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Structural Behavior of Simply Reinforced LRC Beams in Shear Failure

Dr. Ephraim, M. E.

Associate Professor, Department of Civil Engineering,
Rivers-State University of Science and Technology, Port-Harcourt, Nigeria

Engr. Asigo, P. M. M.

Deputy Director, Directorate of Utilities, Infrastructural Development and Waterways,
Deputy Director, Department of Design, Niger Delta Development Commission (NDDC), Nigeria

Abstract:

Structural behaviour of simply reinforced LRC beams in shear failure was studied to compliment other studies on the LRC as suitable material for structural use in the construction industry. A concrete mix of 1:1.35:3.5 at water cement ratio of 0.85 yielding 28days strength of 15N/mm^2 was adopted for the study. Four beams were cast with three reinforced with 10mm, 12mm and 16mm bars while one beam plain. The beams were tested to failure and the experimental data were analyzed to obtain a stress-strain and load – deflection relationship of the beams. The results show that the LRC aggregates used in the experiment were well graded with a uniformity coefficient ($D_{60}/D_{10} = 9.63$) greater than 7 and a coefficient of curvature of $(D_{30})^2/D_{60}/D_{10} = 0.654$. It was observed that, failure of the LRC beams started by the development of inclined or diagonal cracks within the point of support and point of loading. The contribution of web reinforcement, LRC concrete and longitudinal/ interface forces were found to be 38.8%, 23.81% and 37.39% respectively in restraining the beams failing in shear.

Keywords: LRC-laterite rock concrete compressive strength, stress-strain, shear stress-strain, tensile zone, compression zone, steel ratio. Web and longitudinal reinforcement etc.

1. Introduction

1.1. Background

The quest for alternative material for replacement of conventional aggregate in concrete technology has generated a lot of interest in recent past to researchers. Concrete is the most widely used construction material worldwide and it is weak in tension, hence introduction of reinforcement bars or rods in the reinforced and pre-stressed variants results in higher strength, economy and durability. In normal concrete, about 70% of its constituent is made of aggregates, which implies that the overall cost of concrete is largely influenced by the cost of aggregates. Studies have revealed the use of various locally available concrete materials such as, laterized concrete containing partial replacement of fine sand with laterite fines, use of fly ash, periwinkle, and other pozzolans, fibre glass to mention but a few. Adepegba (1975), (1979), Balogun and Adepegba (1984), Salau (2003), and Ukpata *et al.*, (2012), on the other hand, studied the use of quarry dust as partial or total replacement for conventional river sand fine aggregate. There is paucity of studies on use of laterite rock aggregate (LRA) as replacement for conventional aggregates and the structural behaviour of LRC in flexure and shear that have been examined. The experimental work of few cases include the works of Madu (1980), Okoli (1987), Amobi (1992), Akpokojie (1992) on the suitability of laterite rock aggregates. The study carried out by Ephraim and Adoga, (2016) on strength of laterite rock concrete gave credence to the suitability for the use of LRC in structural works. The flexural behavior of simple LRC beams, Shear behavior of simple LRC beams were studied respectively by Fakare and Bonny, (1988) and Asigo (1988). The knowledge of structural behavior of concrete made with laterite rock aggregate is important for the development of structural design parameters and significant for construction of structural elements. However the dearth of study in shear behavior of LRC is recognized, hence this work is an attempt towards a more elaborate investigation of the shear strength and failure of LRC.

1.2. Materials and Methods

The materials used for this study include the following; Portland cement: manufactured by the Eastern Bulkem Company Limited, now Eagle cement, Port Harcourt and was supplied in bags of 50kg. Water which is fit for drinking sourced from the structural laboratory of Rivers State University of Science and Technology Port Harcourt, that conforms to BS3148 requirement. Reinforcement Bars: 10mm, 12mm and 16mm were used. All bars were mild steel with an approximate yield stress of 250N/mm^2 . Mild steel reinforcement bars were chosen because the yield stress is relatively lower than high yield thus, lighter loading and instrumentation. It is less susceptible to sudden collapse of a structural element which means adequate warning is given before collapse; and finally, it is

readily available and cheaper to afford. Aggregates used were composed of both coarse and Laterite Fines. These aggregates however, were sourced from Idah in Benue State. The mould used in casting the beams were fabricated in the wood section of Engineering workshop of the Faculty of Engineering, Rivers state University of Science and Technology, Port Harcourt.

