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Performance Comparison Between Volume Weighted Average Priceand Multiple Moving Average For Derivativeswith Volatility Based Modeling

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Abstract:

Volatility and averages are one of the most essential parameters in the new era of financial market. They have profound and significant impact on modeling various financial asset pricebehaviors. Main purpose of modeling these parameters is to predict future profit and loss of the portfolio. Our endeavors areto tune these parameters more accurately and have shown comparison of volume weighted average price and multiple moving average priceson S&P 500 over last 15 years of data along with the algorithm that we have proposed.

Keywords: *weighted average price,volatility, risk management,mathematical modeling, moving average*

Introduction

Mathematical modeling of financial asset is very complex issue. Tuning right parameters withappropriate values are highly desirable in creating successful algorithmic strategies. More over risk management is the key for a successful strategy. This has raised the need of modeling various parameters. Volatility is one of these parameters which will help to know future profit and loss of a portfolio. The GARCH (p, q) model, introduced by Bollerslev (1986), often provides a parsimonious representation of the volatility dynamics in financial time series[2].

One traditional difficulty in constructing GARCH based models is that the volatility process is inherently unobservable. We surmount this problem by using a proxy of monthly volatility calculated using daily data. Moreover GARCH models treat heteroscedasticityas a variance to be modeled. As a result, not only are the deficiencies of least squares corrected, but a prediction is computed for the variance for each error term[7].We have more faith in the reliability of these volatility estimates. We are using volume weighted average price and multi moving average price along with volatility output. This makes the strategy more robust.

Need For Forecasting Volatility For A Model

The main purposes of forecasting volatility are measuring the potential future profit and losses of a portfolio of financial assets. Moreover it helps a lot in asset pricing phenomenon. One of the most common use of volatility for any commodity, options, stocks are to find out next day's volatility based on historical volatility. This helps to know approximate value of reruns over investment. Several recent studies have found that the volatility of daily U.S. dollar exchange rates tends to be highly persistent and well approximated by an integrated or long memory-type GARCH process[6].In asset allocation, the Markowitz approach of minimizing risk for a given level of expected returns has become a standard approach, and of course an estimate of the variance-covariance matrix is required to measure risk. Perhaps the most challenging application of volatility forecasting, Seasonality in financial-market volatility is pervasive. The historical variance of the Standard and Poor's composite stock-price index in October is almost ten times the variance for March [8].So in today's highly volatile market,it is important to specify various parameters which help to derive volatility more accurately. High kurtosis exists within financial time series of high frequencies (observed on daily or weekly basis). This confirms the fact that distribution of returns generated by

GARCH(p,q) model is always leptokurtic, even when normality assumption is introduced. Right combination of volatility parameter will help to give more reliable and accurate value of volatility. It is important to note that kurtosis is both a measure of peak and fat tails of the distribution. So we have tried to make it as accurate as possible.In the vast empirical finance literature models are well known within the GARCH framework where alternative assumptions on the conditional distribution have been suggested and extensively analyzed[12].

Kurtosis Of Garch(1,1) Process

GARCH models are very popular for representing the dynamic evaluation ofvolatility of financial returns.(see, e.g.,Bollerslev, Engle, and Nelson 1994, Engle 1994, Bera and Higgins 1995, Diebold and Lo´pez 1995, and McAleer and Oxley 2003, among many others [4]).

GARCH(1,1) process:[1] has been assumed

$$\begin{aligned} r_t &= \varepsilon_t \\ \varepsilon_t &= u_t\sqrt{\sigma_t^2} \quad ; \quad u_t \approx i.i.d.\, N(0,1), \\ \sigma_t^2 &= \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2 \end{aligned} \tag{1}$$

The second moment of innovation process $\{\varepsilon_t\}$ equals:

$$E\left[\varepsilon_t^2\right] = Var\left[\varepsilon_t\right] = \frac{\alpha_0}{1-\alpha_1-\beta_1}, \tag{2}$$

