THE INTERNATIONAL JOURNAL OF
SCIENCE & TECHNOLEDGE

Vibration of Uniform Rayleigh Beam Clamped-Clamped Carrying Concentrated Masses Undergoing Tractions

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Abstract:
This paper is sequel to [1], the initial –boundary –value problem of vibration of a uniform Rayleigh beam resting on a constant elastic foundation and with axial force and traversed by masses travelling at a uniform velocity is investigated. The dynamical problem, methods of solution and its closed form solution are as alluded to in [1]. Except that in [1], a simply-supported boundary condition was illustrated this is the simplest and the commonest boundary condition in literature. The novelty in this paper is that we considered boundary condition other than simply-supported one. One of the known but cumbersome boundary conditions in literature is the clamped-clamped boundary condition. A closed form solution of our dynamical problem with clamped –clamped boundary condition is obtained , numerical calculations and discussions of results reveals that; the response amplitude of the uniform Rayleigh beam clamped at both ends decreases as the value of the axial force N increases. It is further observed that higher values of an axial force N and foundation modulli when the rotatory inertial (r) is fixed are required for a more noticeable effect in the case of clamped-clamped time dependent boundary condition than those of simply supported boundary conditions as in [1,3] for both moving force and moving mass problems. Conclusively, this study has shed more light on the reliability of the moving force solution as a safe approximation to the moving mass problem.

Keywords*: Rayleigh Beam, Moving force, Moving mass, Critical Speed, Velocity, Time-Department and Resonance*

1. Introduction

This paper is concerned with the calculation of the dynamic response of structural members carrying one or more traveling loads which is very important in Engineering and Applied Mathematics as applications relate, for example, to the analysis and design of highway and railway bridges, cable- railways and the like. Generally, emphasis is placed on the dynamics of the structural members rather than on that of the moving loads: moving mass and moving force models. Common examples of structural members include beams, plates, and shells while traveling loads include moving trains, trucks, cars, bicycles, cranes etc. A structural member may be elastic, inelastic or viscoelastic. As such we have elastic structural members, inelastic structural members and viscoelastic structural configurations on which one or more loads may travel. Simple examples of these structural members are bridges, railroads, rails, decking slab, elevated roadways to moving vehicles, girders, belt-drive (carrying machine chains) and even floppy disks/cassette players' heads carrying tape. Pertinent to investigation in the field is the response of an elastic structure under the cases of moving concentrated loads with time dependent boundary conditions.

Several other researchers have made tremendous feat in the study of dynamics of structures under moving loads. In all of these, considerations have been limited to cases involving homogeneous boundary conditions and no considerations have been given to the class of dynamical problems in which the boundaries are constrained to undergo displacements or tractions which vary with time. In such cases boundary conditions are no longer homogeneous and boundary conditions become non-classical.

In many practical problems that concern the structural response to moving loads of elastic systems, the supports at the boundaries are not stationary but undergo different motions. Often the motions are in the form of lateral displacement, oscillations or tractions. As such, the boundary conditions are not homogeneous but are time dependent. These classes of non-classical boundary value problems are, in general, resistant to the classical methods of solving dynamical problems. In fact, it becomes more cumbersome, when the dynamical problems involve moving loads with or without consideration of the inertial effect of the moving loads is taken into consideration.

One of the earliest problems of this type was considered by Mindlin and Goodman [19] who described a procedure for extending the method of separation of variables to the solution of Bernoulli – Euler beam vibration problems with time-dependent boundary conditions

Thus, this study concerns the response of Rayleigh beams when it is under the actions of moving concentrated masses. Typical examples of time-dependent boundary conditions are used to illustrate the dynamical configurations.

2. Governing Equation

The problem of the vibration of a uniform Rayleigh beam under the action of a moving concentrated load $P(x,t)$ is considered. The transverse displacement $U(x,t)$, of a uniform Rayleigh beam of Length L transversed by a mass M traveling at a uniform velocity u, is governed by the fourth order partial differential equation.

$$\frac{\partial^2}{\partial x^2}\left[\frac{EI\,\partial^2 U(x,t)}{\partial x^2}\right]-\frac{N\partial^2 U(x,t)}{\partial x^2}+\frac{\mu\partial^2 U(x,t)}{\partial t^2}+KU(x,t)-\mu r^2\frac{\partial^4 U(x,t)}{\partial x^2\partial t^2}=P(x,t) \qquad (1.00)$$

where

x is the spatial co-ordinate, t is the time, $U(x,t)$ is the transverse displacement, E is the Young Modulus, I is the moment of inertial, μ is the mass per unit length of the beam, r is the radius of gyration, N is the axial force, K is the elastic foundation as EI is the flexural rigidity of the beam. For the problem under consideration, the moving load has mass that is commensurable with the mass of the beam. Consequently, the load inertia is not negligible but significantly affects the behavior of the dynamical system. In this case, load function $P(x,t)$ takes the form.

$$P(x,t)=P_f(x,t)\left[1-\frac{1}{g}\frac{d^2U(x,t)}{dt^2}\right] \qquad (1.01)$$

Where the continuous moving force $P_f(x,t)$ acting on the beam model is given by

$$P_f(x,t)=Mg\delta(x-f(t)) \qquad (1.02)$$

And $\frac{d^2}{dt^2}$ is a convective acceleration operator defined as becomes

$$\frac{d^2}{dt^2}=\frac{\partial^2}{\partial t^2}+2\frac{df(t)}{dt}\frac{\partial^2}{\partial x\partial t}+\left(\frac{df(t)}{dt}\right)^2\frac{\partial^2}{\partial x^2}+\frac{d^2f(t)}{dt^2}\frac{\partial}{\partial x} \qquad (1.03)$$

In this work, the moving load is assumed to move with constant speed, consequently, equation (1.03) becomes.

$$\frac{d^2}{dt^2}=\frac{\partial^2}{\partial t^2}+\frac{2u\partial^2}{\partial x\partial t}+\frac{u^2\,\partial^2}{\partial x^2} \qquad (1.04)$$

Now, on substituting equations (1.01), (1.02) and (1.04) into (1.00) and assuming that the flexural rigidity EI, and mass per unit length μ, do not vary with position x along the span L, equation (1.00) becomes.

$$EI\frac{\partial^4 U(x,t)}{\partial x^4}-\frac{N\partial^2 U(x,t)}{\partial x^4}+\frac{\mu\,\partial^2 U(x,t)}{\partial t^2}+KU(x,t)-\mu r^2\frac{\partial^4 U(x,t)}{\partial x^2\partial t^2}$$
$$=Mg\delta(x-ut)\left[1-\frac{1}{g}\left(\frac{\partial^2 U(x,t)}{\partial t^2}+\frac{2u\partial^2 u(x,t)}{\partial x\partial t}+\frac{u^2\partial^2 u(x,t)}{\partial x^2}\right)\right] \qquad (1.05)$$

The boundary conditions of this problem are taken to be time dependent. Thus, at each of the boundary points, there are two boundary conditions written as;

$$D_i\left[U(0,t)\right]=F_i(t) \quad i=1,2 \quad \text{and} \quad D_i\left[U(L,t)\right]=F_i(t) \quad i=3,4 \qquad (1.06)$$

Where D_i 's are linear homogeneous differential operators of order less than or equal to three.

The initial conditions of the motion at time $t=0$ may in general be specified by two arbitrary functions thus:

$$U(x,0)=U_0(x) \quad \text{and} \quad \frac{\partial U(x,0)}{\partial t}=\dot{U}_0(x) \qquad (1.07)$$

2.1. Operational Simplification of Equation

In this work, the analytical solution to the non-homogeneous initial boundary value problem (1.00) with non-homogeneous boundary condition (1.06) and non-homogenous initial conditions (1.07) is sought. To this end, an approach due to Mindlin and Goodman [19] is extended to obtain a robust technique which is capable of solving this class of problems for all variants of support conditions.

