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Dynamic Response of Double Rayleigh Uniform Beams Systems Clamped at Both Ends under Moving Concentrated Loads with Classical Boundry Condition

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Abstract:
The dynamic response behavior of double uniform Rayleigh beams system resting on a constant elastic foundation and with axial force traversed by masses travelling at a uniform velocity is investigated. As alluded to in [3, 4] a clamped-clamped boundary conditions for moving force was considered. The dynamical problem is solved using Mindlin Goodman, Generalized Finite Integral Fourier, and Laplace Integral transformations then convolutiontheory. Using numerical example, various plots of the deflections for the beams are presented and discussed for different values of axial force N, foundation modulli K and at fixed rotatory Inertial (r) and also for fixed axial force N and foundation moduli K but at various rotatory inertial (r) for moving force.

Keywords*: Double uniform Rayleigh Beam, Moving Force, Critical Speed, Velocity, Time-Dependent and Resonance*

1. Introduction

This research work is concerned with the calculation of the dynamic response of structural members carrying one or more traveling loads which is very important in Engineering and Applied Mathematics as applications relate, for example, to the analysis and design of highway and railway bridges, cable- railways and the like. Generally, emphasis is placed on the dynamics of the structural members rather than on that of the moving loads: moving mass and moving force models. Common examples of structural members include beams, plates, and shells while traveling loads include moving trains, trucks, cars, bicycles, cranes etc. A structural member may be elastic, inelastic or viscoelastic. As such we have elastic structural members, inelastic structural members and viscoelastic structural configurations on which one or more loads may travel. Simple examples of these structural members are bridges, railroads, rails, decking slab, elevated roadways to moving vehicles, girders, belt-drive (carrying machine chains) and even floppy disks/cassette players' heads carrying tape. Pertinent to investigation in the field is the response of an elastic structure under the cases of moving concentrated loads with time dependent boundary conditions.

Several other researchers have made tremendous feat in the study of dynamics of structures under moving loads. In all of these, considerations have been limited to cases involving homogeneous boundary conditions and no considerations have been given to the class of dynamical problems in which the boundaries are constrained to undergo displacements or tractions which vary with time. In such cases boundary conditions are no longer homogeneous and boundary conditions become non-classical.

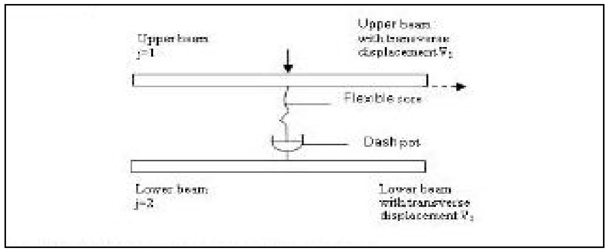
In many practical problems that concern the structural response to moving loads of elastic systems, the supports at the boundaries are not stationary but undergo different motions. Often the motions are in the form of lateral displacement, oscillations or tractions. As such, the boundary conditions are not homogeneous but are time dependent. These classes of non-classical boundary value problems are, in general, resistant to the classical methods of solving dynamical problems. In fact, it becomes more cumbersome, when the dynamical problems involve moving loads with or without consideration of the inertial effect of the moving loads is taken into consideration.

One of the earliest problems of this type was considered by Mindlin and Goodman [14] who described a procedure for extending the method of separation of variables to the solution of Bernoulli – Euler beam vibration problems with time-dependent boundary conditions

Thus, this study concerns the response of Rayleigh beams when it is under the actions of moving concentrated masses. Typical examples of time-dependent boundary conditions are used to illustrate the dynamical configurations. The solution technique employed is based on Mindlin and Goodman, Generalized Finite Integral Fourier, and Laplace Integral transformations then convolution theory. Finally, the analysis is illustrated by numerical examples.

2. Governing Equation

The structural model of an elastically connected double Rayleigh beam system under the action of a moving concentrated load $P(x,t)$ is considered. The transverse displacement $U_j(x,t),\ j=1,2$, of double uniform Rayleigh beam of Length L traversed by mass M traveling at a uniform velocity u, is governed by the fourth order partial differential equations.



$$\frac{EI\,\partial^4 U_1(x,t)}{\partial x^4}-\frac{N\partial^2 U_1(x,t)}{\partial x^2}+\frac{\mu\partial^2 U_1(x,t)}{\partial t^2}+KU_1(x,t)-\mu r^2\frac{\partial^4 U_1(x,t)}{\partial x^2\partial t^2}=P(x,t)$$

and

$$\frac{EI\,\partial^4 U_2(x,t)}{\partial x^4}-\frac{N\partial^2 U_2(x,t)}{\partial x^2}+\frac{\mu\partial^2 U_2(x,t)}{\partial t^2}+KU_2(x,t)-\mu r^2\frac{\partial^4 U_2(x,t)}{\partial x^2\partial t^2}=0 \qquad (1.00)$$

where

x is the spatial co-ordinate, t is the time, $U_j(x,t)$ is the transverse displacement, E is the Young Modulus, I is the moment of inertial, μ is the mass per unit length of the beam, r is the radius of gyration, N is the axial force, K is the elastic foundation as EI is the flexural rigidity of the beam. For the problem under consideration, the moving load has mass that is commensurable with the mass of the beam. Consequently, the load inertia is not negligible but significantly affects the behavior of the dynamical system. In this case, load function $P(x,t)$ takes the form.

$$P(x,t)=P_f(x,t)\left[1-\frac{1}{g}\frac{d^2U(x,t)}{dt^2}\right] \qquad (1.01)$$

Where the continuous moving force $P_f(x,t)$ acting on the beam model is given by

$$P_f(x,t)=Mg\delta(x-f(t)) \qquad (1.02)$$

And $\frac{d^2}{dt^2}$ is a convective acceleration operator defined as becomes

$$\frac{d^2}{dt^2}=\frac{\partial^2}{\partial t^2}+2\frac{df(t)}{dt}\frac{\partial^2}{\partial x\partial t}+\left(\frac{df(t)}{dt}\right)^2\frac{\partial^2}{\partial x^2}+\frac{d^2f(t)}{dt^2}\frac{\partial}{\partial x} \qquad (1.03)$$

In this work, the moving load is assumed to move with constant speed, consequently, equation (1.03) becomes.