While the fourth moment is given as:

$$E\left[\varepsilon_t^4\right] = \frac{3\alpha_0^2(1+\alpha_1+\beta_1)}{(1-\alpha_1-\beta_1)(1-3\alpha_1^2-2\alpha_1\beta_1-\beta_1^2)} \tag{3}$$

From covariance stationary condition of GARCH(1,1) process, and strictly positively conditional variance:

$$\begin{aligned} 1-\alpha_1-\beta_1 &> 0 \\ \alpha_0 &> 0 \end{aligned}, \tag{4}$$

Follows that the second moment of $\{\varepsilon_t\}$ process exists. To assure the existence of the fourth moment, apart from conditions in (4), it is necessary in relation (3) to satisfy this restriction:

$$3\alpha_1^2 + 2\alpha_1\beta_1 + \beta_1^2 < 1. \qquad (5)$$

Since kurtosis is defined as:

$$k = \frac{E\left[\varepsilon_t^2\right]}{\left(E\left[\varepsilon_t^4\right]\right)^2}, \qquad (6)$$

then expression (6) becomes:

$$k = \frac{3(1+\alpha_1+\beta_1)(1-\alpha_1-\beta_1)}{1-3\alpha_1^2-2\alpha_1\beta_1-\beta_1^2}. \qquad (7)$$

After some rearrangement in (7) we can write:

$$k = 3 + \frac{6\alpha_1^2}{1-3\alpha_1^2-2\alpha_1\beta_1-\beta_1^2}. \qquad (8)$$

From relation (8) follows that distribution of returns generated from GARCH(1,1) process always results in excess kurtosis, i.e. Fisher's kurtosis ($k > 3$) even normality assumption is introduced, if and only if conditions in (4) are satisfied. These conditions also could be satisfied when parameter $\alpha_1 = 0$. Only in that case innovations distribution would be normally shaped ($k = 3$). Therefore, the kurtosis is very sensitive on value of parameter α_1.In general kurtosis increases much intensively with larger parameter α_1 in comparison to parameter β_1.

Degrees Of Freedom Estimation

Generally, there are three parameters that define a probability density function: (a) location parameter, (b) scale parameter and (c) shape parameter. The most common measure of location parameter is the mean. The scale parameter measure variability of probability density function (*pdf*), and the most commonly used is variance or standard deviation. The shape parameter (kurtosis and/or skewness) determines how the variations are distributed about the location parameter.

If the data are heavy tailed, the VaR calculated using normal assumption differs significantly from Students t-distribution. Therefore, we find that kurtosis and degrees of freedom from Student's distribution are closely related.

The density function of no central Student t-distribution is given as:

$$f(x)=\frac{\Gamma\left(\frac{df+1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)\sqrt{\pi\cdot\beta\cdot df}}\left(1+\frac{(x-\mu)}{\beta\cdot df}\right)^{-\frac{1+df}{2}} \qquad (9)$$

Where μ is location parameter, β scale parameter and df shape parameter, i.e. degrees of freedom, and $\Gamma(\cdot)$ is gamma function. Standard Student's t-distribution assumes that $\mu=0,\ \beta=1$, with integer degrees of freedom. However, degrees of freedom can be estimated as non-integer, relating to kurtosis:

$$k=\frac{6}{df-4}+3 \qquad \forall df>4. \qquad (10)$$

From relation (10) it's obvious that standard t-distribution has heavier tails than normal distribution when $4<df\leq 30$. Hence, if empirical distribution is more leptokurtic estimated degrees of freedom would be smaller.