First, an auxiliary variable $z(x,t)$ in the form

$$U(x,t)=Z(x,t)+\sum_{i=1}^{4}F_i(t)g_i(x) \qquad (1.09)$$

is introduced. Now, substituting equation (1.09) into (1.05) transforms the boundary-value-problem in terms of $U(x,t)$ into the boundary value problem in terms of $Z(x,t)$. The functions $g_i(x)$ are called the displacement influence functions while $f_i(t)$ are the pertinent displacements at the respective boundaries. The functions $g_i(x)$ are to be chosen so as to render the boundary conditions for the boundary value problems in $Z(x,t)$ homogeneous.

Thus, substituting (1.09) into equation (1.05) one obtains

$$\frac{EI}{\mu}\frac{\partial^4 Z(x,t)}{\partial x^4}-\frac{N}{\mu}\frac{\partial^2}{\partial x^2}Z(x,t)+\frac{\partial^2}{\partial t^2}Z(x,t)+\frac{K}{\mu}Z(x,t)$$
$$-\frac{r^2\partial^4 Z(x,t)}{\partial x^2\partial t^2}+\frac{M}{\mu}\delta(x-ut)\left[\frac{\partial^2 Z(x,t)}{\partial t^2}+\frac{2U\partial^2 Z(x,t)}{\partial x\partial t}+\frac{U^2\partial^2 Z(x,t)}{\partial x^2}\right]$$
$$=\frac{M}{\mu}g\delta(x-ut)-\frac{EI}{\mu}\sum_{i=1}^{4}f_i(t)g_i''''(x)+\frac{N}{\mu}\sum_{i=1}^{4}f_i(t)g_i(x)$$
$$-\sum_{i=1}^{4}\ddot{f}_i(t)g_i(x)-\frac{K}{\mu}\sum_{i=1}^{4}f_i(t)g_i(x)+r^2\sum_{i=1}^{4}f_i''(t)g_i''(x)$$
$$+\frac{M}{\mu}\delta(x-ut)\left[\sum_{i=1}^{4}\left(\ddot{f}_i(x)g_i(x)+2U\dot{f}_i(x)g_i'(x)+U^2 f_i(t)g_i''(x)\right)\right] \quad (1.10)$$

Where dot $(\cdot)$ represents the derivative with respect to time, while slash $(')$ represents the derivative with respect to space coordinate.

Now the expression in equation (1.09) must satisfy the boundary conditions in equation (1.06); consequently, we have

$$D_i[Z(o,t)]+\sum_{j=1}^{4}f_i(t)D_i[g_i(o)]=f_i(t), \qquad i=1,2. \quad (1.11)$$

$$D_i[Z(L,t)]+\sum_{j=1}^{4}f_i(t)D_i[g_i(L)]=f_i(t), \qquad i=3,4. \quad (1.12)$$

Substituting equation (1.09) into the initial equation (1.07) and (1.08) one obtains.

$$Z(x,o)=U(x,o)-\sum_{i=1}^{4}f_i(o)g_i(x) \quad (1.13)$$

$$\frac{\partial}{\partial t}z(x,o)=\dot{U}_0(x)-\sum_{i=1}^{4}\dot{f}_i(o)g_i(x) \quad (1.14)$$

2.2. Solution Procedure

It is observed that the initial – boundary – value problem in equation (1.10) is a fourth order partial differential equation having some coefficients which are not only variable but are also singular. These coefficients are the Dirac delta functions which multiply each term of the convective acceleration operator associated with the inertia of the mass of the moving load. It is remarked at this juncture that this transformed equation is now amenable to the method of generalized finite integral transform used extensively in S.T. Oni and Ajibola.S.O [3,6,9,20,21,22].

2.3. The Generalized Finite Integral Transform Method

The generalized finite integral transform method is one of the best methods used in handling problems involving mechanical vibrations. This integral transform method is given by

$$\bar{z}(m,t)=\int_o^l z(x,t)V_m(x)dx \quad (1.15)$$

With the inverse

$$z(x,t)=\sum_{m=1}^{\infty}\frac{\mu}{V_m}\bar{z}(m,t)V_m(x) \quad (1.16)$$

Where

$$\bar{V}_m=\int_o^l \mu V_m^2(x)dx \quad (1.17)$$

$V(x,t)$, is any function such that the pertinent boundary conditions are satisfied. An appropriate selection of functions for beam problems are beam mode shape. Thus the m^{th} normal mode of vibrations of a uniform beam given by

$$V_m(x) = Sin\frac{\lambda_m x}{L} + A_m Cos\frac{\lambda_m x}{L} + B_m Sinh\frac{\lambda_m x}{L} + C_m Cosh\frac{\lambda_m x}{L} \qquad (1.18)$$

is chosen as a suitable kernel of the integral (1.15) where λ_m is the mode frequency, A_m, B_m and C_m are constant. An important feature of the use of this kernel is that it makes the transformation suitable for all variants of the boundary conditions of the dynamical problems. The parameter λ_m, A_m, B_m and C_m are obtained when the equation (1.18) is substituted into the appropriate boundary conditions.

By applying the generalized finite integral transform (1.15), equation (1.10) takes the form

$$\overline{Z}_{tt}(m,t) = B_1 Q_A(t) + B_2 Q_B(t) + B_3 Z(m,t) + B_1 Z(0,L,t) - r^2 Q_C(t) + Q_D(t) + Q_E(t) + Q_F(t)$$
$$PV_m(Ut) - \left[G_a(t) - G_b(t) + G_c(t) + G_d(t) + G_e(t) + G_f(t) + G_g(t) + G_h(t)\right] \qquad (1.19)$$

where

$$B_1 = \frac{EI}{\mu},\ B_2 = \frac{N}{\mu},\ B_3 = \frac{K}{\mu},\ P = \frac{mg}{\mu},\ \text{and}\ \varepsilon = \frac{M}{\mu L} \qquad (1.20)$$

$$Q_A(t) = \int_o^l \frac{\partial^4}{\partial x^4} Z(x,t) V_m(x) dx \ , \quad Q_B(t) = \int_o^l \frac{\partial^2}{\partial x^2} Z(x,t) V_m(x) dx$$

$$Q_C(t) = \int_o^l \frac{\partial^4}{\partial x^2 \partial t^2} Z(x,t) V_m(x) dx \ , \quad Q_D(t) = \int_o^l \frac{M}{\mu} \delta(x-ut) \frac{\partial^2}{\partial t^2} Z(x,t) V_m(x) dx$$

$$Q_E(t) = \int_o^l \frac{2MU}{\mu} \delta(x-ut) \frac{\partial^2}{\partial x \partial t} Z(x,t) V_m(x) dx, \qquad (1.21)$$

$$Z(0,L,t) = \left[V_m(x) \frac{\partial^3}{\partial x^3} Z(x,t) - V_m'(x) \frac{\partial^2}{\partial x^2} Z(x,t) + V_m''(x) \frac{\partial}{\partial x} Z(x,t) - V_m'''(x) Z(x,t) \right]_0^L$$

(1.22)

$$G_a(t) = B_1 \sum_{i=1}^4 f_i(t) \int_0^L \left(\frac{d^4}{dx^4} g_i(x) \right) V_m(x) dx,\ G_b(t) = B_2 \sum_{i=1}^4 f_i(t) \int_0^L \left(\frac{d^2}{dx^2} g_i(x) \right) V_m(x) dx$$

$$G_c(t) = \sum_{i=1}^4 \ddot{f}_i(t) \int_0^L g_i(x) V_m(x) dx \ , \quad G_d(t) = B_3 \sum_{i=1}^4 \ddot{f}_i(t) \int_0^L g_i(x) V_m(x) dx$$

$$G_e(t) = r^2 \sum_{i=1}^4 \ddot{f}_i(t) \int_0^L \left(\frac{d^2}{dx^2} g_i(x) \right) V_m(x) dx, G_f(t) = \frac{M}{\mu} \sum_{i=1}^4 \ddot{f}_i(t) \int_0^L \delta(x-ut) g_i(x) V_m(x) dx$$

$$G_g(t) = \frac{2MU}{\mu} \sum_{i=1}^4 \dot{f}_i(t) \int_0^L \delta(x-ut) g_i'(x) V_m(x) dx \ G_h(t) = \frac{MU^2}{\mu} \sum_{i=1}^4 f_i(t) \int_0^L \delta(x-ut) g_i''(x) V_m(x) dx$$

