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A Comparative Analysis Of The Cobb Douglas Model And The Linear Regression Model Of Order(2)

Halid O.Y
Department of Mathematical Sciences
Ekiti State University
Ado Ekiti, Nigeria

Ogunwale O.D
Department of Mathematical Sciences
Ekiti State University
Ado Ekiti, Nigeria

Babalola B.T
Department of Mathematical and Physical Sciences
Afe Babalola University
Ado Ekiti,Nigeria

Abstract:

This paper carried out a comparative analysis of the Cobb-Douglas model and the Linear regression model of order (2) on the data of recharge cards consumption in Nigeria. Results showed that consumption of recharges cards is well explained by the price of the recharge cards and the consumer's income in both models but the two-variable model gives a higher co-efficient of determination 0.96 as compared to that of the Cobb Douglas model of 0.94. We therefore conclude that the two-variable linear regression fits the data better.

Keywords: *Regression, Cobb-Douglas model, Recharge Cards, Income, Consumer.*

Introduction

In the wake of deregulation in the telecommunication industries in Nigeria in the year 2000, there has been an astronomical growth in the acquirement of cell phones by every group of individuals in the country. The Nigerian Communications Commission (NCC) recorded a rise from 0.73% in December 2001 to 26.47% in August 2007 in tele-density. Telecommunication in Nigeria has been monopolized for long by Nigeria Telecommunications (NITEL) since 1985 before it became available in an open market in the advent of Global System of Mobile Communications (GSM) mobile operator providers. The total number of subscribers to telephone lines as at the end of December 1986 was put at around 230,000 while Telex subscribers were only 5,300 in number. Total installed capacity for telephone then was 320,834 and telex 11,577. Total installed capacity for telephone then was 320,834 and telex 11,577. The percentage utilisation for telephone therefore was 71.6 per cent while telex was approximately 45.7 per cent. However, modernity in telecommunications has provided facilities that make for new class of service, improved revenue generation with properly reviewed tariff policy. Nigeria embraced Digital Technology since the 1980s with the introduction of Digital Switches and Transmission Systems (Radio and Optic fibre) into the network. Since the beginning of the 90s, Mobile Telephone Services (Cellular), Paging and Electronic Mail have also been part of the services offered by NITEL. The importance of communication cannot be over-emphasized and when telephones are involved, it has a direct link with recharge cards. In Nigeria, the recharge card has become a vital item and is probably the most sort after product due to how Nigerians have embraced telecommunication in the 21st century. The Recharge card is a paper card that contains pins in form of digits that allows the end user to load credit or top up airtime on his/her mobile phone. They are of different prices ranging from 100 to 1,500 Naira. In Nigeria, people more commonly use the GSM as a reliable means of communication than fixed lines. GSM phones are everywhere and services supporting it are readily available than when compared to fixed lines that have become less popular due to its past history or unreliability. Even some Fixed Wireless telecoms operators also use recharge cards to enable customers load airtime on their phone by dialing the activation code and recharge card pin. Some recharge card is used by subscribers of GSM companies in Nigeria are; MTN, Globacom, Multilinks, Airtel, Etisalat,Visafone and Mobitel e.t.c. Even some Fixed Wireless telecoms operators also use recharge cards to enable customers load airtime on their phone by dialing the activation code and recharge card pin.

Materials And Methods

The models adopted are the Cobb-Douglas model and the two-variable regression model. The two-variable regression model represents a logical extension of the simple linear regression analysis. Two independent variables are used to estimate the value of a dependent variable. The regression equation for this analysis is given as;

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon_i \quad (1)$$

Where Y is the dependent variable, X1 and X2 are the independent variables and εi is the disturbance **term. This model can be transformed to;**

$$C = \beta 0 + \beta 1 P + \beta 2 I + \varepsilon i \quad (2)$$

Where C is the Consumption of recharge cards (Y), P is the Price of the recharge cards (X1),I is the Consumers income (X2), β0 is the intercept, β1 is the slope of the relationship C and P , β2 is the slope of the relationship C and I while ε is the disturbance term.