$$\frac{d^2}{dt^2}=\frac{\partial^2}{\partial t^2}+\frac{2u\partial^2}{\partial x\partial t}+\frac{u^2\partial^2}{\partial x^2} \qquad (1.04)$$

Now, on substituting equations (1.01), (1.02) and (1.04) into (1.00) and assuming that the flexural rigidity EI, and mass per unit length μ, do not vary with position x along the span L, equation (1.00) becomes.

$$EI\frac{\partial^4 U_1(x,t)}{\partial x^4}-\frac{N\partial^2 U_1(x,t)}{\partial x^4}+\frac{\mu\,\partial^2 U_1(x,t)}{\partial t^2}+KU_1(x,t)-\mu r^2\frac{\partial^4 U_1(x,t)}{\partial x^2\partial t^2}$$

$$=Mg\delta(x-ut)\left[1-\frac{1}{g}\left(\frac{\partial^2 U_1(x,t)}{\partial t^2}+\frac{2u\partial^2 U_1(x,t)}{\partial x\partial t}+\frac{u^2\partial^2 U_1(x,t)}{\partial x^2}\right)\right] \qquad (1.05)$$

and

$$\frac{EI\,\partial^4 U_2(x,t)}{\partial x^4}-\frac{N\partial^2 U_2(x,t)}{\partial x^2}+\frac{\mu\partial^2 U_2(x,t)}{\partial t^2}+KU_2(x,t)-\mu r^2\frac{\partial^4 U_2(x,t)}{\partial x^2\partial t^2}=0 \qquad (1.06)$$

Equation(1.05) is for the upper beam while (1.06) is for the lower beam and the boundary conditions of these problems are taken to be time dependent. Thus, at each of the boundary points, there are two boundary conditions written as;

$$D_i\left[U(0,t)\right] = F_i(t) \qquad i = 1,\ 2 \text{ and } D_i\left[U(L,t)\right] = F_i(t) \qquad i = 3,\ 4 \tag{1.07}$$

Where D_i 's are linear homogeneous differential operators of order less than or equal to three.

The initial conditions of the motion at time $t = 0$ may in general be specified by two arbitrary functions thus:

$$U(x,0) = U_0(x) \qquad \text{and } \frac{\partial U(x,0)}{\partial t} = \dot{U}_0(x) \tag{1.08}$$

2.1. Operational Simplification of Equation

In this work, the analytical solution to the non-homogeneous initial boundary value problems (1.00) with non-homogeneous boundary conditions (1.07) and non-homogenous initial conditions (1.08) is sought. To this end, an approach due to Mindlin and Goodman [19] is extended to obtain a robust technique which is capable of solving this class of problems for all variants of support conditions.

First, an auxiliary variable $z(x,t)$ in the form

$$U_j(x,t) = Z_j(x,t) + \sum_{i=1}^{4} f_i(t) g_i(x),\ j = 1,2 \tag{1.09}$$

is introduced. Now, substituting equation (1.09) into (1.05)and (1.06) transforms the boundary-value-problem in terms of $U_j(x,t)$ into the boundary value problem in terms of $Z_j(x,t)$. The functions $g_i(x)$ are called the displacement influence functions while $f_i(t)$ are the pertinent displacements at the respective boundaries. The functions $g_i(x)$ are to be chosen so as to render the boundary conditions for the boundary value problems in $Z_j(x,t)$ homogeneous.

Thus, substituting (1.09) into equation (1.05) the upper beam, one obtains

$$\begin{aligned}
&\frac{EI}{\mu}\frac{\partial^4}{\partial x^4} Z_1(x,t) - \frac{N}{\mu}\frac{\partial^2}{\partial x^2} Z_1(x,t) + \frac{\partial^2}{\partial t^2} Z_1(x,t) + \frac{K}{\mu} Z_1(x,t) \\
&- \frac{r^2\partial^4}{\partial x^2 \partial t^2} Z_1(x,t) + \frac{M}{\mu}\delta(x-ut)\left[\frac{\partial^2}{\partial t^2} Z_1(x,t) + \frac{2u\partial^2}{\partial x \partial t} Z_1(x,t) + \frac{u^2\partial^2}{\partial x^2} Z_1(x,t)\right] \\
&= \frac{M}{\mu} g\delta(x-ut) - \frac{EI}{\mu}\sum_{i=1}^{4} f_i(t) g_i''''(x) + \frac{N}{\mu}\sum_{i=1}^{4} f_i(t) g_i(x) \\
&- \sum_{i=1}^{4} \ddot{f}_i(t) g_i(x) - \frac{K}{\mu}\sum_{i=1}^{4} f_i(t) g_i(x) + r^2 \sum_{i=1}^{4} f_i''(t) g_i''(x) \\
&+ \frac{M}{\mu}\delta(x-ut)\left[\sum_{i=1}^{4}\left(\ddot{f}_i(x) g_i(x) + 2u\dot{f}_i(x) g_i'(x) + u^2 f_i(t) g_i''(x)\right)\right]
\end{aligned} \tag{1.10a}$$

and substituting (1.09) into equation (1.07) the lower beam, one obtains

$$\begin{aligned}
&\frac{EI}{\mu}\frac{\partial^4}{\partial x^4} Z_2(x,t) - \frac{N}{\mu}\frac{\partial^2}{\partial x^2} Z_2(x,t) + \frac{\partial^2}{\partial t^2} Z_2(x,t) + \frac{K}{\mu} Z_2(x,t) - \frac{r^2\partial^4}{\partial x^2 \partial t^2} Z_2(x,t) \\
&+ \sum_{i=1}^{4} \ddot{f}_i(t) g_i(x) - \frac{K}{\mu}\sum_{i=1}^{4} f_i(t) g_i(x) + r^2 \sum_{i=1}^{4} f_i''(t) g_i''(x) = 0
\end{aligned} \tag{1.10b}$$

Where dot $(\cdot)$ represents the derivative with respect to time, while slash $(')$ represents the derivative with respect to space coordinate.