(1.23)

It is well know that the natural mode in Equation (1.18) satisfies the homogeneous differential equation

$$EI \frac{d^4}{dx^4} V_m(x) - \mu \omega_m^2 V_m(x) = 0 \qquad (1.24)$$

for the Euler beam. The parameter (ω) is the natural circular frequency defined by

$$\omega_m^2 = \frac{\lambda^4}{L^4} \frac{EI}{\mu} \qquad (1.25)$$

Equation (1.24) implies

$$EI \int_o^L \left(\frac{d^4}{dx^4} V_m(x) \right) Z(x,t) dx = \mu \omega_m^2 \int_0^L V_m(x) Z(x,t) dx \qquad (1.26)$$

Thus, by (1.15)

$$Q_A(t) = \frac{\mu}{EI} \omega_m^2 \overline{Z}(m,t) \qquad (1.27)$$

Since

$\overline{Z}(m,t)$ is just the coefficient of the generalized finite integral transform, equation (1.16) yields

$$Z(x,t)=\sum_{k=0}^{\infty}\frac{\mu}{V_k}\overline{Z}(k,t)V_k(x) \quad (1.28)$$

Thus $$\frac{\partial^2}{\partial x^2}Z_{tt}(x,t)=\sum_{k=1}^{\infty}\frac{\mu}{V_k}\overline{Z}(k,t)\frac{d^2}{dx^2}V_k(x) \quad (1.29)$$

And the integral (1.21) can be written as

$$Q_c(t)=\sum_{k=1}^{\infty}\frac{\mu}{V_k}\overline{Z}_{tt}(x,t)\int_0^L\left(\frac{d^2}{dx^2}V_k(x)\right)V_m(x)dx \quad (1.30)$$

Now using the property of Dirac-Delta function as an even function, it can be expressed it in Fourier cosine series namely

$$\delta(x-\mu t)=\frac{1}{L}+\frac{2}{L}\sum_{n=1}^{\infty}\cos\frac{n\pi\mu t}{L}\cos\frac{n\pi x}{L} \quad (1.31)$$

When use is made of equations (1.28) to (1.31), one obtains

$$Q_d(t)=\frac{M}{\mu L}\sum_{k=1}^{\infty}\frac{\mu}{V_k}\overline{Z}_{tt}(k,t)\left[\int_0^L V_k(x)V_m(x)dx+2\sum_{n=1}^{\infty}Cos\frac{n\pi ut}{L}\int_0^L Cos\frac{n\pi x}{L}V_k(x)V_m(x)dx\right] \quad (1.32)$$

Using similar argument in equations (1.21). It is straight forward to show that

$$Q_e(t)=\frac{2MU}{\mu L}\sum_{k=1}^{\infty}\overline{Z}_t(k,t)\left[\frac{\mu}{V_k}\int_0^L V_k'(x)V_m(x)dx+2\sum_{n=1}^{\infty}Cos\frac{n\pi ut}{L}\frac{\mu}{V_k}\int_0^L Cos\frac{n\pi x}{L}V_k'(x)V_m(x)dx\right] \text{ and} \quad (1.33)$$

$$Q_f(t)=\frac{MU^2}{\mu L}\sum_{k=1}^{\infty}\overline{Z}(k,t)\left[\frac{\mu}{V_k}\int_0^L V_k(x)V_m(x)dx+2\sum_{n=1}^{\infty}Cos\frac{n\pi ut}{L}\frac{\mu}{V_k}\int_0^L Cos\frac{n\pi x}{L}V_k(x)V_m(x)dx\right] \quad (1.34)$$

Substituting equations (1.27) to (134), into (1.19), after simplifications and arrangements yields

$$\overline{Z}_{tt}(m,t)+\alpha_m^2\overline{Z}_t(m,t)-\frac{N}{\mu}\sum_{k=1}^{\infty}\overline{Z}(k,t)S_1^*(k,m)-r^2\sum_{k=1}^{\infty}\overline{Z}_{tt}(k,t)S_1^*(k,m)+\varepsilon\left[\sum_{k=1}^{\infty}\overline{Z}_{tt}(k,t)S_2^*(k,m)\right.$$

$$+2\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}Cos\frac{n\pi ut}{L}\overline{Z}_{tt}(k,t)S_{2c}^*(k,m,n)+2U\sum_{k=1}^{\infty}\overline{Z}(k,t)S_3^*(k,m)$$

$$+4U\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}Cos\frac{n\pi ut}{L}\overline{Z}_t(k,t)S_{3c}^*(k,m,n)+U^2\sum_{k=1}^{\infty}\overline{Z}(k,t)S_1^*(k,m)$$

$$+2U^2\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}Cos\frac{n\pi ut}{L}\overline{Z}_t(k,t)S_{1c}^*(k,m,n)$$

$$=P\left[Sin\frac{\lambda_m ut}{L}+A_mCos\frac{\lambda_m ut}{L}+B_mSinh\frac{\lambda_m ut}{L}+C_mCosh\frac{\lambda_m ut}{L}\right]$$

$$-\left[G_a(t)-G_b(t)+G_c(t)+G_d(t)-G_e(t)+G_f(t)+G_g(t)+G_h(t)\right] \quad (1.35)$$

where G_a, G_b, G_c..............................G_h are as defined in equations (1.23) ,

$$\alpha_m^2=\left(\omega_m^2+\frac{k}{\mu}\right) \quad (1.36)$$

First, we shall obtain the particular functions $g_i(x)$, where $i=1,2,3,4.$ which ensure zeros of the right hand sides of the boundary conditions for a clamped-clamped beam. Going through the same process discussed in [1,3,6], one obtains

$$g_1(x)=1-3\left(\frac{x}{L}\right)^2+2\left(\frac{x}{L}\right)^3,\ g_2(x)=x-\frac{x^2}{L},\ g_3(x)=3\left(\frac{x}{L}\right)^2-2\left(\frac{x}{L}\right)^3,\ g_4(x)=-\frac{x^2}{L}+\frac{x^3}{L^2} \quad (1.37)$$

It is only necessary to compute those of the $g_i(x)$ for which the corresponding $f_i(t)$ do not vanish. Thus, we need only $g_1(x)$ and $g_3(x)$ for our boundary displacement functions $f_1(t)$ and $f_3(t)$ as defined in [1,3,6,19,20,21,22,23].

Thus we can write

$$f_1 = BSin\Omega t \text{ and } f_3 = Ae^{-\beta t} Sin\Omega t \quad (1.41)$$

Where A, B are amplitudes, Ω is frequency and β is parameter.
The initial conditions are, again

$$\overline{Z}(x,0) = 0 \text{ and } \overline{Z}_t(x,0) = -\Omega \quad (1.42)$$

which when transformed yield

$$\overline{Z}(m,0) = 0 \text{ and } \overline{Z}_t(m,0) = \eta_2 \quad (1.43)$$

where

$$\eta_2 = \eta_{or}\left[(1-\cos\lambda_m) + B_m(\cosh\lambda_m - 1) + A_m \sin\lambda_m + C_m \sinh\lambda_m\right] \quad (1.44)$$

and

$$\eta_{or} = -\frac{L\Omega}{\lambda_m} \quad (1.45)$$

In view of equations (1.37),(1.42) and (1.43); the transformed equation of our problem, reduces to