1.3. Experimental Procedure

The method for the investigation includes carrying out a particle size distribution (PSD) analysis of the all-in-aggregates through sieve analysis with the aid of a sieve shaker fitted with a time switch to ensure that there is uniformity of the sieving operation. Results tabulated and plotted with grading. From the grading curve, mix ratio was designed using the absolute volume method and a mix proportion of 1:1.35:3.15 was determined using cube sizes of 100mmx100mmx100mm. From the design, the constituent materials summed up to 22816.256kg with all-in-aggregates representing about 50% of the total. The experimental details included casting five set of cubes of 150mmx 150mmx 150 mm with water/ cement ratios ranging from 0.65, 0.75 to 0.85. 0.65, 0.75 and 0.85 were used for the first set of 3 cubes with 30% Laterite fines; 0.75 and 0.85 were used for the last set of 2 cubes with 40% laterite fines. The aim was to determine the adequate mix proportion that will produce a dense laterite rock concrete with sufficient strength that could satisfy workability in the fresh state and durability in the hardened form. Four beams were cast using W/C ratio of 0.85 with 30% Laterite fines and cured for 28days. The four beams had labels B-plain, B₁₀, B₁₂ and B₁₆. All reinforced beams were provided with vertical web reinforcement bars of 6mm diameter at a spacing not exceeding 0.75d. Testing of beams was instrumented in line with the 60° rosette disposition. Strain gages used were that of the DEMEC type with accuracy of 0.001. Beams were aligned properly using the physical axis on the testing device with 100 kN AVERY crushing machine and loads applied in steps of 0.2f_{cu}. The two point loads were applied at a distance of 150mm from each support which consisted a_v/d between 1 and 1.25. Strain measurements were conducted twice to record the elastic and plastic deformation at intervals of 10minutes. Also the mid span deflections were measured by a dial gauge of accuracy of 0.002. Data collected were analyzed using the Rossette formula from theory of Elasticity to determine the principal strains, stresses, shear strain and the maximum shear stress, when a point is subjected to shear loading. All relevant tables and figures are presented in the results and discussions.

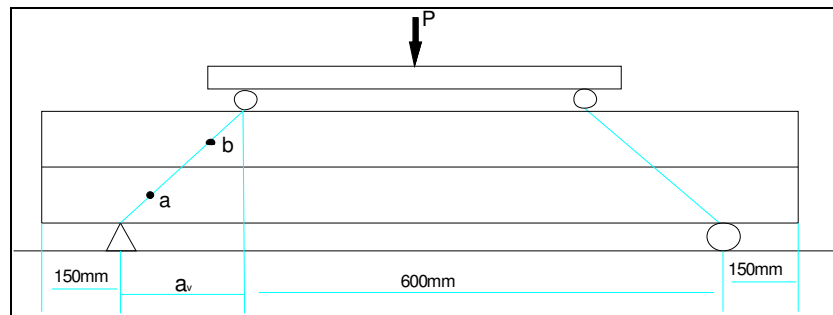


Figure 1: Loading and instrumentation profile of beam specimen

2. Results and Discussion

2.1. Physical Properties of Material

Particle Size Distribution (PSD) and Grading of Laterite Rock Aggregates (LRA): The PSD through sieve analysis of LRA of laterite fines and laterite rock with its grading are shown in table 1 and figure 2 respectively. From the grading curve, it was observed that the aggregates were well graded with a uniformity co-efficient D_{60}/D_{10} to be 9.663 greater than 7. This implies therefore that the aggregates have a high packing density.

Particle size (mm)	37.50	20.0	14.00	10.00	6.30	5.00	2.36	1.18	0.85	0.60	0.30	0.15	0.075
% passing	100.00	99.80	78.30	49.80	31.40	29.20	15.60	7.20	4.50	2.50	0.60	0.30	0.20

Table 1: Particle size distribution of laterite rock Aggregate (LRA)

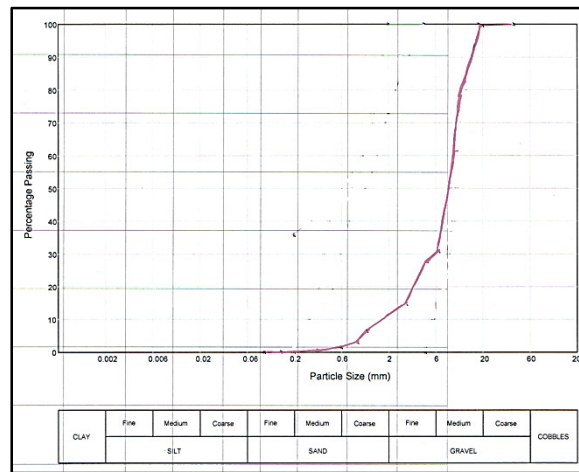


Figure 2: grading curve of laterite rock Aggregates LRA

Compressive Strength of LRC; The compressive strength of LRC at 28days curing revealed that at W/C ratio of 0.75 and 0.85, the strength of the hardened concrete was 15.1N/mm² which is considered low since LRC is normal weight concrete comparable to normal concrete. This low strength is characterized by lack of adequate compaction and the presence of voids in the dense LRC and the design mix ratio used.