Now the expression in equation (1.09) must satisfy the boundary conditions in equation (1.07); consequently, we have

$$D_i\left[Z(o,t)\right] + \sum_{j=1}^{4} f_i(t) D_i\left[g_i(o)\right] = f_i(t), \qquad i = 1,2. \tag{1.11}$$

$$D_i\left[Z(L,t)\right] + \sum_{j=1}^{4} f_i(t) D_i\left[g_i(L)\right] = f_i(t), \qquad i = 3,4. \tag{1.12}$$

Substituting equation (1.09) into the initial equation (1.07) and (1.08) one obtains.

$$Z(x,o) = U(x,o) - \sum_{i=1}^{4} f_i(o) g_i(x) \tag{1.13}$$

$$\frac{\partial}{\partial t} z(x,o) = \dot{U}_0(x) - \sum_{i=1}^{4} \dot{f}_i(o) g_i(x) \tag{1.14}$$

2.2. Solution Procedure

For slip damping to take place, both the upper and the lower beams must retain physical contact along the interface so as to remain as one structure. Thus, $Z_1(x,t) = Z_2(x,t) = Z(x,t)$ on adding the upper and the lower beams together, we have

$$2\left(\frac{EI}{\mu} \frac{\partial^4}{\partial x^4} Z(x,t) - \frac{N}{\mu} \frac{\partial^2}{\partial x^2} Z(x,t) + \frac{\partial^2}{\partial t^2} Z(x,t) + \frac{K}{\mu} Z(x,t) - \frac{r^2 \partial^4}{\partial x^2 \partial t^2} Z(x,t) \right)$$
$$+ \frac{M}{\mu} \delta(x-ut) \left[\frac{\partial^2}{\partial t^2} Z(x,t) + \frac{2u\partial^2}{\partial x \partial t} Z(x,t) + \frac{u^2 \partial^2}{\partial x^2} Z(x,t) \right]$$
$$= \frac{M}{\mu} g\delta(x-ut) - \frac{2EI}{\mu} \sum_{i=1}^{4} f_i(t) g_i''(x) + \frac{2N}{\mu} \sum_{i=1}^{4} f_i(t) g_i(x)$$
$$2\left(-\sum_{i=1}^{4} \ddot{f}_i(t) g_i(x) - \frac{K}{\mu} \sum_{i=1}^{4} f_i(t) g_i(x) + r^2 \sum_{i=1}^{4} f_i''(t) g_i''(x) \right)$$
$$+ \frac{M}{\mu} \delta(x-ut) \left[\sum_{i=1}^{4} \left(\ddot{f}_i(x) g_i(x) + 2u\dot{f}_i(x) g_i'(x) + u^2 f_i(t) g_i''(x) \right) \right] \quad 1.10c$$

It is observed that the initial – boundary – value problem in equation (1.10c) is a fourth order partial differential equation having some coefficients which are not only variable but are also singular. These coefficients are the Dirac delta functions which multiply each term of the convective acceleration operator associated with the inertia of the mass of the moving load. It is remarked at this juncture that this transformed equation is now amenable to the method of generalized finite integral transform used extensively in S.T. Oni [1,2,5,6,11,14].

2.3. The Generalized Finite Integral Transform Method

The generalized finite integral transform method is one of the best methods used in handling problems involving mechanical vibrations. This integral transform method is given by

$$\bar{z}(m,t) = \int_o^l z(x,t) V_m(x) dx \tag{1.15}$$

With the inverse

$$z(x,t) = \sum_{m=1}^{\infty} \frac{\mu}{V_m} \bar{z}(m,t) V_m(x) \tag{1.16}$$

Where

$$\bar{V}_m = \int_o^l \mu V_m^2(x) dx \tag{1.17}$$

$V(x,t)$, is any function such that the pertinent boundary conditions are satisfied. An appropriate selection of functions for beam problems are beam mode shape. Thus the m^{th} normal mode of vibrations of a uniform beam given by

$$V_m(x) = Sin\frac{\lambda_m x}{L} + A_m Cos\frac{\lambda_m x}{L} + B_m Sinh\frac{\lambda_m x}{L} + C_m Cosh\frac{\lambda_m x}{L} \tag{1.18}$$

is chosen as a suitable kernel of the integral (1.15) where λ_m is the mode frequency, A_m, B_m and C_m are constant. An important feature of the use of this kernel is that it makes the transformation suitable for all variants of the boundary conditions of the dynamical problems. The parameter λ_m, A_m, B_m and C_m are obtained when the equation (1.18) is substituted into the appropriate boundary conditions.

By applying the generalized finite integral transform (1.15), equation (1.10c) takes the form

$$\bar{Z}_{tt}(m,t) = B_1 Q_A(t) + B_2 Q_B(t) + B_3 Z(m,t) + B_1 Z(0,L,t) - r^2 Q_C(t) + Q_D(t) + Q_E(t) + Q_F(t)$$
$$PV_m(Ut) - \left[G_a(t) - G_b(t) + G_c(t) + G_d(t) + G_e(t) + G_f(t) + G_g(t) + G_h(t) \right] \tag{1.19}$$

where

$$B_1 = \frac{2EI}{\mu},\ B_2 = \frac{2N}{\mu},\ B_3 = \frac{2K}{\mu},\ P = \frac{mg}{\mu},\ \text{and}\ \varepsilon = \frac{M}{\mu L} \tag{1.20}$$

$$Q_A(t)=\int_o^l \frac{\partial^4}{\partial x^4}Z(x,t)V_m(x)dx \ , \quad Q_B(t)=\int_o^l \frac{\partial^2}{\partial x^2}Z(x,t)V_m(x)dx$$

$$Q_C(t)=\int_o^l \frac{\partial^4}{\partial x^2\partial t^2}Z(x,t)V_m(x)dx \ , \quad Q_D(t)=\int_o^l \frac{M}{\mu}\delta(x-ut)\frac{\partial^2}{\partial t^2}Z(x,t)V_m(x)dx$$

$$Q_E(t)=\int_o^l \frac{2MU}{\mu}\delta(x-ut)\frac{\partial^2}{\partial x\partial t}Z(x,t)V_m(x)dx, \qquad (1.21)$$

$$Z(0,L,t)=\left[V_m(x)\frac{\partial^3}{\partial x^3}Z(x,t)-V_m'(x)\frac{\partial^2}{\partial x^2}Z(x,t)+V_m''(x)\frac{\partial}{\partial x}Z(x,t)-V_m'''(x)Z(x,t)\right]_0^L$$
(1.22)