$$\overline{Z}_{tt}(m,t) + \left(\omega_m^2 + \frac{k}{\mu}\right)\overline{Z}(m,t) - \frac{N}{\mu}\sum_{K=1}^{\infty}\overline{Z}(k,t)S_1^*(K,m) - r^2\sum_{K=1}^{\infty}\overline{Z}_{tt}(k,t)S_1^*(K,m)$$
$$+\varepsilon\left\{\sum_{K=1}^{\infty}\overline{Z}_{tt}(k,t)S_2^*(K,m) + 2\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\cos\frac{n\pi ut}{L}\overline{Z}_{tt}(k,t)S_{2c}^*(k,m,n)\right.$$
$$+2U\sum_{K=1}^{\infty}\overline{Z}(k,t)S_3^*(K,m) + 4U\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\cos\frac{n\pi ut}{L}\overline{Z}_t(k,t)S_{3c}^*(k,m,n)$$
$$\left.+U^2\sum_{K=1}^{\infty}\overline{Z}(k,t)S_3^*(K,m) + 2U^2\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\cos\frac{n\pi ut}{L}\overline{Z}_t(k,t)S_{1c}^*(k,m,n)\right\}$$
$$= P\left[Sin\frac{\lambda_m ut}{L} + A_m Cos\frac{\lambda_m ut}{L} + B_m Sinh\frac{\lambda_m ut}{L} + C_m Cosh\frac{\lambda_m ut}{L}\right]$$
$$-\left[\ddot{f}_1(t)\,H_1 + \left(\ddot{f}_3(t) - \ddot{f}_1(t)\right)H_9 + f_1(t)\frac{K}{\mu}H_1 + \left(f_3(t) - f_1(t)\right)H_{10}\right.$$
$$\left.+\left(\dot{f}_3(t) - \dot{f}_1(t)\right)\left(H_{13} + H_{14}\sum_{n=1}^{\upsilon}\cos\frac{n\upsilon t}{L}\right) + \left(f_3(t) - f_1(t)\right)\left(H_{15} + H_{16}\sum_{n=1}^{\upsilon}\cos\frac{n\upsilon t}{L}\right)\right\}\Bigg] \quad (1.46)$$

Where

$$\varepsilon = \frac{M}{\mu L} \quad (1.47)$$

$$H_1 = N_1 - N_3 + A_m(N_2 - N_4) \quad ; \quad H_2 = N_5 - N_7 + A_m(N_6 - N_8)$$
$$H_3 = N_9 - N_{11} + A_m(N_{10} - N_{12}) \quad ; \quad H_4 = N_{13} - N_{15} + A_m(N_{14} - N_{16})$$
$$H_5 = N_{17} - N_{19} + A_m(N_{18} - N_{20}) \quad ; \quad H_6 = N_{21} - N_{23} + A_m(N_{22} - N_{24})$$
$$H_7 = N_{25} - N_{27} + A_m(N_{26} - N_{28}) \quad ; \quad H_8 = N_{29} - N_{31} + A_m(N_{30} - N_{32})$$
$$H_9 = \left[\frac{3}{L^2}H_3 - \frac{2}{L^3}H_4 - \sigma^2\left(\frac{6}{L^2}H_1 - \frac{12}{L^3}H_2\right)\right] \quad , \quad H_{10} = \left[\frac{K}{\mu}\left(\frac{3}{L^2}H_3 - \frac{2}{L^3}H_4\right) - \frac{N}{\mu}\left(\frac{6}{L^2}H_1 - \frac{12}{L^3}H_2\right)\right] \quad ,$$
$$H_{11} = \frac{3}{L^2}H_3 - \frac{2}{L^3}H_4 \quad ; \quad H_{12} = \frac{6}{L^2}H_7 - \frac{4}{L^3}H_8, \quad H_{13} = 2U\left(\frac{6}{L^2}H_2 - \frac{6}{L^3}H_3\right)$$
$$; H_{14} = 2U\left(\frac{12}{L^2}H_6 - \frac{12}{L^3}H_7\right)$$
$$H_{15} = U^2\left(\frac{6}{L^2}H_1 - \frac{12}{L^3}H_2\right) \quad ; \quad H_{16} = U^2\left(\frac{12}{L^2}H_5 - \frac{24}{L^3}H_6\right) \quad (1.48)$$

Equation (1.46) is the transformed equation governing the problem of a uniform Rayleigh beam resting on a constant elastic foundation. Two special cases of equation (1.46) are considered namely moving force and moving mass problems.

3.The Clamped-Clamped Moving Force Problem

In equation (1.46), ε is set to zero, using Strubles asymptotic techniques as alluded to in [1,3,6,19,20,21,22,23]. On this consideration, Hence, the entire equation (1.46) takes the form

$$\overline{Z}_{tt}(m,t)+\gamma_m^2\overline{Z}(m,t)=$$

$$\left(1+\varepsilon_* S_1^*(m,m)\right)\left[P\left(Sin\frac{\lambda_m ut}{L}+A_m Cos\frac{\lambda_m ut}{L}+B_m Sinh\frac{\lambda_m ut}{L}+C_m Cosh\frac{\lambda_m ut}{L}\right)\right.$$

$$\left.-H_{17}\sinh\Omega t-H_{18}\ell^{-\beta t}\sin\Omega t+H_{19}\ell^{-\beta t}\cos\Omega t\right] \quad . \qquad (1.49)$$

Where

$$\gamma_m=-\frac{\omega_{mf}}{2}\left[\varepsilon_*\left(S_1^*(m,m)+\varepsilon_*\frac{N}{\mu r^2\omega_{mf}^2}S_1^*(m,m)\right)\right] \qquad (1.50)$$

represents the modified frequency due to the effect of rotatory inertia.

$$\frac{r^2}{L}=\lambda_0 \qquad (1.51)$$

In order to obtain the solution to equation (1.49), it is subjected to a Laplace transformation in conjunction with the initial condition. The equation, after simplifications and rearrangements then we obtain

$$\overline{Z}(m,t)=\frac{P_{OR}}{\gamma_m\left(\gamma_m^4-z_0^4\right)}\left[\left(\gamma_m^2+z_0^2\right)\left(\left(\gamma_m\sin z_0t-z_0\sin\gamma_m t\right)+A_m\left(\cos z_0t-\cos\gamma_m t\right)\right)\right.$$

$$\left.+\left(\gamma_m^2-z_0^2\right)\left(B_m\left(\gamma_m\sinh z_ot-z_0\sin\gamma_m t\right)+C_m\left(\cosh z_0t-\cos\gamma_m t\right)\right)\right]$$

$$+\frac{H_{17}^{OR}\Omega}{2\gamma_m\left(\gamma_m^2-\Omega_m^2\right)}\left[(\gamma_m-\Omega)\sin\gamma_m t\cos(\Omega+\gamma_m)-(\gamma_m-\Omega)\cos\gamma_m t\sin(\Omega+\gamma_m)\right.$$

$$\left.-(\Omega+\gamma_m)\sin\gamma_m t\cos(\Omega-\gamma_m)-(\Omega+\gamma_m)\cos\gamma_m t\sin(\Omega-\gamma_m)+2\Omega\sin\gamma_m t\right]$$

$$+\frac{H_{18}^{OR}e^{-\beta t}}{2\gamma_m\left[\beta^2+(\gamma_m+\Omega)^2\right]}\left[-\beta\sin\gamma_m t\sin(\gamma_m+\Omega)t-\beta\cos\gamma_m t-(\gamma_m+\Omega)\beta\sin\gamma_m t\cos(\gamma_m+\Omega)t\right.$$

$$\left.+(\gamma_m+\Omega)\sin\gamma_m t-\beta\cos\gamma_m t\cos(\gamma_m+\Omega)t+(\gamma_m+\Omega)\cos\gamma_m t\sin(\gamma_m+\Omega)t\right]$$

$$+\frac{H_{18}^{OR}e^{-\beta t}}{2\gamma_m\left[\beta^2+(\gamma_m-\Omega)^2\right]}\left[\beta\cos\gamma_m t\cos(\gamma_m-\Omega)t-(\gamma_m-\Omega)\left[\cos\gamma_m t\sin(\gamma_m-\Omega)t\right.\right.$$

$$\left.+\beta\sin\gamma_m t\sin(\gamma_m-\Omega)t+(\gamma_m-\Omega)\sin\gamma_m t\cos(\gamma_m-\Omega)t-(\gamma_m-\Omega)\sin\gamma_m t-\beta\cos\gamma_m t\right]$$