2.2. Load-Deflection of LRC Simply reinforced Beams

The beams deflected on application of load were of the same pattern. On linearization, the deflection becomes proportional to increase in load. The linearization was done from the regression analysis of the data resulting to R, R², t, and f values pointing to a close relationship and that the data fits the model which is a further indication that LRC is suitable for structural works. The strength behavior of reinforced concrete members is dependent on the size, shape of the members and stress – strain properties of the concrete and the reinforcement.

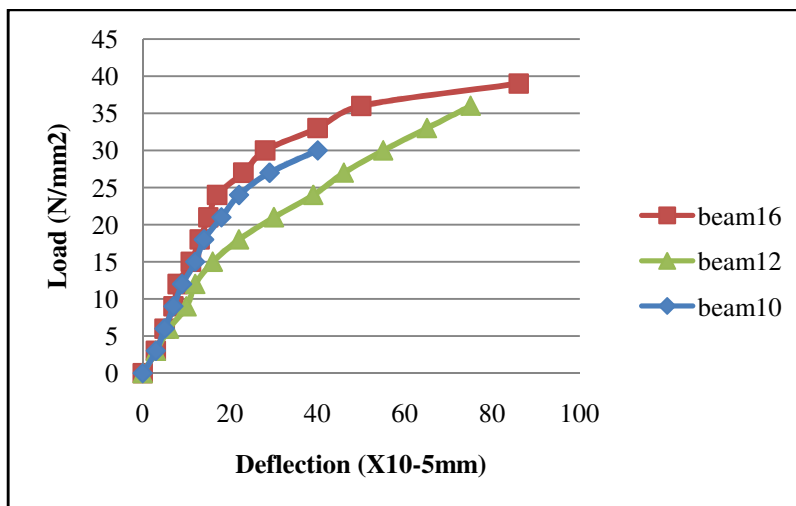


Figure 3: Relationship between load and deflection for 3 LRC simply supported beams under direct tension

Beam Types	Initial crack Load (kN)	Ultimate Failure Load (kN)	Deflection at initial crack load (x10 ⁻⁴)	Deflection at ultimate Failure (x10 ⁻⁴)
B ₁ -10	12	30	9.0	40.0
B ₂ -12	21	36	30.0	75.0
B ₃ -16	33	39	65.0	86.0

Table 2: Load – Deflection parameters for simply reinforced LRC Beams

Shear Behaviour of LRC Beams; All reinforced beams failed by development of initial flexural cracks and later shear cracks which propagate from the point of support to point of loading. Primarily, shear compression failure, splitting and bond failure were observed in the reinforced beams. Experimental loads recorded were 36 kN for beams reinforced with 10mm and 12mm rods, while the beam reinforced with 16mm had 39 kN.



Figure 4a: Showing the failure of reinforced Beam 10mm & Beam 12mm.

Figure 4b: Showing the failure of reinforced Beam 16mm.

Figure 4c: Showing the failure of reinforced Plain Beam.

2.3. Effect of Web Reinforcement and Dowel Action

The introduction of web reinforcement as well as longitudinal bars, effectively restrained the quick development of cracks and ultimate failure. This means that increase in longitudinal reinforcement in LRC beams would increase its strength and considerably reduce the deformation suffered. The web and longitudinal reinforcement significantly affect the strength of LRC beams. The web reinforcement has a great restraining effect in the development or formation of diagonal cracks, because they help to carry part of the shear force. Also, increases the shear strength of LRC beams and formation of cracks is delayed. For the three reinforced LRC beams, initial cracks were observed at 12KN, 21KN and 33KN respectively for 10mm, 12mm and 16mm reinforcement bars.