$$G_a(t)=B_1\sum_{i=1}^{4}f_i(t)\int_0^L\left(\frac{d^4}{dx^4}g_i(x)\right)V_m(x)dx,\ G_b(t)=B_2\sum_{i=1}^{4}f_i(t)\int_0^L\left(\frac{d^2}{dx^2}g_i(x)\right)V_m(x)dx$$

$$G_c(t)=2\sum_{i=1}^{4}\ddot{f}_i(t)\int_0^L g_i(x)V_m(x)dx \ , \quad G_d(t)=B_3\sum_{i=1}^{4}\ddot{f}_i(t)\int_0^L g_i(x)V_m(x)dx$$

$$G_e(t)=2r^2\sum_{i=1}^{4}\ddot{f}_i(t)\int_0^L\left(\frac{d^2}{dx^2}g_i(x)\right)V_m(x)dx, G_f(t)=\frac{M}{\mu}\sum_{i=1}^{4}\ddot{f}_i(t)\int_0^L\delta(x-ut)g_i(x)V_m(x)dx$$

$$G_g(t)=\frac{2MU}{\mu}\sum_{i=1}^{4}\dot{f}_i(t)\int_0^L\delta(x-ut)g_i'(x)V_m(x)dx\ G_h(t)=\frac{MU^2}{\mu}\sum_{i=1}^{4}f_i(t)\int_0^L\delta(x-ut)g_i''(x)V_m(x)dx$$
(1.23)

It is well know that the natural mode in Equation (1.18) satisfies the homogeneous differential equation

$$EI\frac{d^4}{dx^4}V_m(x)-\mu\omega_m^2V_m(x)=0 \qquad (1.24)$$

for the Euler beam. The parameter (ω) is the natural circular frequency defined by

$$\omega_m^2=\frac{\lambda^4}{L^4}\frac{EI}{\mu} \qquad (1.25)$$

Equation (1.24) implies

$$EI\int_o^L\left(\frac{d^4}{dx^4}V_m(x)\right)Z(x,t)dx=\mu\omega_m^2\int_0^L V_m(x)Z(x,t)dx \qquad (1.26)$$

Thus, by (1.15)

$$Q_A(t)=\frac{\mu}{EI}\omega_m^2\overline{Z}(m,t) \qquad (1.27)$$

Since

$\overline{Z}(m,t)$ is just the coefficient of the generalized finite integral transform, equation (1.16) yields

$$Z(x,t)=\sum_{k=0}^{\infty}\frac{\mu}{V_k}\overline{Z}(k,t)V_k(x) \qquad (1.28)$$

Thus $$\frac{\partial^2}{\partial x^2}Z_{tt}(x,t)=\sum_{k=1}^{\infty}\frac{\mu}{V_k}\overline{Z}(k,t)\frac{d^2}{dx^2}V_k(x) \qquad (1.29)$$

And the integral (1.21) can be written as

$$Q_c(t)=\sum_{k=1}^{\infty}\frac{\mu}{V_k}\overline{Z}_{tt}(x,t)\int_0^L\left(\frac{d^2}{dx^2}V_k(x)\right)V_m(x)dx \qquad (1.30)$$

Now using the property of Dirac-Delta function as an even function, which can be expressed in Fourier cosine series namely

$$\delta(x-\mu t)=\frac{1}{L}+\frac{2}{L}\sum_{n=1}^{\infty}\cos\frac{n\pi\mu t}{L}\cos\frac{n\pi x}{L} \qquad (1.31)$$

When use is made of equations (1.28) to (1.31), one obtains

$$Q_d(t) = \frac{M}{\mu L}\sum_{k=1}^{\infty}\frac{\mu}{V_k}\overline{Z}_{tt}(k,t)\left[\int_0^L V_k(x)V_m(x)dx + 2\sum_{n=1}^{\infty} Cos\frac{n\pi ut}{L}\int_0^L Cos\frac{n\pi x}{L}V_k(x)V_m(x)dx\right] \quad (1.32)$$

Using similar argument in equations (1.21). It is straight forward to show that

$$Q_e(t) = \frac{2MU}{\mu L}\sum_{k=1}^{\infty}\overline{Z}_t(k,t)\left[\frac{\mu}{V_k}\int_0^L V_k'(x)V_m(x)dx + 2\sum_{n=1}^{\infty} Cos\frac{n\pi ut}{L}\frac{\mu}{V_k}\int_0^L Cos\frac{n\pi x}{L}V_k'(x)V_m(x)dx\right] \text{ and}$$

(1.33)

$$Q_f(t) = \frac{MU^2}{\mu L}\sum_{k=1}^{\infty}\overline{Z}(k,t)\left[\frac{\mu}{V_k}\int_0^L V_k(x)V_m(x)dx + 2\sum_{n=1}^{\infty} Cos\frac{n\pi ut}{L}\frac{\mu}{V_k}\int_0^L Cos\frac{n\pi x}{L}V_k(x)V_m(x)dx\right] (1.34)$$

Substituting equations (1.27) to (134), into (1.19), after simplifications and arrangements yields

$$\overline{Z}_{tt}(m,t) + \alpha_m^2\overline{Z}_t(m,t) - \frac{N}{\mu}\sum_{k=1}^{\infty}\overline{Z}(k,t)S_1^*(k,m) - r^2\sum_{k=1}^{\infty}\overline{Z}_{tt}(k,t)S_1^*(k,m) + \varepsilon\left[\sum_{k=1}^{\infty}\overline{Z}_{tt}(k,t)S_2^*(k,m)\right.$$

$$+2\sum_{k=1}^{\infty}\sum_{n=1}^{\infty} Cos\frac{n\pi ut}{L}\overline{Z}_{tt}(k,t)S_{2c}^*(k,m,n) + 2u\sum_{k=1}^{\infty}\overline{Z}(k,t)S_3^*(k,m)$$

$$+4u\sum_{k=1}^{\infty}\sum_{n=1}^{\infty} Cos\frac{n\pi ut}{L}\overline{Z}_t(k,t)S_{3c}^*(k,m,n) + u^2\sum_{k=1}^{\infty}\overline{Z}(k,t)S_1^*(k,m)$$

$$+2u^2\sum_{k=1}^{\infty}\sum_{n=1}^{\infty} Cos\frac{n\pi ut}{L}\overline{Z}_t(k,t)S_{1c}^*(k,m,n)$$