$$+\frac{H_{19}^{OR}e^{-\beta t}}{2\gamma_m\left[\beta^2+(\gamma_m+\Omega)^2\right]}\left[-\beta\sin\gamma_m t\cos(\gamma_m+\Omega)t+\beta\sin\gamma_m t+(\gamma_m+\Omega)\sin\gamma_m t\sin(\gamma_m+\Omega)t\right.$$

$$\left.+\beta\cos\gamma_m t\sin(\gamma_m+\Omega)t+(\gamma_m+\Omega)\cos\gamma_m t\cos(\gamma_m+\Omega)t-(\gamma_m+\Omega)\cos\gamma_m t\right]$$

$$+\frac{H_{19}^{OR}e^{-\beta t}}{2\gamma_m\left[\beta^2+(\gamma_m-\Omega)^2\right]}\left[-\beta\sin\gamma_m t\cos(\gamma_m-\Omega)t+\beta\sin\gamma_m t+(\gamma_m-\Omega)\sin\gamma_m t\sin(\gamma_m-\Omega)t\right.$$

$$\left.(\Omega-\gamma_m)\cos\gamma_m t-\beta\cos\gamma_m t\sin(\Omega-\gamma_m)t-(\Omega-\gamma_m)\cos\gamma_m t\cos(\Omega-\gamma_m)t\right]+\frac{\eta_2}{\gamma_m}\sin\gamma_m t \qquad (1.52)$$

on inversion yields

$$\overline{Z}(x,t)=\frac{2}{L}\sum_{m=1}^{\infty}\overline{Z}(m,t)\left[\cosh\frac{\lambda_m x}{L}-\cos\frac{\lambda_m x}{L}-\sigma_m\left(\sinh\frac{\lambda_m x}{L}-\sin\frac{\lambda_m x}{L}\right)\right]$$

$$U(x,t)=\overline{Z}(x,t)+\left(1-3\left(\frac{x}{L}\right)^2+2\left(\frac{x}{L}\right)^3\right)\sin\Omega t+\left(3\left(\frac{x}{L}\right)^2-2\left(\frac{x}{L}\right)^3\right)e^{-\beta t}\sin\Omega t \qquad (1.53)$$

Equation (1.53) is the transverse response of a thin beam under the action of a moving force whose two clamped edges are constrained to undergo displacements which vary with time.

4. The Clamped-Clamped Moving Mass Problem

If the mass of the moving load is commensurable with that of the structure, the inertia effect of the moving mass is not negligible. Thus, $\varepsilon_0 \neq 0$ and the solution of the entire equation (1.46) is desired. Using Strubles asymptotic technique after simplifications and rearrangements, we obtain

$$\overline{Z}_{tt}(m,t)+\gamma_{mf}^{2}\overline{Z}(m,t)=H_{28}\sin\frac{\lambda_m u t}{L}+H_{29}\cos\frac{\lambda_m u t}{L}+H_{30}\sinh\frac{\lambda_m u t}{L}+H_{31}\cosh\frac{\lambda_m u t}{L}$$

$$H_{32}\sin\Omega t+H_{33}\cos\Omega t+H_{34}e^{-\beta t}\sin\Omega t+H_{35}e^{-\beta t}\cos\Omega t+H_{36}\frac{n\pi u t}{L}\sin\Omega$$

$$H_{37}\cos\frac{n\pi u t}{L}\cos\Omega t+H_{38}e^{-\beta t}\frac{n\pi u t}{L}\sin\Omega t+H_{39}e^{-\beta t}\cos\frac{n\pi u t}{L}\cos\Omega t+H_{40}\cos\frac{n\pi u t}{L}\sin\frac{\lambda_m u t}{L}$$

$$H_{41}\cos\frac{n\pi u t}{L}\cos\frac{\lambda_m u t}{L}+H_{42}\cos\frac{n\pi u t}{L}\sinh\frac{\lambda_m u t}{L}+H_{43}\cos\frac{n\pi u t}{L}\cosh\frac{\lambda_m u t}{L} \qquad (1.54)$$

where : $P_{OR}^{*}=P\left[1-\lambda\left(S_2^{*}(m,m)+2\sum_{n=1}^{\infty}\cos\frac{n\pi ut}{L}S_{2C}^{*}(m,m,n)\right)\right]$, $H_{28}=P_{OR}^{*}$, $H_{29}=P_{OR}^{*}A_m$,

$H_{30}=P_{OR}^{*}B_m$, $H_{31}=P_{OR}^{*}C_m$; $H_{32}=\left[H_{17}+\lambda\left(H_{20}-H_{17}S_2^{*}(m,m)\right)\right]$, $H_{33}=\lambda H_{21}$;

$$H_{34}=-\left[H_{18}+\lambda\left(H_{22}-H_{18}S_2^{*}(m,m)\right)\right]$$

$$H_{35}=-\left[H_{19}+\lambda\left(H_{23}-H_{19}S_2^{*}(m,m)\right)\right],\ H_{36}=-\lambda\sum_{n=1}^{\infty}\left[H_{24}-2H_{17}S_{2C}^{*}(m,m,n)\right],\ H_{37}=-\lambda H_{25}$$

$$H_{38}=-\lambda\sum_{n=1}^{\infty}\left(H_{26}-2H_{18}S_{2C}^{*}(m,m,n)\right)\ ,\ H_{39}=-\lambda\sum_{n=1}^{\infty}\left(H_{27}-2H_{19}S_{2C}^{*}(m,m,n)\right)$$

$$H_{40}=-2\lambda P\sum_{n=1}^{\infty}S_{2C}^{*}(m,m,n),\quad H_{41}=-2\lambda PA_m\sum_{n=1}^{\infty}S_{2C}^{*}(m,m,n),$$

$$H_{42}=-2\lambda PB_m S_{2c}^{*}(m,m,n)\quad \text{and}\quad H_{43}=-2\lambda PC_m\sum_{n=1}^{\infty}S_{2c}^{*}(m,m,n) \qquad (1.55)$$

To obtain the solution of equation (1.53), it is subjected to Laplace transform in conjunction with the initial conditions, after some simplifications and rearrangements ,we obtain

$$\begin{aligned}\overline{Z}(m,t)&=A_{29}Sin\gamma_{mf}t+A_{30}SinZ_0t+A_{31}CosZ_0t+A_{32}SinhZ_0t+A_{33}CoshZ_0t\\&+A_{34}Sin\Omega t+A_{35}Cos\Omega t+A_{36}Cos\gamma_{mf}t+A_{37}SinZ_3t+A_{38}SinZ_4t+A_{39}CosZ_3t\\&+A_{40}CosZ_4t+A_{41}SinZ_7t+A_{42}SinZ_8t+A_{43}CosZ_7t+A_{44}CosZ_8t+A_{45}e^{-\beta t}Sin\Omega t\\&+A_{46}e^{\beta t}Cos\Omega t+A_{47}e^{-\beta t}SinZ_3t+A_{48}e^{-\beta t}CosZ_3t\\&+A_{49}e^{\beta t}SinZ_4t+A_{50}e^{-\beta t}CosZ_4t+A_{51}CosZ_2tCosht\\&+A_{52}CosZ_2tSinht+A_{53}SinZ_2tSinht+A_{54}SinZ_2tCosht\ ..\end{aligned} \qquad (1.56)$$