The second and fourth central moment of function (9) are defined as:

$$\mu_2=E\left[(x-\mu)^2\right]=\frac{\beta\cdot df}{df-2}$$
$$\mu_4=E\left[(x-\mu)^4\right]=\frac{3\beta^2 df^2}{(df-2)(df-4)}, \qquad (11)$$

with Fisher's kurtosis:

$$k^*=\frac{\mu_4}{\mu_2^2}-3=\frac{6}{df-4}. \qquad (12)$$

Therefore, we may apply method of moments and get consistent estimators:

$$\hat{df}=4+\frac{6}{\hat{k}^*}$$
$$\hat{\beta}=\left(\frac{3+\hat{k}^*}{3+2\hat{k}^*}\right)\hat{\sigma}^2, \qquad (13)$$

Where the sample variance is biased estimator of β.To get unbiased estimator of standard deviation we use correction factor:

$$\sqrt{\frac{3+\hat{k}^*}{3+2\hat{k}^*}} \qquad (14)$$

which is equivalent to:

$$\sqrt{\frac{\hat{df}-2}{\hat{df}}}. \qquad (15)$$

In practice, the kurtosis is often larger than six, leading to estimation of non-integer degrees of freedom between four and five. However, kurtosis will depend on volatility persistence. Volatility persistence is defined as the sum of parameters $\alpha_1 + \beta_1$ in GARCH(1,1) model.

If we rearrange condition variance equation of GARCH(1,1) model as follows:

$$\sigma_t^2 = \alpha_0 + \alpha_1\left(\varepsilon_{t-1}^2 - \sigma_{t-1}^2\right) + \left(\alpha_1 + \beta_1\right)\sigma_{t-1}^2, \quad (16)$$

Then the sum of parameters $\alpha_1 + \beta_1$ shows the time which is needed for shocks in volatility to die out. If this sum is close to 1 long time is needed for shocks to die out. However, if the sum is equal to unity the covariance stationary condition is not satisfied and GARCH(1,1) model follows integrated GARCH process of order one, i.e. IGARCH(1,1).

If we substitute $\sigma_t^2 = \varepsilon_t^2 - v_t$ than stationary condition occurs from ARMA(1,1) representation of GARCH(1,1) model:

$$\varepsilon_t^2 = \alpha_0 + \left(\alpha_1 + \beta_1\right)\varepsilon_{t-1}^2 + v_t - \beta_1 v_{t-1}. \quad (17)$$

Proposed Algorithm

Volatility Calculation

Volatility parameters of GARCH(1,1) like variance – covariance matrix, Kurtosis, probability density function are calculated on basis of historical data. We have shown empirical results for last 14 years for S&P 500 in Table 1.

Boundary Value Calculation

Once volatility parameters are calculated, they are fed to the equation from which boundary value is obtained. Boundary values are calculated based on volume weighted average price and multiple weighted moving averages which is shown in figure 1.

Decision Support System

Economic theory frequently suggests that economic agents respond not only to the mean, but also to higher moments of economic random variable[9].If the opening value is greater than boundary value the conclusion is reached that the instrument is overvalued. Hence short position is taken with selling point being the difference between opening value and predicted volatility. On the other hand if the opening value is less than the boundary value the conclusion is reached that the instrument is undervalued. Hence, long position is taken with the selling point being the sum of volatility and opening. It can be noted that for both cases the profit is volatility.

We take multiple moving average prices and assign weights to that. In this algorithm we have taken 5,10,20,50,130,260 day moving averages. We have divided them into three different categories. Those are short term, medium term and long term averages. We have assigned those 30%, 30% and 40% weights. Out of those moving averages 5 and 10 day moving averages are treated as short term, 20 and 50 are considered as medium term moving averages. And 130 and 260 is considered as long term average. These are fed to the equation from which boundary value is obtained. This boundary value acts as the reference for the decision support system. Figure 1 shows the flow to find out boundary value.

