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A Theoretical Study of Proton Lifetimes under GUTs Taking Fourth Generation of Leptons and Quarks into Account

A. K. Bhaskar
Ph.D, Associate Professor, Department of Physics ,College of Commerce, Patna, India

Abstract:
In this paper one most prolific problem – namely, proton lifetimes under SO(10) Grand Unified Theories (GUTs) has been discussed. Proton lifetimes have been calculated in three kinetic models of Kane & Karl[1]: (a) the static model(NR) in which the q^c is taken at rest (p = 0),(b) the recoil model(REC) in which p/m_q~ 3/4, and (c)the relativistic model(R) in which the q^c mass is neglected(m_q = 0). The result has been improved by introducing the contribution of higher order generation of leptons and quarks and proton lifetimes came outof the order of 10^{32} yr.

***Keywords**: Proton lifetimes, Generation problem, SO(10) GUTs*

1. Introduction
The brilliant idea of unified theory of physics was muted by none other than Albert Einstein the greatest scientist of all times. But his dream project was not executed in his lifetime because of his allergic attitude towards quantum logic.
But in the late 1950s a host of physicists throughout the world started working on this ambitious project of unification of physics. Theinitial success of the gauge theories in unifying weak(W) and electromagnetic(EM) interactions, known as Glashow-Weinberg-Salam (GWS) model[2,3,4]and quantum chromodynamics (QCD)[5,6] successful gauge theory of strong interaction(SI) propelled to explore the feasibility of constructing a single gauge theory of all the three interactions – the SI, the WI and EMI.Such unification is known as "Grand Unification". Under grand unified theroies (GUTs), it is the leptoquarks that bring the interaction between the quarks and leptons and their mass is bout 10^{15} GeV, the unification energy.The most extraordinary prediction of GUTs – the instability of the proton (and the bound neutron) violating B-number conservation has broken the myth of proton stability before the advent of GUTs.
Early GUTs such as Geogi-Glashow(GG) model[7], which were the first consistent theories allowing protons to decay via the X bosons and Higgs bosons (H^0) postulated that the proton's half-life would beτ_P> 10^{31}yr. The rate at which these events occur is governed by the mass of leptoquarks and the intermediate H^0 particles and a large volume of material will occassionaly exhibit a spontaneous proton decay. Diagrams which can mediate proton decay via leptoquarks and Higgs bosons are shown in figures 1,2,3.

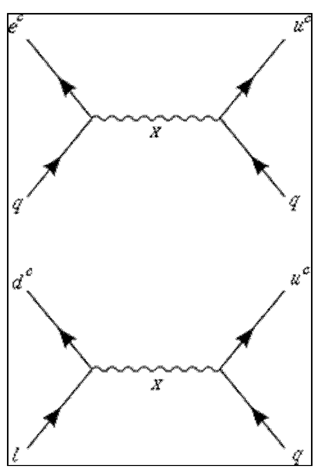

Figure 1: Proton decay mediated by $(3,2)_{-5/6}$ in SU(5) GUT

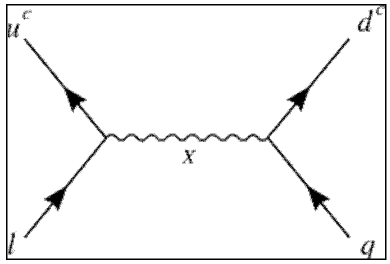

Figure 2: Proton decay mediated by $(3,2)_{1/6}$ in SU(5) GUT

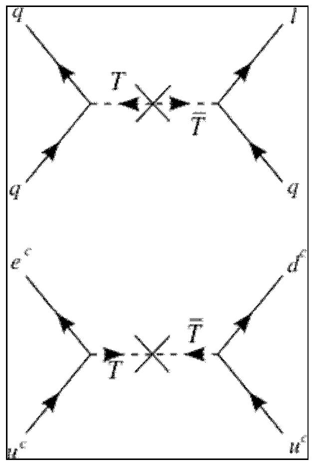

Figure 3: Proton decay mediated by the triplet Higgs $T(3, 1)_{-1/3}$ and anti-triplet $\bar{T}(\bar{3},1)_{1/3}$ in SU(5) GUT

Recent experiments at the Super-Kamiokande water cerenkov radiation detected in Japan gave lower limits for proton half-life at 90% confidence level of 6.6×10^{33}yr via anti-muon decay and 8.2×10^{33}yr via positron decay[8].