Where

$$\begin{aligned}A_{29}=&\left[\frac{-H_{28}Z_0}{\left(\gamma_{mf}^2-Z_0^2\right)\gamma_{mf}}-\frac{H_{30}Z_0}{\left(\gamma_{mf}^2+Z_0^2\right)\gamma_{mf}}-\frac{H_{32}\Omega}{\left(\gamma_{mf}^2-\Omega^2\right)\gamma_{mf}}\right.\\&+\frac{\left(\gamma_{mf}^2-\Omega^2+\beta^2\right)\gamma_{mf}H_{34}+\left(\gamma_{mf}^2+\Omega^2+\beta^2\right)H_{35}}{\gamma_{mf}\left[\left(\gamma_{mf}^2+\Omega^2+\beta^2\right)^2-4\gamma_{mf}^2\Omega^2\right]}\\&-\frac{H_{36}Z_3}{2\gamma_{mf}\left(\gamma_{mf}^2-Z_3^2\right)}+\frac{H_{36}Z_4}{2\gamma_{mf}\left(\gamma_{mf}^2-Z_4^2\right)}+\frac{\left(\gamma_{mf}^2-Z_3^2+\beta^2\right)H_{38}+2\left(\gamma_{mf}^2+\beta^2+Z_3^2\right)H_{39}}{2\left[\left(\gamma_{mf}^2+Z_3^2+\beta^2\right)^2-4\gamma_{mf}^2Z_3^2\right]}\\&-\frac{\gamma_{mf}\left(\gamma_{mf}^2-Z_4^2+\beta^2\right)H_{38}+\left(\gamma_{mf}^2+\beta^2+Z_4^2\right)H_{39}}{2\gamma_{mf}\left[\left(\gamma_{mf}^2+Z_2^2+\beta^2\right)^2-4\gamma_{mf}^2Z_4^2\right]}-\frac{H_{4\cup}Z_7}{2\gamma_{mf}\left(\gamma_{mf}^2-Z_7^2\right)}+\frac{H_{41}Z_8}{2\left(\gamma_{mf}^2-Z_8^2\right)}\end{aligned}$$

$$+\frac{\left(\gamma_{mf}^2+Z_2^2+1\right)\left(H_{45}-H_{44}\right)}{\gamma_{mf}\left[\left(\gamma_{mf}^2+Z_2^2+1\right)^2-4\gamma_{mf}^2Z_2^2\right]}+\frac{\eta_2}{\gamma_{mf}}\Bigg]$$

$$A_{30}=H_{28}/\left(\gamma_{mf}^2-Z_0^2\right), A_{31}=H_{29}/\left(\gamma_{mf}^2-Z_0^2\right);\ A_{32}=H_{30}/\left(\gamma_{mf}^2-Z_0^2\right), A_{33}=H_{31}/\left(\gamma_{mf}^2-Z_0^2\right)$$

$$A_{34}=H_{32}/\left(\gamma_{mf}^2-\Omega^2\right), A_{35}=H_{33}/\left(\gamma_{mf}^2-\Omega^2\right);\ A_{36}=\Bigg[\frac{-H_{29}}{\left(\gamma_{mf}^2-Z_0^2\right)}-\frac{H_{31}}{\left(\gamma_{mf}^2+Z_0^2\right)}-\frac{H_{33}}{\gamma_{mf}^2-\Omega^2}$$

$$-\frac{2\Omega\beta H_{34}-\left(\gamma_{mf}^2-\Omega^2+\beta^2\right)H_{35}}{\left[\left(\gamma_{mf}^2+\Omega^2+\beta^2\right)^2-4\gamma_{mf}^2\Omega^2\right]};\ \frac{H_{37}}{2\left(\gamma_{mf}^2-Z_3^2\right)}-\frac{H_{37}}{2\left(\gamma_{mf}^2-Z_4^2\right)}$$

$$-\frac{2Z_3\beta H_{38}-\left(\gamma_{mf}^2-Z_3^2+\beta^2\right)H_{39}}{2\left[\left(\gamma_{mf}^2+Z_3^2+\beta^2\right)^2-4\gamma_{mf}^2Z_3^2\right]}+\frac{2Z_4\beta H_{38}-\left(\gamma_{mf}^2-Z_4^2+\beta^2\right)H_{39}}{2\left[\left(\gamma_{mf}^2+Z_4^2+\beta^2\right)^2-4\gamma_{mf}^2Z_4^2\right]}$$

$$-\frac{H_{41}}{2\left(\gamma_{mf}^2-Z_7^2\right)}+\frac{H_{41}}{2\left(\gamma_{mf}^2-Z_8^2\right)}-\frac{\left(\gamma_{mf}^2-Z_2^2+1\right)\left(H_{44}+H_{45}\right)}{\left[\left(\gamma_{mf}^2+Z_2^2+1\right)^2-4\gamma_{mf}^2Z_2^2\right]}\Bigg]$$

$$A_{37}=\frac{H_{36}}{2\left(\gamma_{mf}^2-Z_3^2\right)},\ A_{38}=-\frac{H_{36}}{2\left(\gamma_{mf}^2-Z_4^2\right)},\ A_{39}=\frac{H_{37}}{2\left(\gamma_{mf}^2-Z_3^2\right)},\ A_{40}=\frac{H_{37}}{2\left(\gamma_{mf}^2-Z_4^2\right)}$$

$$A_{41}=\frac{H_{40}}{2\left(\gamma_{mf}^2-Z_7^2\right)},\ A_{42}=-\frac{H_{41}}{2\left(\gamma_{mf}^2-Z_8^2\right)},\ A_{43}=\frac{H_{41}}{2\left(\gamma_{mf}^2-Z_7^2\right)},\ A_{44}=\frac{H_{41}}{2\left(\gamma_{mf}^2-Z_8^2\right)}$$

$$A_{45}=\frac{\left(\gamma_{mf}^2-\Omega^2+\beta^2\right)H_{34}-2\Omega\beta H_{35}}{\left[\left(\gamma_{mf}^2+\Omega^2+\beta^2\right)^2-4\gamma_{mf}^2\Omega^2\right]};\ A_{46}=\frac{2\Omega\beta H_{34}+\left(\gamma_{mf}^2-\Omega^2+\beta^2\right)H_{35}}{\left[\left(\gamma_{mf}^2+\Omega^2+\beta^2\right)^2-4\gamma_{mf}^2\Omega^2\right]}$$

$$A_{47}=\frac{\left(\gamma_{mf}^2-Z_3^2+\beta^2\right)H_{38}-2Z_3\beta H_{39}}{2\left[\left(\gamma_{mf}^2+Z_3^2+\beta^2\right)^2-4\gamma_{mf}^2Z_3^2\right]};\ A_{48}=\frac{2Z_3\beta H_{38}+\left(\gamma_{mf}^2-Z_3^2+\beta^2\right)H_{39}}{2\left[\left(\gamma_{mf}^2+Z_3^2+\beta^2\right)^2-4\gamma_{mf}^2Z_3^2\right]}$$

$$A_{49}=-\frac{1}{2}\left(\frac{\left(\gamma_{mf}^2-Z_4^2+\beta^2\right)H_{38}+2Z_4\beta H_{39}}{\left[\left(\gamma_{mf}^2+Z_3^2+\beta^2\right)^2-4\gamma_{mf}^2Z_3^2\right]}\right);\ A_{50}=-\frac{2Z_4\beta H_{38}+\left(\gamma_{mf}^2-Z_4^2+\beta^2\right)H_{39}}{2\left[\left(\gamma_{mf}^2+Z_4^2+\beta^2\right)^2-4\gamma_{mf}^2Z_4^2\right]}$$

$$A_{51}=\frac{H_{43}\left(\gamma_{mf}^2-Z_2^2+1\right)}{\left[\left(\gamma_{mf}^2+Z_2^2+1\right)^2-4\gamma_{mf}^2Z_2^2\right]};\ A_{52}=\frac{H_{42}\left(\gamma_{mf}^2-Z_2^2+1\right)}{\left[\left(\gamma_{mf}^2+Z_2^2+1\right)^2-4\gamma_{mf}^2Z_2^2\right]}$$

$$A_{53}=\frac{2H_{43}Z_2}{\left[\left(\gamma_{mf}^2+Z_2^2+1\right)^2-4\gamma_{mf}^2Z_2^2\right]}\quad\text{and}\quad A_{54}=\frac{2H_{42}Z_2}{\left[\left(\gamma_{mf}^2+Z_2^2+1\right)^2-4\gamma_{mf}^2Z_2^2\right]}\qquad(1.57)$$

Equation (1.56) is then inverted to obtain

$$\bar{Z}(x,t)=\frac{2}{L}\sum_{m=1}^{\infty}\bar{Z}(m,t)\left[\cosh\frac{\lambda_m x}{L}-\cos\frac{\lambda_m x}{L}-\sigma_m\left(\sinh\frac{\lambda_m x}{L}-\sin\frac{\lambda_m x}{L}\right)\right]$$

Thus,

$$U(x,t)=\bar{Z}(x,t)+\left(1-3\left(\frac{x}{L}\right)^2+2\left(\frac{x}{L}\right)^3\right)\sin\Omega t+\left(3\left(\frac{x}{L}\right)^2-2\left(\frac{x}{L}\right)^3\right)e^{-\beta t}\sin\Omega t\qquad(1.58)$$

Equation (1.58) is the transverse response of a Rayleigh beam under the action of a moving mass whose two clamped edges are constrained to undergo displacements which vary with time.