The Cobb-Douglas functional form of production function is widely used to represent the relationship between an output and inputs. The function is;

$$Y = AL\alpha K\beta \quad (3)$$

Where Y is the Total production, L is the Labour input, K is the Capital input, A is the Total factor productivity and α, β are the elasticties of Labour and Capital respectively. The above model can be **related to;**

$$C = \beta_0 P^{\beta_1} I^{\beta_2} \varepsilon \quad (4)$$

The model can be transformed into a linear form by taking the natural logarithm to get.

$$\ln C = \ln \beta_0 + \beta_1 \ln P + \beta_2 \ln I + \varepsilon \quad (5)$$

Anova Test

The analysis of variance (ANOVA) for two-variable regression analysis appraises the overall significance of the regression coefficients. It tests the null hypothesis that all the true population regression coefficients (slope) equal zero.

The Ordinary Least Square of Regression of Order (2)

From equation (1), we can minimize the error term and obtain the estimates of β_0, β_1 and β_2.**Since**

$$C = \beta_0 + \beta_1 P + \beta_2 I + \varepsilon_i$$

$$\sum \varepsilon_i = \sum (C - \beta_0 - \beta_1 P - \beta_2 I)^2 \quad (6)$$

To minimize the error term, we differentiate equation (6) with respect to β_0, β_1 and β_2 **and equate to zero. That is:**

$$\frac{\partial \sum \varepsilon_i^2}{\partial \beta_0} = 2\sum (C - \beta_0 - \beta_1 P - \beta_2 I) = 0$$

$$\frac{\partial \sum \varepsilon_i^2}{\partial \beta_1} = 2\sum (C - \beta_0 - \beta_1 P - \beta_2 I)(-P) = 0$$

$$\frac{\partial \sum \varepsilon_i^2}{\partial \beta_2} = 2\sum (C - \beta_0 - \beta_1 P - \beta_2 I)(-I) = 0$$

Consequently, the normal equations are;

$$\sum PC = \beta_1 \sum P^2 + \beta_2 \sum PI \quad (7)$$

$$\sum IC = \beta_1 \sum PI + \beta_2 \sum I^2 \quad (8)$$

Solving equations (7) and (8), will give;

$$\beta_1 = \frac{\sum PC \sum I^2 - \sum IC \sum PI}{\sum P^2 \sum I^2 - (\sum PI)^2} \quad (9)$$

$$\beta_2 = \frac{\sum IC \sum P^2 - \sum PC \sum PI}{\sum P^2 \sum I^2 - (\sum PI)^2} \quad (10)$$

Finally,

$$\beta_0 = \overline{C} - \beta_1 \overline{P} - \beta_2 \overline{I} \quad (11)$$

Analysis And Discussion Of Results

Given the primary data obtained by means of questionnaire, we seek to analysis the data in order to fit the two models stated earlier. Table 1 presents the analysis using Cobb-Douglas model.

C(N10,000)	$\ln C(Y)$	P(N100)	$\ln P(X_1)$	I(N100,000)	$\ln I(X_2)$
2.52	0.92	1.0	0	9.90	2.29
1.47	0.39	1.5	0.41	7.70	2.04
12.80	2.55	2.0	0.70	37.50	3.57
12.96	2.56	4.0	1.39	30.00	3.40
17.55	2.87	5.0	1.61	45.50	3.82
15.60	2.74	7.5	2.01	28.00	3.33
16.80	2.82	10.0	2.30	27.00	3.30
14.40	2.67	15.0	2.71	18.00	2.90

Table 1: Cobb-Douglas Table

the table above, we can create matrix for the dependent variable Y (consumption) and the explanatory variables X (price and income)

$$X = \begin{pmatrix} 1 & 0 & 2.29 \\ 1 & 0.41 & 2.04 \\ 1 & 0.7 & 3.57 \\ 1 & 1.39 & 3.40 \\ 1 & 1.61 & 3.82 \\ 1 & 2.01 & 3.33 \\ 1 & 2.30 & 3.30 \\ 1 & 2.70 & 2.90 \end{pmatrix} \quad Y = \begin{pmatrix} 0.9 \\ 0.39 \\ 2.55 \\ 2.56 \\ 2.87 \\ 2.74 \\ 2.82 \\ 2.67 \end{pmatrix}$$

Figure 1

With the knowledge of regression, $\beta = (X^1X)^{-1}X^1Y$ **. Therefore,**

$\beta = \begin{pmatrix} -2.061 \\ 0.4416 \\ 1.1676 \end{pmatrix}$ **. That is,** $\beta_0 = -2.061, \beta_1 = 0.4416$ **and** $\beta_2 = 1.1676$ **.**