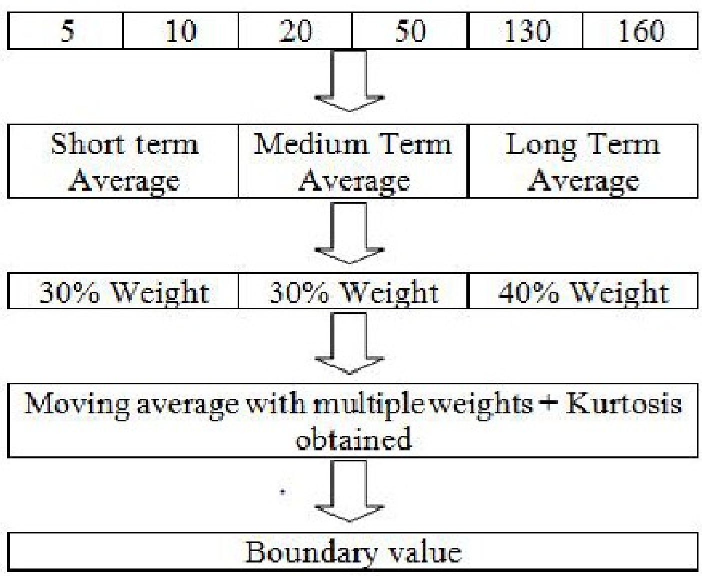

Figure 1: Boundary value calculation

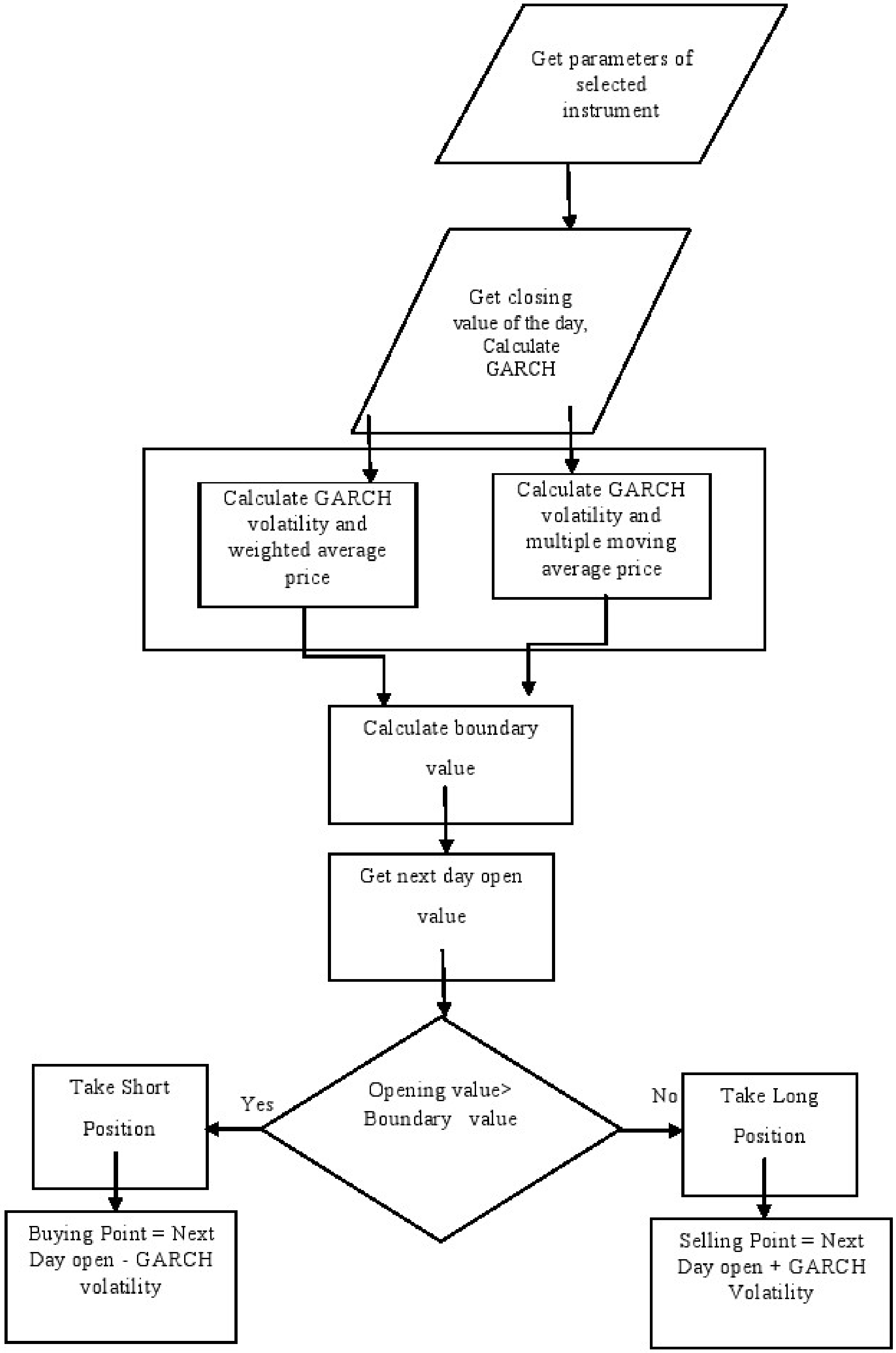

Figure 2: Algorithm flowchar

Empirical Results

The findings with the algorithms are presented here. The performance of the S&P 500 has been analyzed. Monte-Carlo simulation has been performed over random subset of stocks. We use daily data to make forecasts for the next day and have overnight time interval from the close of trading open of the next day, risk-free rates are used. The transaction cost incurred is 1% when we change our position. It will be based on tomorrow's closing price because we focus on out-of-sample prediction and we assume that we place our order to buy or sell immediately before the close of trading tomorrow. Of course we may use tomorrow's opening price or high frequency data in practice. We believe that here the strategy will be more profitable because of more flexibility and less delay. As shown in figure 2, multiple moving average or volume weighted average price can be used. To make decision more precise some short of neural network or genetic algorithm with use of fuzzy logic will definitely improve the decision criteria.

The leverage feedback effect has magnified the fluctuation in the market caused by the extreme events. For example, after Lehman Brothers declared its bankruptcy on September 14th, 2008, a series of bank and insurance company failures triggered the global financial crisis in which the market fluctuates dramatically. It is the extreme event, i.e., the declaration of Lehman Brothers' bankruptcy, together with the volatility clustering plus the leverage feedback effect caused by Lehman Brothers bankruptcy news result in the catastrophic financial crisis in 2008. So leverage and transaction cost cannot be neglected. Our empirical