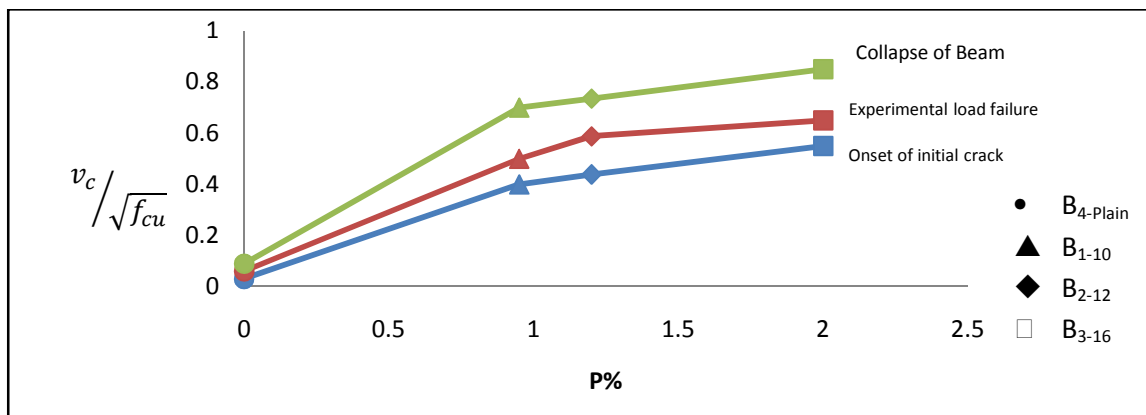


Figure 5: Effect of Steel Ratio on the Strength of LRC Beams under Shear Failure

Type	Web reinforcement %	concrete	Longitudinal interface forces
B ₁ – 10	43.10	22.45	34.45
B ₂ – 12	40.67	23.91	35.42
B ₃ – 16	32.63	25.07	42.30
B ₄ - plain	-----	18.81	-----

Table 3: Contribution of Web, Concrete and Longitudinal Reinforcement Bars

The Table 3 showed that in all the reinforced LRC beams, web reinforcement is seen to be contributing largely to the restraining effect in B₁₋₁₀ and B₂₋₁₂ though B₃₋₁₆ was lowest with no effect on B₄ – plain. This explains the fact that since there was no reinforcement, the effect of web cannot be noticed except the concrete shear resistance. However, in B₃₋₁₆, it was observed that there is an increase in the shear capacity of concrete and longitudinal/interface forces in restraining formation of cracks and ultimate failure but with the lowest web influence. This behavior aptly explains the mode of failure mechanism as illustrated in figure 5 above.

2.4. Application of Strain Equation

Strain at an angle is denoted as ε_θ which is expressed as follows;

$$\epsilon_{\theta} = \frac{(\epsilon_x + \epsilon_y)}{2} + \frac{(\epsilon_x - \epsilon_y)\cos 2\theta}{2} + \frac{\phi \sin 2\theta}{2} \tag{1}$$

The above formula was used in calculating the maximum shear stress at all points where strain measurements were read for each increment of load. Readings of strain at a point across the diagonal when initial cracks developed for a location at ‘a’ and ‘b’ were evaluated. In the tension zone ε₀ = 62, ε₆₀ = 94 and ε₁₂₀ = 87, the application of these values in the strain equation given above results to three simultaneous equations with which when solved gives the principal strains ε₁, ε₂; principal stresses σ₁, σ₂ and maximum shear stress T_{max}. Similarly, in the compression zone at point ‘b’.

ε₀ = -85, ε₆₀ = -28 and ε₁₂₀ = -104 the principal strains, stresses and maximum shear stress can be evaluated.

Parameters	Point a	Point b
ϵ_1	100.42	-44.73
ϵ_2	61.58	-99.95
τ_1	21.39N/mm ²	-15.73N/mm ²
τ_2	19.84N/mm ²	-26.46N/mm ²
τ	3.78N/mm ²	5.37N/mm ²

Table 4: Computed Values of Stress, Strain and Maximum Shear Stress at point 'a' and 'b'.

The table above showed that point 'a' along the diagonal of shear failure is in tensile stresses while point 'b' is in compressive stresses. It further indicated that when the horizontal direct stress is positive, tension is found to be maximum on a steeper plane (angle of inclination or direction of the tensile stress), however, when the horizontal direct stress is compression, the maximum tensile stress occurs at a flatter plane Fig 6a and Fig 6b. Illustrate the orientation of the maximum tensile stresses discussed above and the respective proofs are presented below.

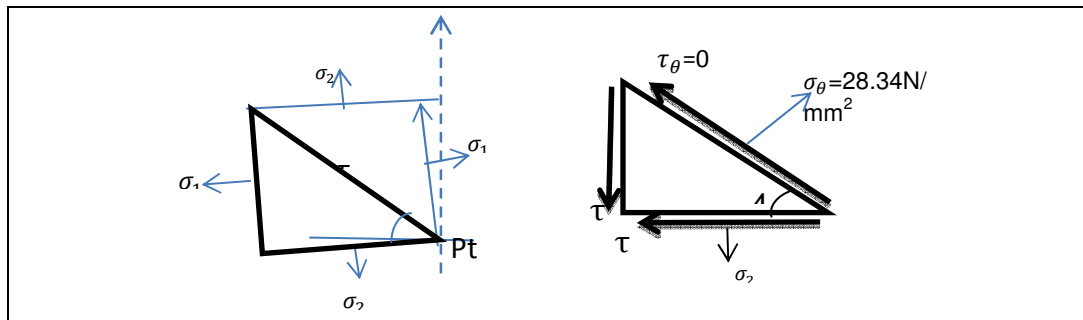


Figure 6a: Orientation of principal stress and their planes in tension zone of reinforced LRC beam.