$$= P\left[Sin\frac{\lambda_m ut}{L} + A_m Cos\frac{\lambda_m ut}{L} + B_m Sinh\frac{\lambda_m ut}{L} + C_m Cosh\frac{\lambda_m ut}{L}\right]$$

$$-\left[G_a(t) - G_b(t) + G_c(t) + G_d(t) - G_e(t) + G_f(t) + G_g(t) + G_h(t)\right] \quad (1.35)$$

where G_a, G_b, G_c............................G_h are as defined in equations (1.23) ,

$$\alpha_m^2 = \left(\omega_m^2 + \frac{k}{\mu}\right) \quad (1.36)$$

First, we shall obtain the particular functions $g_i(x)$, where $i = 1,2,3,4.$ which ensure zeros of the right hand sides of the boundary conditions for a clamped-clamped beam. Going through the same process discussed in [1,3,6], one obtains

$$g_1(x) = 1 - 3\left(\frac{x}{L}\right)^2 + 2\left(\frac{x}{L}\right)^3, g_2(x) = x - \frac{x^2}{L}, g_3(x) = 3\left(\frac{x}{L}\right)^2 - 2\left(\frac{x}{L}\right)^3, g_4(x) = -\frac{x^2}{L} + \frac{x^3}{L^2} \quad (1.37)$$

It is only necessary to compute those of the $g_i(x)$ for which the corresponding $f_i(t)$ do not vanish. Thus, we need only $g_1(x)$ and $g_3(x)$ for our boundary displacement functions $f_1(t)$ and $f_3(t)$ as defined in [1,2,3,4].

Thus we can write

$$f_1 = BSin\Omega t \text{ and } f_3 = Ae^{-\beta t}Sin\Omega t \quad (1.41)$$

Where A, B are amplitudes, Ω is frequency and β is parameter.

The initial conditions are, again

$$\overline{Z}(x,0) = 0 \text{ and } \overline{Z}_t(x,0) = -\Omega \quad (1.42)$$

which when transformed yield

$$\overline{Z}(m,0) = 0 \text{ and } \overline{Z}_t(m,0) = \eta_2 \quad (1.43)$$

where

$$\eta_2 = \eta_{or}\left[(1-\cos\lambda_m) + B_m(\cosh\lambda_m - 1) + A_m\sin\lambda_m + C_m\sinh\lambda_m\right] \quad (1.44)$$

and

$$\eta_{or} = -\frac{L\Omega}{\lambda_m} \quad (1.45)$$

In view of equations (1.37),(1.42) and (1.43); the transformed equation of our problem, reduces to

$$\overline{Z}_{tt}(m,t) + \left(\omega_m^2 + \frac{k}{\mu}\right)\overline{Z}(m,t) - \frac{N}{\mu}\sum_{K=1}^{\infty}\overline{Z}(k,t)S_1^*(K,m) - r^2\sum_{K=1}^{\infty}\overline{Z}_{tt}(k,t)S_1^*(K,m)$$

$$+\varepsilon\left\{\sum_{K=1}^{\infty}\overline{Z}_{tt}(k,t)S_2^*(K,m)+2\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\cos\frac{n\pi ut}{L}\overline{Z}_{tt}(k,t)S_{2c}^*(k,m,n)\right.$$
$$+2u\sum_{K=1}^{\infty}\overline{Z}(k,t)S_3^*(K,m)+4u\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\cos\frac{n\pi ut}{L}\overline{Z}_t(k,t)S_{3c}^*(k,m,n)$$
$$\left.+u^2\sum_{K=1}^{\infty}\overline{Z}(k,t)S_3^*(K,m)+2u^2\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\cos\frac{n\pi ut}{L}\overline{Z}_t(k,t)S_{1c}^*(k,m,n)\quad\right\}$$
$$=P\left[Sin\frac{\lambda_m ut}{L}+A_m Cos\frac{\lambda_m ut}{L}+B_m Sinh\frac{\lambda_m ut}{L}+C_m Cosh\frac{\lambda_m ut}{L}\right]$$
$$-\left[\ddot{f}_1(t)\ \mathrm{H}_1+\left(\ddot{f}_3(t)-\ddot{f}_1(t)\right)\mathrm{H}_9+f_1(t)\frac{\mathrm{K}}{\mu}\mathrm{H}_1+\left(f_3(t)-f_1(t)\right)\mathrm{H}_{10}\right.$$
$$\left.+\left(\dot{f}_3(t)-\dot{f}_1(t)\right)\left(\mathrm{H}_{13}+\mathrm{H}_{14}\sum_{n=1}^{\upsilon}\cos\frac{n\upsilon t}{L}\right)+\left(f_3(t)-f_1(t)\right)\left(\mathrm{H}_{15}+\mathrm{H}_{16}\sum_{n=1}^{\upsilon}\cos\frac{n\upsilon t}{L}\right)\Bigg\}\right]\ (1.46)$$

Where

$$\varepsilon=\frac{M}{\mu L}\ (1.47)$$

$$H_1=N_1-N_3+A_m(N_2-N_4)\quad;\quad H_2=N_5-N_7+A_m(N_6-N_8)$$
$$H_3=N_9-N_{11}+A_m(N_{10}-N_{12})\quad;\quad H_4=N_{13}-N_{15}+A_m(N_{14}-N_{16})$$
$$H_5=N_{17}-N_{19}+A_m(N_{18}-N_{20})\quad;\quad H_6=N_{21}-N_{23}+A_m(N_{22}-N_{24})$$
$$H_7=N_{25}-N_{27}+A_m(N_{26}-N_{28})\quad;\quad H_8=N_{29}-N_{31}+A_m(N_{30}-N_{32})$$
$$H_9=\left[\frac{3}{L^2}H_3-\frac{2}{L^3}H_4-\sigma^2\left(\frac{6}{L^2}H_1-\frac{12}{L^3}H_2\right)\right]\quad,\quad H_{10}=\left[\frac{K}{\mu}\left(\frac{3}{L^2}H_3-\frac{2}{L^3}H_4\right)-\frac{N}{\mu}\left(\frac{6}{L^2}H_1-\frac{12}{L^3}H_2\right)\right]$$
$$,H_{11}=\frac{3}{L^2}H_3-\frac{2}{L^3}H_4\quad;\quad H_{12}=\frac{6}{L^2}H_7-\frac{4}{L^3}H_8,\quad H_{13}=2U\left(\frac{6}{L^2}H_2-\frac{6}{L^3}H_3\right)$$
$$;H_{14}=2U\left(\frac{12}{L^2}H_6-\frac{12}{L^3}H_7\right)$$
$$H_{15}=U^2\left(\frac{6}{L^2}H_1-\frac{12}{L^3}H_2\right)\quad;\quad H_{16}=U^2\left(\frac{12}{L^2}H_5-\frac{24}{L^3}H_6\right)\qquad(1.48)$$

Equation (1.46) is the transformed equation governing the model of double uniform Rayleigh beams resting on a constant elastic foundation. Two special cases of equation (1.46) can be considered namely moving force and moving mass modelsbut in this report only the moving force model is considered.