5. Discussion of the Analytical Solution

If the undamped system such as this is studied, it is desirable to examine the response amplitude of the dynamical system which may grow without bound. This is termed resonance when it occurs. Equation (1.36) clearly shows that the **Clamped-Clamped** elastic Rayleigh beams transverse by a moving force will be in state of resonance whenever

$$\alpha_{mf}^2 = Z_3^2 \quad \text{which implies that} \quad \alpha_{mf} = Z_3 \qquad (1.59)$$

and equation ((1.54) shows that the same beam under the action of moving mass experiences resonance effect when

$$\beta_{mf}^2 = Z_3^2 \quad \text{which implies that} \quad \beta_{mf} = Z_3 \qquad (1.60)$$

from equation (1.50)

$$\beta_{mf} = \alpha_{mf}\left[1 - \frac{\lambda}{2}\left(\frac{H_1(m,k)}{\alpha_o(m,k)} - \frac{C^2 H_3(m,k)}{\alpha_o(m,k)\alpha_{mf}^2}\right)\right] \qquad (1.61)$$

This implies,

$$\gamma_{mf}^2 = \frac{\alpha_1^*(m,k)}{\alpha_0^*(m,k)} \qquad (1.62)$$

From equations (1.60) and (1.62), we deduced for the same natural frequency, the critical speed for the system of a Clamped-Clamped elastic beam on an elastic foundation and traversed by a moving force is greater than that traversed by moving mass. Thus, resonance is reached earlier in the moving mass system than in the moving force system.

6. Numerical Calculation and Discussion of the Results

Again, to illustrate the analytical results, the uniform Rayleigh beam of length L=12.192m is considered, the load velocity u = 8.123 and $E = 2.109 \times 10^9 kg/m$ .The values of the foundation moduli K varied between 0 and 400000 and for fixed values of rotatory inertia r=1.The traverse deflections of the uniform Rayleigh beam are calculated and plotted against time for values of rotatory inertia and foundation stiffness K.

Fig.1 displayed the transverse displacement response of clamped-clamped moving force of a uniform Rayleigh beam moving with variable velocities for various values of axial force N and fixed value of foundation moduli K =40000. The graph shows that the response amplitude decreases as the values of the axial force N increases while in fig 2 one observed that deflection profile of the clamped-clamped moving force of a uniform Rayleigh beam moving with variable velocities for various values of foundation moduli K and for fixed value of axial force N=20000. The graph shows that the response amplitude decreases as the values of the foundation moduli K increases. However, fig3 shows the transverse displacement response of clamped-clamped moving mass of a uniform Rayleigh beam moving with variable velocities for various values of axial force N and fixed value of foundation moduli K =40000. From the graph it shows that the response amplitude decreases as the values of the axial force N increases. More so, fig.4 depicts the deflection profile of the clamped-clamped moving mass of a uniform Rayleigh beam moving with variable velocities for various values of foundation moduli K when the axial force is fixed at N=20000. It is clearly seen from the graph that the response amplitude decreases as the values of the foundation K increases. Lastly, fig.5 shows the comparison of the moving force and moving mass clamped-clamped uniform Rayleigh beams moving with variable velocities for fixed value of foundation moduli K and axial force N respectively. however, it shows that the response amplitudes of moving force is lower than that of the moving mass. Consequently, going by this result it confirmed that moving force problem cannot be a good approximation to a moving mass problem rather it is misleading.

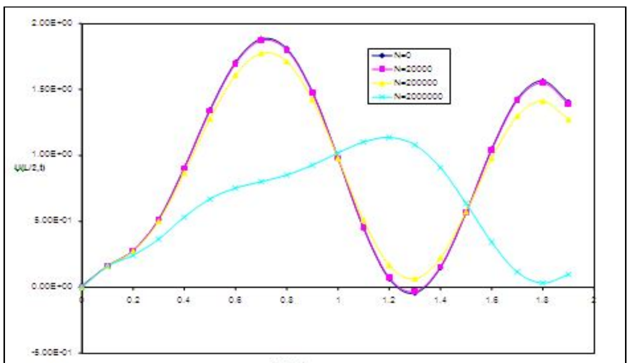

Fig 1: Transverse displacement response of clamped-clamped uniform beam for various Values of axial force N and fixed value of foundation moduli K=40000

Figure 1

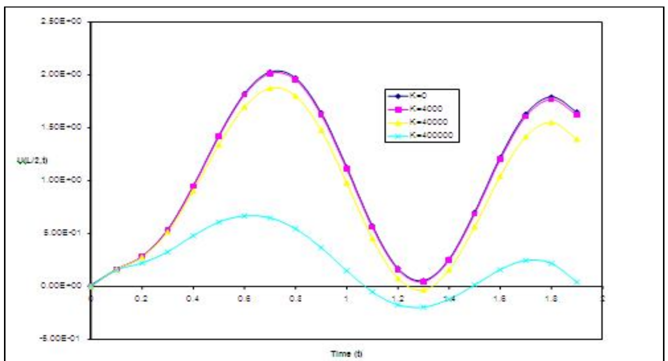

Fig 2: Deflection profile of clamped-clamped uniform beam for various values of foundation moduli K and for fixed value of axial force n=20000

Figure 2

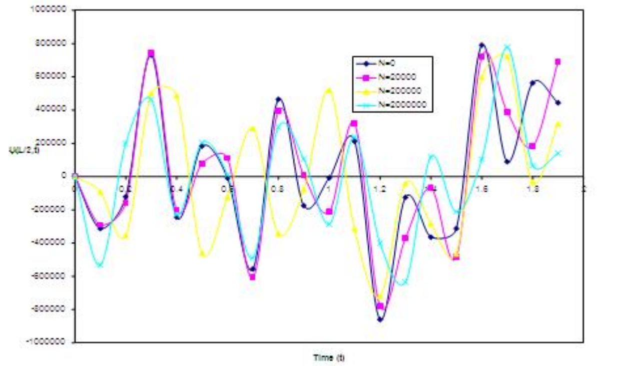

Fig 3: transverse displacement response of clamped-clamped moving mass of uniform beam for various values of axial force N and fixed value of foundation moduli K=40000

Figure 3

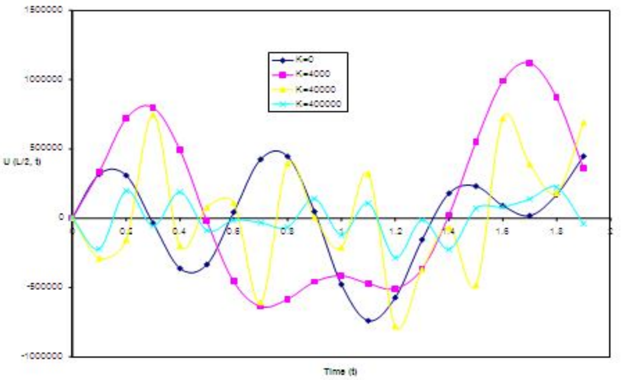

Fig 4: Deflection profile of the clampe-clamped moving mass uniform beam for various Values of foundation moduli K and for fixed value of axial force N=20000