From Table 5;

$$\sigma_1 = 27.39\text{N/mm}^2, \sigma_2 = -19.84\text{N/mm}^2, T = 3.78\text{N/mm}^2$$

Resolving along σ_θ

$$\begin{aligned} \sigma_\theta &= \sigma_2 \cos\theta \sin\theta + \sigma_1 \sin\theta \cos\theta - T \cos^2\theta + T \sin^2\theta \\ &= 9.67 + 18.7 - 0 = 28.34\text{N/mm}^2 \end{aligned}$$

Resolving along T_θ

$$\begin{aligned} T_\theta &= \sigma_2 \cos 2\theta - \sigma_1 \sin^2\theta + T \sin 2\theta \\ &= 9.92 - 13.7 \cdot 3.78 = 0 \end{aligned}$$

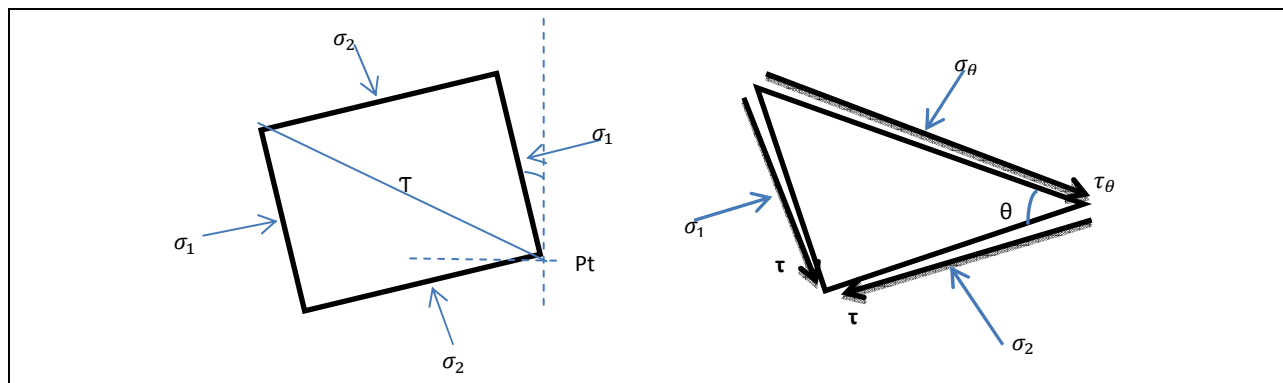


Figure 6b: Orientation of Principal Stresses and their Planes in Compression Zone of Re-inforced LRC beam

From the Table;

$$\sigma_1 = -15.73\text{N/mm}^2, \sigma_2 = -26.46\text{N/mm}^2, \tau_{max} = 5.37\text{N/mm}^2, \theta = 45^\circ$$

$$\begin{aligned} \sigma_\theta &= \sigma_2 \cos\theta \sin\theta + \sigma_1 \sin\theta \cos\theta - T \cos^2\theta + T \sin^2\theta \\ &= -13.23 - 7.87 = -21.1\text{N/mm}^2. \end{aligned}$$

Resolving along τ_θ

$$\begin{aligned} T_\theta &= \sigma_2 \cos 2\theta - \sigma_1 \sin^2\theta + T \sin 2\theta \\ &= -13.24 + 7.865 + 5.31 = 0 \end{aligned}$$

The Figures sketched above give an indication that an element located at the N.A shall experience only pure shearing stress because unit tensile and compressive stresses of the same magnitude are generated. However, at point 'a' the tensile stress combines with shear

stress to reduce the diagonal compression on the neutral axis thus increases the diagonal tensile stresses in the tension zone and point 'b' shall develop compressive stresses in addition to shear, thus adding a diagonal compression. Result of analysis showed that point 'a' located at the tensile zone experiences tensile stresses which is equally true for point 'b' in the compression zone.