2. Proton Lifetimes

There has been a lot of discussion of the gauge models based on SU(5) and SO(10) chosen as the grand unified gauge. Proton lifetimes have been calculated in the SO(10) GUT which accommodate SU(5) sub-group by including the form-factor effects due to the finite sizes of the nucleons and mesons, and the generation mixing based on the assumption that $\Delta B = -1$ decay interactions is generated by the exchange of superheavyleptoquark gauge bosons, D[X(4/3,1/3), Y(1/3,1/3)] and E[X'(2/3,1/3), Y'(-1/3,1/3)] and their anti-particles:

$SO(10) \rightarrow SU(5) \rightarrow SU^C(3) \times SU(2) \times U(1) \rightarrow SU^C(3) \times U(1)_{EM}$

Several experimental groups obtained the proton lifetime limit $\tau_P > 10^{31}$ yr. Present study has been based particularly on those of Langacker[9], Ellis et al[10], Matsuki et al[11] and Hara[12]. The mass of the leptoquark M_D depends on leptoquark mass ratio $R = M_E/M_D$ and QCD parameter Λ_{MS} is given by

$M_D = (1.86-0.167R^{-2})\,[\Lambda_{MS}/0.4(\text{GeV})]^{1.06} \times 10^{14}$GeV

for $0.05 < \Lambda_{MS} < 1.00$ (GeV) and $1.5 < R < 10$. The higher value of Λ_{MS} = 0.4-0.5 GeV is attributed to two generations of fermions. For three or more generations of fermions the present world value of Λ_{MS} = 0.08-0.26 GeV is only suitable. The results for Λ_{MS} = 0.08 - 0.26 GeV are shown in Table 2.

The lifetimes of the protons due to two body decays in three kinetic models for Λ_{MS} =0.08GeV, 0.16GeV, 0.26 GeV and for various R.

$\tau_P(\times10^{30}$yr)										
Kinetic models	**R / Λ_{MS}**	**1/10**	**¼**	**½**	**2/3**	**1**	**3/2**	**2**	**4**	**10**
Static	0.08	7.0×10^{-7}	6.6×10^{-6}	2.1×10^{-4}	6.8×10^{-4}	2.1×10^{-3}	3.7×10^{-3}	4.6×10^{-3}	6.1×10^{-3}	6.7×10^{-3}
	0.16	5.1×10^{-7}	1.3×10^{-4}	3.9×10^{-3}	1.3×10^{-2}	3.9×10^{-2}	0.07	0.09	0.11	0.13
	0.26	4.0×10^{-6}	9.8×10^{-4}	3.1×10^{-2}	0.10	0.31	0.55	0.68	0.90	1.0
Recoil	0.08	1.2×10^{-8}	2.8×10^{-6}	8.9×10^{-5}	3.1×10^{-4}	9.7×10^{-4}	1.8×10^{-3}	2.3×10^{-3}	3.0×10^{-3}	3.5×10^{-3}
	0.16	2.3×10^{-7}	5.3×10^{-5}	1.7×10^{-3}	6.0×10^{-3}	1.8×10^{-2}	3.5×10^{-2}	4.3×10^{-2}	5.7×10^{-2}	6.6×10^{-2}
	0.26	1.8×10^{-6}	4.2×10^{-4}	1.3×10^{-2}	4.7×10^{-2}	0.14	0.27	0.34	0.45	0.52
Relativistic	0.08	6.5×10^{-9}	1.6×10^{-6}	5.0×10^{-5}	1.7×10^{-4}	5.7×10^{-4}	1.1×10^{-3}	1.4×10^{-3}	1.8×10^{-3}	2.1×10^{-3}
	0.16	1.2×10^{-7}	3.1×10^{-5}	9.4×10^{-4}	3.3×10^{-3}	1.1×10^{-2}	2.1×10^{-2}	2.7×10^{-2}	3.5×10^{-2}	3.9×10^{-2}
	0.26	9.7×10^{-7}	2.4×10^{-4}	7.4×10^{-3}	2.6×10^{-2}	8.4×10^{-2}	0.16	0.21	0.27	0.31

Table 2

By making use of the relation M_D α $[\Lambda_{MS}]^{1.06}$,in three kinetic models for Λ_{MS} = 0.08-0.26 GeV, the proton lifetimes are calculated to be very much shorter than the experimental lower limit, $\boldsymbol{\tau_P} > 10^{31}$ yr. This lower limit was adopted at the ICOBAN 84 (International Colloquium on Baryon Non-Conservation) held at Park City, Utah.