Figure 4

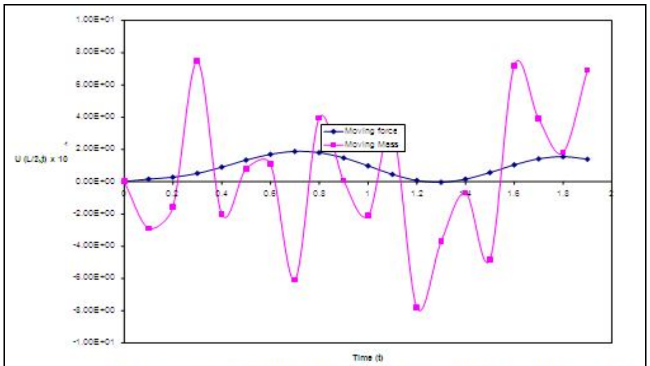

Fig 5: Comparison of the Moving force and Moving mass of the clamped-clamped uniform Beam for fixed values of axial force N and foundation moduli K

Figure 5

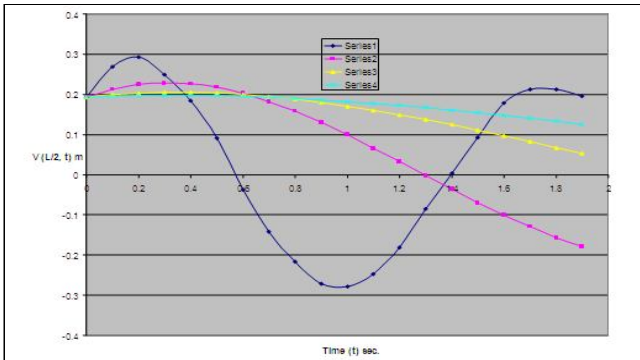

Fig.6: Deflection profile of clamped-clamped uniform Rayleigh beam under the action of moving force for Various values of rotatory inertia and fixed value of foundation modulus k=20000 and fixed value of axial Force N=40000.

Figure 6

fig 6 shows the deflection profile for clamped-clamped uniform Rayleigh beam under the action of moving mass for various values of rotatory inertia and for fixed value of axial force N=200000 and for fixed value of foundation modulus K=200000. it was found out that as the values of roatory inertia increases the deflection profile reduces. Consequently,fig7 depicts that the deflection profile for clamped-clamped uniform Rayleigh beam under the action of moving force for various values of rotatory inertia and for fixed value of axial force N=20000 and for fixed value of foundation modulus K=20000. it was found out that as the values of roatory inertia increases the deflection profile reduces.

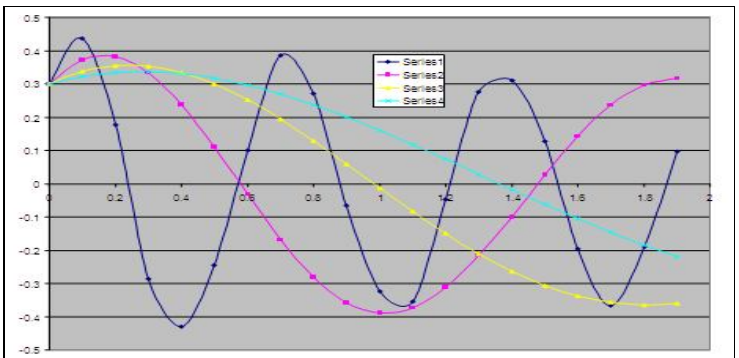

Fig 7: Deflection profile of clamped-clamped uniform Rayleigh beam under the action of moving mass for Various values of rotatory inertia R and for fixed value of foundation modulus K=400000 and for fixed Axial force N=400000.

Figure 7

7. References

1. Oni.S.T and Ajibola.S.O: Dynamical Analysis under moving Concentrated Loads of Rayleigh Beams with Time-Dependent Boundary Conditions: *Journal of Engineering ResearchVol.14, No.4.* (2010)
2. Sadiku S.and Leipholz. H: On the dynamics of elastic systems with moving concentrated masses Ing. Archiv. 57, 223-242. (1981)
3. Oni S.T.: On the dynamic response of elastic structures to moving multi-mass systems Ph.D Thesis,University of Ilorin, Ilorin, Nigeria (1991).
4. Gbadeyan J and Oni S.T.: Dynamic behaviour of beams and rectangular plates under moving loads. Journal of Sound and Vibrations. 182 (5), pp. 677-695 (1995).
5. Oni S.T. and Omolofe B: Dynamic analysis of prestressed elastic beam with general boundary conditions under moving loads traveling at varying velocities. Journal of Engineering and Engineering Tech, FUTA.4 (1):55-74.(2005)
6. Oni S.T.and Awodola T: Vibrations under a moving load of a non-Rayleigh beam on variable elastic foundation, Journal of Nigerian Association of Mathematical Physics. Vol.7, pp 191-206. (2003)
7. Omer Civalek1 and Aitung Yauas2. Large Deflection Static Analysis of Rectangular Plates on two Parameter Elastic Foundations. International Journal of Sciense and Technology. Vol.1. No.1, 43-50. (2006).
8. Oni S.T. and Omolofe B:Dynamic Behaviour of Non-Uniform Bernoulli-Euler Beams Subjected to Concentrated loads travelling at Varying Velocities. Abacus, Journal of Mathematical Association of Nigeria. Vol.32, no2A, 2005.
9. Savin E: Dynamics amplification factor and response spectrum for the evaluation of vibrations of beams under successive moving loads. Journal of Sound and Vibrations (2001). 248(2), 267-288.
10. Stanisic, Euler,and Montgomeny,: On a theory concerning the dynamical behaviour of structures carrying moving masses. Ing. Archiv, 43. pp 295-305, (1974).
11. Oni: S.T. Response of a non-uniform beam resting on an elastic foundation to several moving masses. Abacus, Journal of Mathematical Association of Nigeria. Vol.24, no2, 1996.
12. Jia-Jang Wu: Vibration Analysis of a Portal Frame Under the Action of a Moving Distributed mass using Moving Mass Element. Int. Jor.For Numerical Methods in Engineering, Int. Jor. Num. Meth. Engng. (2006): 62. 2028-2052.
13. Fryba.L: Vibration of solids and structures under moving loads.Groanigen, Noordhoff 1972.
14. Bishop, R.E.D. and Johnson, D.C.: The mechanics of vibration. Cambridge University press, 1960. Reissued with minor revisions 1979
15. Hilderbrand, F.B.: Advanced calculus for applications, Second Edition, Prentice-Hall of India Private Limited. New Delhi 110001, 1977.
16. Jaeger, J.C. and Strarfield, A.M.: An introduction to applied mathematics, Second edition, Oxford University Press, 1974.
17. Clough Ray. W and Penzien, J.: Dynamics of Structures: McGraw-Hill, Inc. 1975.
18. Craig R.R.(Jr): Structural Dynamics : John Willy and Sons, pp. 187-234, 1981.
19. Mindlin, R. D. and Goodman, L. E.: Beam Vibrations with time-dependent boundary conditions, J. Appl. Mech. 17, pp377-380, 1950.
20. Oni, S. T. and Ajibola. S. O. (2011). Motion of moving concentrated loads of simply – supported non – uniform Rayleigh beams with non – classical boundary conditions. 2011. Vol. 18. No 1. JNAMP .
21. Ajibola, S. O. (2011). Transverse Displacement of Clamped – Clamped non – uniform Rayleigh beams under moving concentrated masses resting on a constant Elastic foundation 2011. Vol. 18. No 1. JNAMP
22. B. Omolofe, [2]A. Adedowole, [3]AjibolaS.O and[4]Jimoh Ahmed(2011), . Exact Vibration solution for Initially Stressed beams resting on Elastic Foundation and Subjected to Partially Distributed Masses. Vol. 18. No 1. JNAMP
23. Ajibola.S.O. (2012)Transverse Response To Moving Distributed Loads of Structurally Damped Beam Resting On Elastic Foundation With Exponential Stiffness. Nigerian Journal Of Mathematics and Applications. Vol.21. pp. 27-41