The estimated Cobb-Douglas model is then;

$C = -2.061P^{0.4416}I^{1.1676}$

Source Of Variation	Degree Of Freedom	Sum Of Square	Mean Sum Of Square	F-Ratio
Regression	2	6.121	3.0605	39.24
Error	5	0.39	0.078	
Total	7	6.511		

Table 2: ANOVA TABLE (THE COBB-DOUGLAS MODEL)

From table 2,we can deduce that at 5% significant level, the critical F-value (F_{tab}) $= F_{0.05}(2,5) = 5.79 <$ **calculated F-value =**39.24 as such, we don't have sufficient evidence to reject the null hypothesis that states that all the β_i **is zero. So we conclude that either the price of recharge cards or the income of consumers is significant to consumption of recharge cards if not both. And since** $R^2 = SSR/SST = 0.941$**, this shows that 94% of the changes in consumption of recharge cards can be explained by the predicators. Upon testing for the significance of the co-efficient of the predictors using the student t-test, we obtained t-values of** β_1 and β_2 **as 3.45 and 5.64 respectively. Both values are greater than the critical t-value** $t_{(0.025,5)}$ **which is 2.571. Therefore, this shows that both predicators have influence on consumption of recharge cards.**

For Regression model of order (2), the matrixes obtained are;

$$X = \begin{pmatrix} 1 & 1 & 9.9 \\ 1 & 1.5 & 7.7 \\ 1 & 2 & 37.5 \\ 1 & 4 & 30 \\ 1 & 5 & 45.5 \\ 1 & 7.5 & 28 \\ 1 & 10 & 27 \\ 1 & 15 & 18 \end{pmatrix} \quad Y = \begin{pmatrix} 2.52 \\ 1.47 \\ 12.80 \\ 12.96 \\ 17.55 \\ 15.60 \\ 16.80 \\ 14.40 \end{pmatrix}$$

Figure 2

The coefficients of the regression are obtained to be:

$$\beta = \begin{pmatrix} -1.9151 \\ 0.7360 \\ 0.37 \end{pmatrix}$$ **That is,** $\beta_0 = -1.9151, \beta_1 = 0.736$ **and** $\beta_2 = 0.37$

The estimated Regression model is;

$$C = -1.9151 + 0.736P + 0.37I$$

This implies that a unit increase in the price of the recharge card with bring about a 0.736 increase in the consumption of it and a unit increase in consumer's income will bring about a 0.37 increase in consumption of recharge cards.

Source Of Variation	Degree Of Freedom	Sum Of Square	Mean Sum Of Square	F-Ratio
Regression	2	262.287	131.194	54.60
Error	5	12.013	2.4026	
Total	7	274.40		

Table 3: ANOVA TABLE (THE TWO-VARIABLE REGRESSION MODEL)

From the ANOVA table, we can deduce that one or both of the predictors have a significant effect on consumption of recharge cards since F-ratio=54.60> $F_{0.05}$ (2,5)=5.79.The R^2 value gives 0.96 and this indicates that 96% of the total variation in consumption can be explained by price and income. The t-values for β_1 and β_2 are 6.088 and 8.222 respectively. Since the values are both greater than the critical t-value $t_{(0.025,5)}$ =2.571, this shows that both the price of the recharge cards and the consumers income have an influence on the amount of recharge card consumed.

Summary

From the Cobb-Douglas analysis, the established model is $C = -2.061P^{0.4416}I^{1.1676}$ and the regression model of order two, the estimated model is $C = -1.9151 + 0.736P + 0.37I$. In both models, there is a positive relationship between price and consumption of recharge cards and there also exists a positive relationship between consumption and consumers income. Another test of significance of the regression coefficient and t-test reveal that both predictors are statistically significant at 5% level. The coefficient of determination of Cobb-Douglas model and the regression model are 0.94 and 0.96 respectively. It follows that both models are ideal for the data.

Conclusion

Based on the analysis, we can conclude that both price of the recharge cards and the consumed income have great significant effect on the consumption of recharge cards in Nigeria and that the regression model of order(2) preferred for this data since it has a higher coefficient of determination. It follows that it can fit the data better.

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