3. The Clamped-Clamped Moving Forcesmodel

In equation (1.46), ε is set to zero, using Strubles asymptotic techniques as alluded to in [1,2,3,4,5,6]. On this consideration, hence, the entire equation (1.46) takes the form.

$$\overline{Z}_{tt}(m,t)+\gamma_m^2\overline{Z}(m,t)=$$
$$\left(1+\varepsilon_*S_1^*(m,m)\right)\left[P\left(Sin\frac{\lambda_m ut}{L}+A_m Cos\frac{\lambda_m ut}{L}+B_m Sinh\frac{\lambda_m ut}{L}+C_m Cosh\frac{\lambda_m ut}{L}\right)\right.$$
$$\left.-H_{17}\sinh\Omega t-H_{18}\ell^{-\beta t}\sin\Omega t+H_{19}\ell^{-\beta t}\cos\Omega t\right].\qquad(1.49)$$

Where

$$\gamma_m=-\frac{\omega_{mf}}{2}\left[\varepsilon_*\left(S_1^*(m,m)+\varepsilon_*\frac{N}{\mu r^2\omega_{mf}^2}S_1^*(m,m)\right)\right]\qquad(1.50)$$

represents the modified frequency due to the effect of rotatory inertia.

$$\frac{r^2}{L} = \lambda_0 \qquad (1.51)$$

In order to obtain the solution to equation (1.49), it is subjected to a Laplace transformation in conjunction with the initial condition. The equation, after simplifications and rearrangements then we obtain

$$\overline{Z}(m,t) = \frac{P_{OR}}{\gamma_m\left(\gamma_m^4 - z_0^4\right)}\left[\left(\gamma_m^2 + z_0^2\right)\left(\left(\gamma_m \sin z_0 t - z_0 \sin\gamma_m t\right) + A_m\left(\cos z_0 t - \cos\gamma_m t\right)\right)\right.$$
$$\left. + \left(\gamma_m^2 - z_0^2\right)\left(B_m\left(\gamma_m \sinh z_o t - z_0 \sin\gamma_m t\right) + C_m\left(\cosh z_0 t - \cos\gamma_m t\right)\right)\right]$$
$$+ \frac{H_{17}^{OR}\Omega}{2\gamma_m\left(\gamma_m^2 - \Omega_m^2\right)}\left[(\gamma_m - \Omega)\sin\gamma_m t\cos(\Omega + \gamma_m) - (\gamma_m - \Omega)\cos\gamma_m t\sin(\Omega + \gamma_m)\right.$$
$$\left. - (\Omega + \gamma_m)\sin\gamma_m t\cos(\Omega - \gamma_m) - (\Omega + \gamma_m)\cos\gamma_m t\sin(\Omega - \gamma_m) + 2\Omega\sin\gamma_m t\ \right]$$
$$+ \frac{H_{18}^{OR}e^{-\beta t}}{2\gamma_m\left[\beta^2 + (\gamma_m + \Omega)^2\right]}\left[-\beta\sin\gamma_m t\sin(\gamma_m + \Omega)t - \beta\cos\gamma_m t - (\gamma_m + \Omega)\beta\sin\gamma_m t\cos(\gamma_m + \Omega)t\right.$$
$$\left. + (\gamma_m + \Omega)\sin\gamma_m t - \beta\cos\gamma_m t\cos(\gamma_m + \Omega)t + (\gamma_m + \Omega)\cos\gamma_m t\sin(\gamma_m + \Omega)t\right]$$
$$+ \frac{H_{18}^{OR}e^{-\beta t}}{2\gamma_m\left[\beta^2 + (\gamma_m - \Omega)^2\right]}\left[\beta\cos\gamma_m t\cos(\gamma_m - \Omega)t - (\gamma_m - \Omega)\left[\cos\gamma_m t\sin(\gamma_m - \Omega)t\right.\right.$$
$$\left. + \beta\sin\gamma_m t\sin(\gamma_m - \Omega)t + (\gamma_m - \Omega)\sin\gamma_m t\cos(\gamma_m - \Omega)t - (\gamma_m - \Omega)\sin\gamma_m t - \beta\cos\gamma_m t\right]$$
$$+ \frac{H_{19}^{OR}e^{-\beta t}}{2\gamma_m\left[\beta^2 + (\gamma_m + \Omega)^2\right]}\left[-\beta\sin\gamma_m t\cos(\gamma_m + \Omega)t + \beta\sin\gamma_m t + (\gamma_m + \Omega)\sin\gamma_m t\sin(\gamma_m + \Omega)t\right.$$
$$\left. + \beta\cos\gamma_m t\sin(\gamma_m + \Omega)t + (\gamma_m + \Omega)\cos\gamma_m t\cos(\gamma_m + \Omega)t - (\gamma_m + \Omega)\cos\gamma_m t\ \right]$$
$$+ \frac{H_{19}^{OR}e^{-\beta t}}{2\gamma_m\left[\beta^2 + (\gamma_m - \Omega)^2\right]}\left[-\beta\sin\gamma_m t\cos(\gamma_m - \Omega)t + \beta\sin\gamma_m t + (\gamma_m - \Omega)\sin\gamma_m t\sin(\gamma_m - \Omega)t\right.$$
$$\left.(\Omega - \gamma_m)\cos\gamma_m t - \beta\cos\gamma_m t\sin(\Omega - \gamma_m)t - (\Omega - \gamma_m)\cos\gamma_m t\cos(\Omega - \gamma_m)t\ \right] + \frac{\eta_2}{\gamma_m}\sin\gamma_m t \quad (1.52)$$

on inversion yields

$$\overline{Z}(x,t) = \frac{2}{L}\sum_{m=1}^{\infty}\overline{Z}(m,t)\left[\cosh\frac{\lambda_m x}{L} - \cos\frac{\lambda_m x}{L} - \sigma_m\left(\sinh\frac{\lambda_m x}{L} - \sin\frac{\lambda_m x}{L}\right)\right]$$

$$U(x,t) = \overline{Z}(x,t) + \left(1 - 3\left(\frac{x}{L}\right)^2 + 2\left(\frac{x}{L}\right)^3\right)\sin\Omega t + \left(3\left(\frac{x}{L}\right)^2 - 2\left(\frac{x}{L}\right)^3\right)e^{-\beta t}\sin\Omega t \qquad (1.53)$$

Equation (1.53) is the transverse response of double uniformRayleigh beam under the action of a moving force whose two clamped edges are constrained to undergo displacements which vary with time.