Stress-Strain of Simply Reinforced and Plain LRC Beams; Fig; 7a, 7b, 7c and 7d represent the graphical relation of the behaviour of reinforced and plain beams made of laterite rock concretes. Data for each beam were modeled using the cubic equation from computer analysis. The cubic equation is a model equation for the stress, strain relationship of beams reinforced with varying reinforcement bars or rods since the LRC is weak in tension and strong in compression. However, the elasticity of the rod or bars is not considered, though they work monolithically as a composite element.

Beam type	Cubic equation	Linear equation
B₁₀	$y = -0.12 + 0.15x + 1.66e^{-4}x^2 - 9.81e^{-7}x^3$	$y = 0.7 + 0.14x$
B₁₂	$y = -0.03 + 0.14x + 7.91e^{-4}x^2 - 6.3e^{-6}x^3$	$y = 0.97 + 0.13x$
B₁₆	$y = 0.21 + 0.136x - 1.49e^{-5}x^2 + 1.46e^{-7}x^3$	$y = 5.04 + 0.09x$
Plain	$y = 0.08 + 0.2x - 1.14e^{-3}x^2 + 4.91e^{-6}x^3$	$y = 0.8 + 0.13x$

Table 5: Summary of Analysis for Stress, Shear Strain of LRC Beams

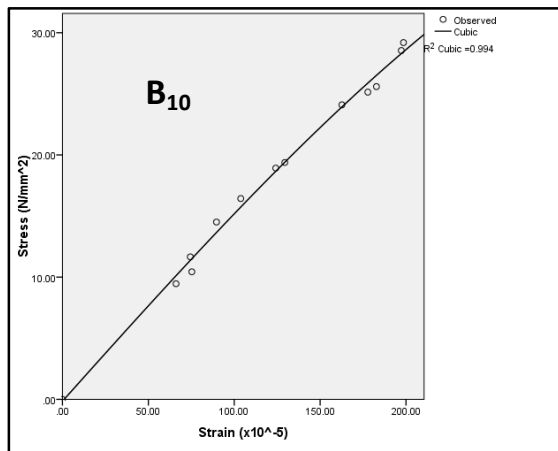


Figure 7a: Stress/ Strain Relationship of simply Reinforced 10 mm Beam.

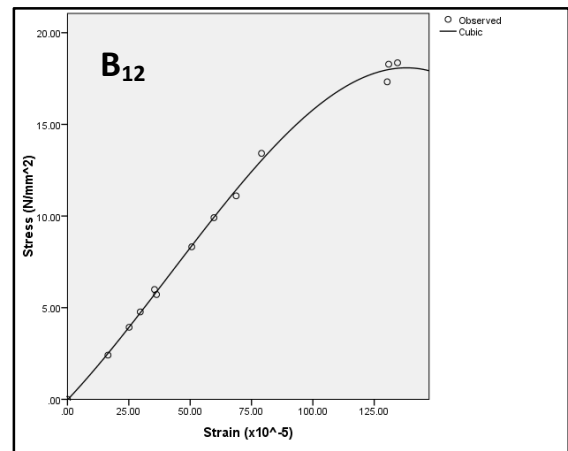


Figure 7b: Stress/ Strain Relationship of simply Reinforced 12 mm Beam.

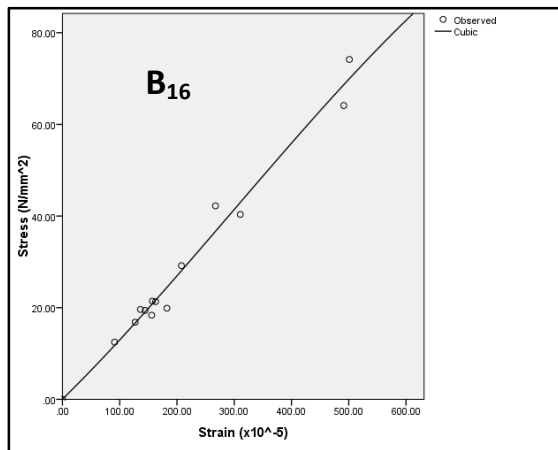


Figure 7c: Stress/ Strain Relationship of simply Reinforced 16 mm Beam.

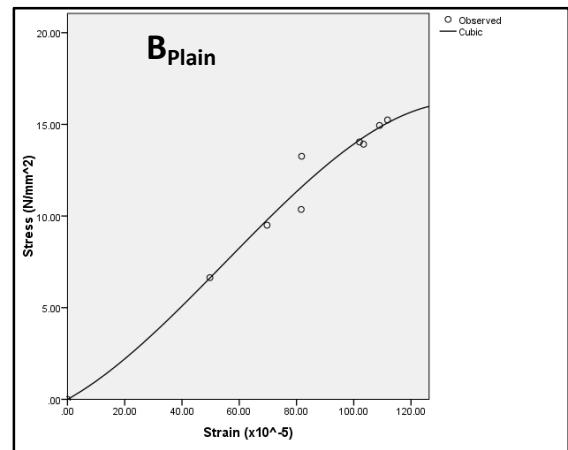


Figure 7d: Stress/ Strain Relationship of simply Reinforced 12 mm Beam.