The theoretical uncertainty in the proton lifetimes is mostly due to the uncertainty of $|\varphi\ (N)|^2$. If we derive the magnitude of $|\varphi\ (N)|^2$ from the S-wave amplitudes in place of P-wave hyperon non-leptonic decay amplitude proton lifetimes increase by 50%.Thus we have an additional multiplication factor of 2 in the lifetimes. $|\varphi\ (N)|^2$ has other theoretical uncertainties, the uncertainty due to possible higher order effects, and the possible effect of varying the masses of superheavy Higgs particles M_H , the uncertaintyin the precise value of $\alpha_{em}(2M_w)$, etc. These effects give M_D an uncertainty of a factor of about $2^{\pm}$. Thus, if we take only 2^{+}uncertainty in M_D, then lifetimes again getenhanced by a factor of 2^4=16.

The renormalization effects in SO(10) model permit a fourth generation (t', b', τ', $\nu_{\tau'}$) of leptons and quarks. From ongoing experiments at PETRA, it is presumably reasonably to assume that $m_{\tau'} \approx$ 17–35GeV, $m_{b'} \approx$ 42–188GeV and $m_{t'} \approx$ 150-230 GeV. If the fourth generation fermions are included in the theory, the estimate of M_D increases by a factor of 1.4.

Hence, taking theoretical uncertainties into account and introducing a fourth generation of light fermions in theory, proton lifetimesincrease by a factor of about $2x2^4x1.4^4$=123.Even this increase in proton lifetime does not either permit Λ_{MS} =0.08GeV or Λ_{MS}=0.16GeV. The proton lifetimes become shorter for $M_E< M_D$ than the experimental lower bound. Hence conclude that these cases are unlikely to be realized in our model. On the other hand the proton lifetimes predicted for $M_E \geq M_D$ and Λ_{MS}=0.26GeV only seems reasonable. In this case $\boldsymbol{\tau_P}$~ (1-12) x 10^{31}yr and the dominant decay modes are $e^+\pi^0$, $\nu_e\pi^+$, $e^+\omega$. The results for Λ_{MS} = 0.26 GeV are shown in Table 3.

$\tau_P(\times10^{31}yr)$									
R=M_E/M_D	**1/10**	**¼**	**½**	**2/3**	**1**	**3/2**	**2**	**4**	**10**
$M_D(\times10^{14}GeV)$	1.5	2.4	3.1	3.5	3.9	4.2	4.3	4.5	4.6
Static	4.9×10^{-5}	1.2×10^{-2}	0.38	1.2	3.8	6.8	8.4	11.1	12.3
Recoil	2.2×10^{-5}	5.2×10^{-3}	0.16	0.58	1.7	3.3	4.2	5.5	6.4
Relativistic	1.2×10^{-5}	3.0×10^{-3}	0.09	0.32	1.0	2.0	2.6	3.3	3.8

Table 3: The lifetimes of the protons due to two body decays in three kinetic models forΛ_{MS} = 0.26 GeV and for various R

3. Conclusion

If latest experimental results are any indication then the proton lifetimes must be greater than 10^{31}yr and the dominant decay mode is not $e^+\pi^0$. In the present study of proton lifetimes under GUTs, we have a free parameter R = M_E/M_D which can be determined by the experiment. Most of the authors have assumed Λ_{MS} = 0.4 GeV. But we have assigned the present
"world Λ_{MS} = 0.26 GeV" and introduction of *fourth generation* of light fermions into the theory has increased proton lifetimes by 123 times and taking uncertainty in M_D also lifts the value by 16 times thus giving$\boldsymbol{\tau_P}$~ (1-12) x 10^{31}yr which is close to the observed values.

4. References

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