4. Numerical Calculation and Discussion of the Results

The analytical results of double uniform Rayleigh beamswith these parameters are considered Length L=12.192m, the load velocity u = 8.123m/s and $E = 2.109\times10^9\,kg/m$ .The values of the foundation moduli K varied between 0 and 400000 and for fixed values of rotatory inertia r=1.The traverse deflections of the uniform Rayleigh beam are calculated and plotted against time for values of rotatory inertia and foundation stiffness K.

Fig. 1. Reveals deflection profile when all the beams parameters are fixed and the displacement was plot against time.Fig.2. Displayed the dynamic response of clamped-clamped moving forcesystem for various values of axial force N and fixed value of foundation moduli K =40000. The graph shows that the response amplitude decreases as the values of the axial force N increases while in fig.3. One observed that deflection profile of the clamped-clamped moving force of a uniform Rayleigh beams moving with variable velocities for various values of foundation moduli K and for fixed value of axial force N=20000. The graph shows that the response amplitude decreases as the values of the foundation moduli K increases,fig.4. shows the deflection profile for clamped-clamped uniform Rayleigh beams under the action of moving force for various values of rotatory inertia and for fixed value of axial force N=200000 and for fixed value of foundation modulus K=200000. It was found out that as the values of rotatory inertia increases the deflection profile reduces.

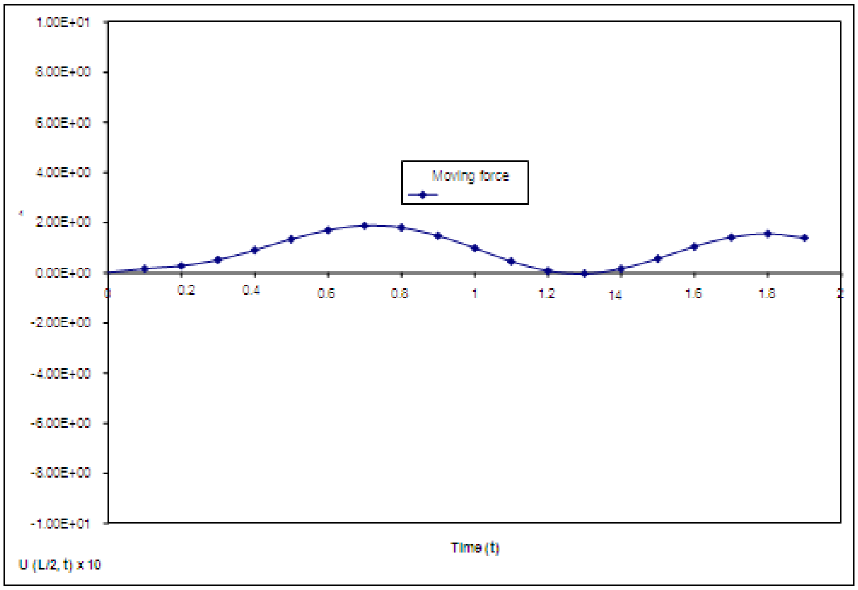


Fig 1: Moving force against Time (t) of the clamped-clamped uniform Rayleigh Beam for fixed values of axial force N and foundation moduli K

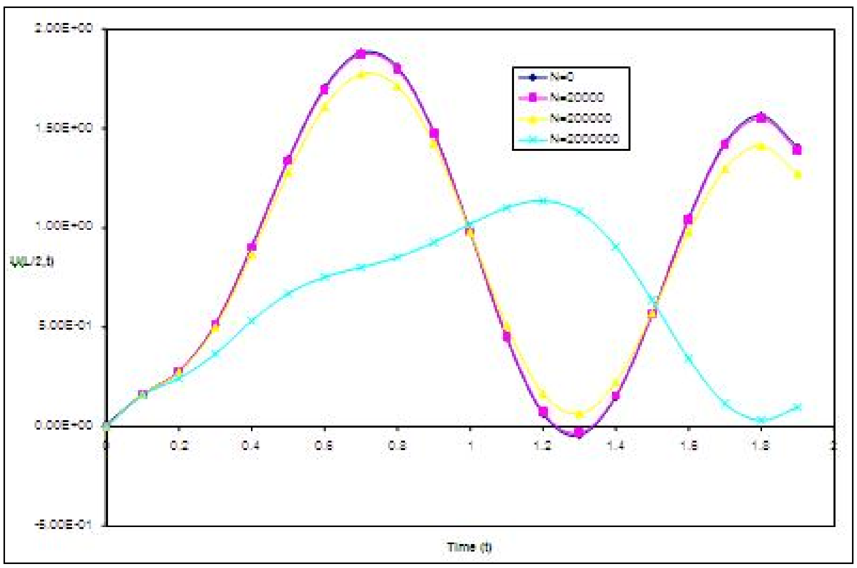


Fig 2: Dynamic response of clamped-clamped uniform beam for various values of axial force N and fixed value of foundation moduli K=40000