Cubic equations for the four beams have been evaluated using the second derivative to determine the value of x when $\frac{d^2y}{dx^2} = 0$. The conurbation of each result has been presented in the table below.

Model	B ₁₀ (KN/mm ²)	B ₁₂ (KN/mm ²)	B ₁₆ (KN/mm ²)	B _{Plain} (KN/mm ²)	Mean (KN/mm ²)
Linear	14	13	9	13	12.25
Cubic	15	16	14	21	16.5

Table 6: Elastic modulus of LRC

The Table above shows that the cubic model equation is more producible than the linear with an average Elastic modulus of 16KN/mm² which tilts to 20KN/mm² of normal concrete. However, the range of modulus of elasticity for LRC could be taken as 12.25 kN/mm² to 16 kN/mm².

Shear Modulus of Deformation; Figures; 8a, 7b, 7c and 7d show the shear stress and shear strain relationship of reinforced and plain beams. Graphically the mode of deformation showed that there is elastic-plastic mode of failure corresponding to either tensile-shear in the tension zone, or compression –shear in the compression zone of the beams. Equally the un-reinforced beam exhibited a pattern that showed its failure mode which is a cleave or a split at experimental load, hence, the stress-strain relationship and that of shear stress –strain are almost linear from analysis. The shear modulus of elasticity for these beams, taking note of the fact that they are made of the same material have been averaged to be 11.7 kN/mm². using a linear model equation. Summarily, the Table below shows the shear modulus of deformation of each beam.

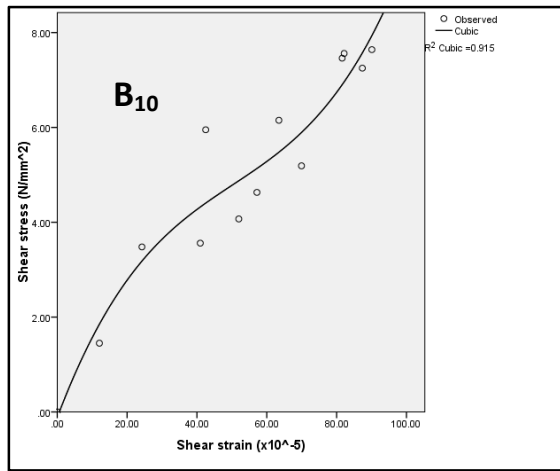


Figure 8a: Shear Stress/ Strain Relationship of simply Reinforced 10 mm Beam.

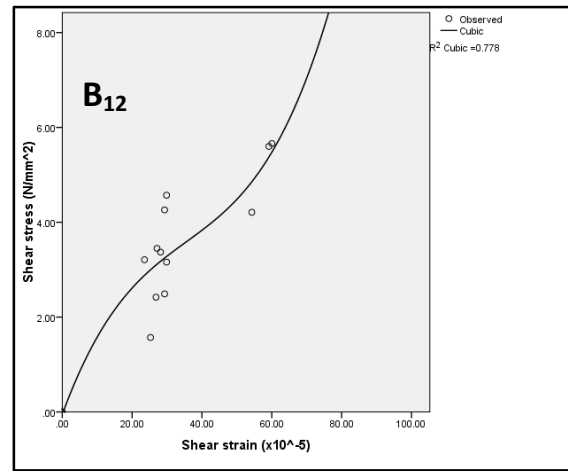


Figure 8b: Shear Stress/ Strain Relationship of simply Reinforced 12 mm Beam.

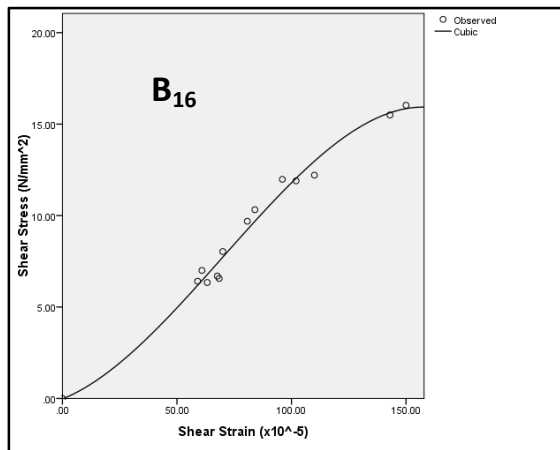


Figure 8c: Shear Stress/ Strain Relationship of simply Reinforced 16 mm Beam.

