α_2 = .1, k=6, n=1,2,3,4,5 and k>n.

a	1	2	3	4	5	6	7	8	9	10
E(T)	4.76	2.38	1.58	1.19	.95	.79	.68	.59	.52	.47
V(T)	22.68	5.67	2.52	1.42	.91	.63	.46	.35	.28	.23

TABLE 3

For a=1,2,3.....10 μ=1, α_1 = .2, α_2 = .3, k=6, n=1,2,3,4,5 and k>n.

a	1	2	3	4	5	6	7	8	9	10
E(T)	1.75	.87	.58	.44	.35	.29	.25	.22	.19	.18
V(T)	3.08	.77	.34	.19	.12	.09	.63	.48	.04	.03

TABLE 4

For a=1,2,3.....10 μ=2, α_1 = .2, α_2 = .3, k=6, n=1,2,3,4,5 and k>n.

6. Conclusion

Since μ is fixed and the amount of wastage increases 'a' there exists an increase in the expected time to recruitment. When the interval is fixed in the policy changing time and with the amount of wastage increases there exists an decreasing in the expected time to recruitment.

7. References

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