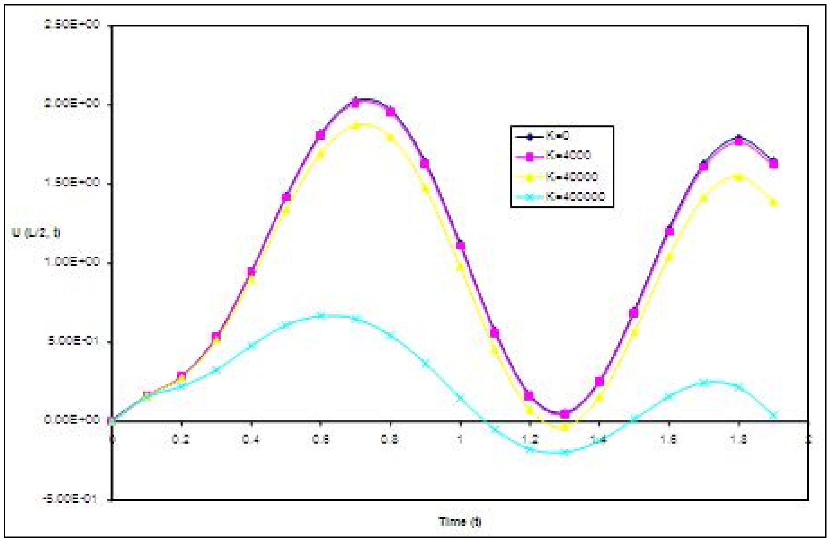


Fig 3: Deflection profile of clamped-clamped uniform beams for various values of foundation moduli K and for fixed value of axial force N=20000

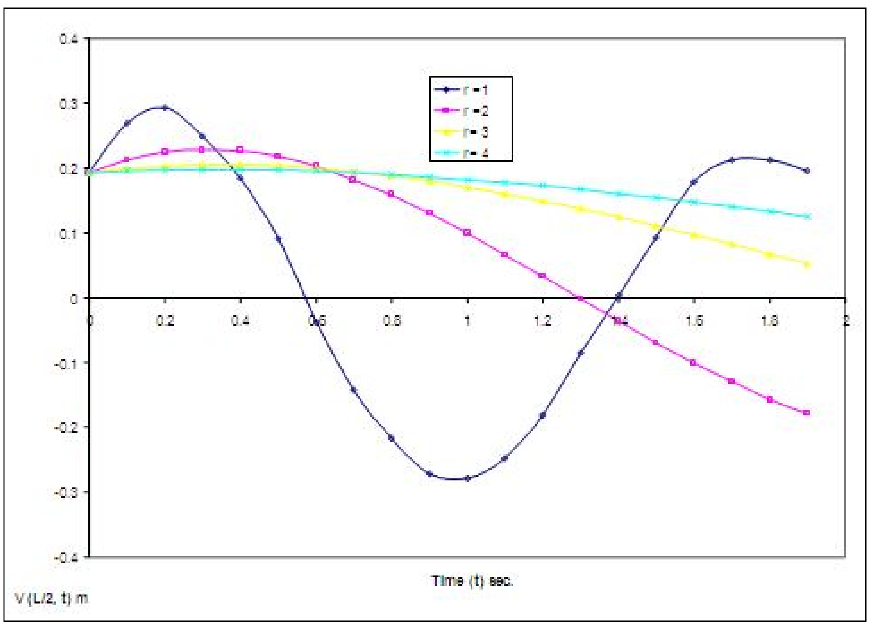


Fig 4:Deflection profile of clamped-clamped uniform Rayleigh beams under the action of moving force for various values of rotatory inertia and fixed value of foundation modulus k=20000 and fixed value of axial force N=40000

5. References

1. S.T Oni. and S.O Ajibola. (2011):Motion of moving concentrated loads of simply-supported non-uniform Rayleigh beam with non-classic boundary conditions.Journal of Nigerian Association of Mathematical Physics.Vol 18.No.1.
2. 2. S.T Oni. and S.O Ajibola. (2009): Dynamical Analysis under moving Concentrated Loads of Rayleigh Beams with Time- Dependent Boundary Conditions:Journal of Engineering ResearchVol.14, No.4.
3. S. O Ajibola, (2011). Transvers Displacement of Clamped-Clamped non-uniform Rayleigh beam under moving concentrated masses resting on a constant Elastic foundation. Journal of Nigerian Association of Mathematical Physics.Vol 18.No.1.
4. S.O Ajibola (2014). "Vibration Of Uniform Rayleigh Beam Clamped-Clamped Carrying Concentrated Masses Undergoing Traction’ ’The International Journal of Science and Technoledge. (IJST), Vol.11, Issue Vi,
5. S.T Oni(1991). On the dynamic response of elastic structures to moving multi-mass systems Ph.DThesis,University of Ilorin, Ilorin, Nigeria
6. J.A Gbadeyan and S.T Oni.(1995): Dynamic behaviour of beams and rectangular plates under moving loads. Journal of Sound and Vibrations. 182 (5), pp. 677-695
7. S.T Oni and B. Omolofe(2005): Dynamic analysis of prestressed elastic beam with general boundary conditions under moving loads traveling at varying velocities. Journal of Engineering and Engineering Tech, FUTA.4 (1):55-74.

8. S.T Oni[1].and T. Awodola[2],(2003): Vibrations under a moving load of a non-Rayleigh beam on variable elastic foundation, Journal of Nigerian Association of Mathematical Physics. Vol.7, pp 191-206.
9. Omer Civalek[1] and Aitung Yauas[2] (2006).. Large Deflection Static Analysis of Rectangular Plates on two Parameter Elastic Foundations. International Journal of Science and Technology. Vol.1. No.1, 43-50.
10. S.T Oni and B. Omolofe(2005):DynamicBehaviour of Non-Uniform Bernoulli-Euler
11. Beams Subjected to Concentrated loads travelling at Varying Velocities.
12. Abacus, Journal of Mathematical Association of Nigeria. Vol.32, no.2A,
13. E. Savin (2001): Dynamics amplification factor and response spectrum for the evaluation of vibrations of beams under successive moving loads. Journal of Sound and Vibrations 248(2), 267-288
14. S.O Ajibola. (2009): Dynamic Analysis under Moving Concentrated Loads of Rayleigh Beam with time dependent boundary conditions. Federal University of Technology Akure (FUTA) Ondo State Ph.D Thesis.
15. J.A. Gbadeyan and O.O Agboola.(2012):Jounal of Applied Sciences and Research. 8(1).Dynamic Behaviour of a double Rayleigh Beam System due to uniform partially distributed Moving Load.571-581.
16. R.D Mindlin and L.E.Goodman (1950): Beam Vibrations with time-dependent boundary conditions. Jounal of Applied Mechanics, 17.Pp377-380