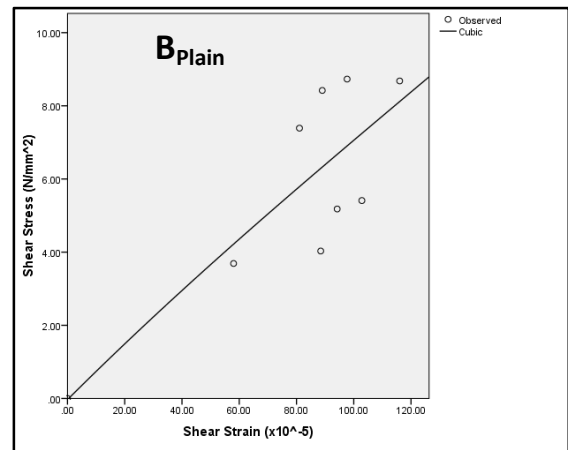


Figure 8d: Shear Stress/ Strain Relationship of simply Reinforced Plain Beam.

Beam type	Cubic equation	Linear equation
B ₁₀	$y = -0.04 + 0.19x - 1.91e^{-3}x^2 + 1.48e^{-5}x^3$	$y = -0.2 + 0.15x$
B ₁₂	$y = 0.05 + 0.2x - 3.88e^{-3}x^2 + 3.57e^{-5}x^3$	$y = 0.72 + 0.08x$
B ₁₆	$y = -0.03 + 0.05x + 1.23e^{-3}x^2 - 5.84e^{-6}x^3$	$y = 0.04 + 0.17x$
Plain	$y = -0.03 + 0.08x - 1.1e^{-4}x^2 + 3.41e^{-7}x^3$	$y = 0.04 + 0.07x$

Table 7: Cubic and Linear Model of Reinforced and Un-reinforced LRC Beams failure.

model	B ₁₀ (kN/mm ²)	B ₁₂ (kN/mm ²)	B ₁₆ (kN/mm ²)	B _{Plain} (kN/mm ²)	Mean (kN/mm ²)
Linear	15	8.0	17	7.0	11.75
Cubic	13.44	10.8	10.70	7.126	10.529

Table 8: Summary Analysis of Shear Modulus of Deformation

Table 6 above showed that the shear modulus of deformation on a linear model gave an average value of 11.75 kN/mm^2 . while from the cubic model a value of 10.529 kN/mm^2 . Variance of the two models shows about 10% which is an indication of 90% confidence interval of the occurrence of the solution used in the computation of the shear modulus of deformation of LRC beams (reinforced or plain) with varying diameter of reinforcements.

3. Conclusion

- Laterite rock aggregates (LRA) used for this study are well graded with a uniformity coefficient $D_{60}/D_{10} = 9.663$ greater than 7 and a coefficient of curvature of 0.654.
- Experimental cube strength of 15.1 mm^2 was recorded after 28 days curing of LRC cubes
- The mix used for the study was designed which satisfies workability, durability and strength requirement with an average density of 2461 kg/m^3 .
- The behaviour of the relationship between load and deflection of the three beams are similar, exhibiting linear – curve relationship which portrays the elastic – plastic characteristic of Normal concrete.
- From Statistical analysis, each of the paired variables, that is load and deflection had R values tending to +1 which is a clear indication that the model proposed for each beam is a fit for the variables.
- Plain LRC beam failed without warning, that is no development of cracks. The beam failed as a cleave or split within the middle third.
- There was minimal or no influence of shear on the elastic and creep properties of the plain beam. Shear strain recorded was 0.0012 for the plain beam while 0.0009 and 0.0006 were computed for B_{10} and B_{12} respectively.
- Failure of all reinforced LRC beams started by development of inclined or diagonal cracks within the support and point of loading, though flexural cracks. Also developed from which sometimes diagonal cracks progress to the point of loading.
- Mode of failure was mostly shear compression for B_{10} and B_{12} , however, B_{16} experienced both bond and shear compression mode of failure.
- The ultimate failure load for beams reinforced with 10 mm, 12 mm and 16 mm were 30 kN, 36 kN and 39 kN respectively.
- The steel ratio showed that the web reinforcement contributed significantly to a proportion of 38.8%. And the contribution of LR concrete and longitudinal bars/interface action corresponds to 23.81% and 37.39% respectively in restraining the failure of the beams.

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