results take both of these parameters in account. Figure 3 shows Cumulative classic return obtained via logarithmic return for S&P 500 from 1994 to 2011.Table 1 shows all evaluation parameters comparison between volumes weighted average price and multiple moving average price of the test in detail.

Parameters	Value for Volume Weighted Avg. Price	Value for multiple moving Avg. Price
Logarithmic Return	195%	198%
Classic Return	580%	589%
Mean Daily Logarithmic return	0.0801%	0.0810%
Standard Deviation	0.0075	0.0073
Skewness	0.36	0.34
Kurtosis	10.085	10.024

Table 1: Performance of the strategy in terms of daily logarithmic returns for S&P 500

Conclusion

Andersen et al.1998 [1] and Christodoulakis and Satchell 2003 [5] have argued that the poor forecasting from GARCH models are too smooth to capture the entire variation of volatility. In our strategy volatility is the main parameter to get boundary value and entry point.

We have also introduced the concept of volume weighted average value of asset with multiple period based boundary conditions along with volatility. Moreover some short of artificial intelligence like neural network,genetic algorithm or fuzzy logic will make boundary value decision more precise. This gives more reliable entry points along with volatility. Moreover we have also included multiple moving average concepts. Both of this can be separately used in the strategy. The result shows that more return has beengained by using multiple moving average compare to volume weightedprice. By only using volume weighted price logarithmic return is approximately 195%, whereas with use of multiple moving average can get 198% of logarithmic return. Mean daily logarithmic return is also higher in case of multiple moving averages.

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