



“Wavelet Based Image Denoising Implementation”

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Abstract:

Magnetic Resonance Imaging (MRI) is a powerful diagnostic technique. However, the incorporated noise during image acquisition degrades the human interpretation, or computer-aided analysis of the images. Time averaging of image sequences aimed at improving the signal-to-noise ratio (SNR) would result in additional acquisition time and reduce the temporal resolution. Therefore, denoising should be performed to improve the image quality for more accurate diagnosis. This project implements different approaches of wavelet based image denoising methods. The wavelet transform is a simple and elegant tool that can be used for many digital signal processing applications. It overcomes some of the limitations of the Fourier transform with its ability to represent a function simultaneously in the frequency and time domains using a single prototype function (or wavelet) and its scales and shifts. In this project, some emerging wavelet domain Denoising methods such as soft and hard thresholding, bayeshrink, visushrink and SUREshrink are considered. The basic idea behind this thesis is the estimation of the uncorrupted image from the distorted or noisy image, and is referred to as image “denoising”.

Keywords: *Image denoising, Soft and hard, thresholding, Bayeshrink, Visushrink, SUREshrink*

1.Introduction

Owing to its rapidly increasing popularity over last few decades, the wavelet transform has become quite a standard tool in numerous research and application domains. This project is about wavelet domain image denoising: we study and develop statistical models and estimators for image wavelet coefficients given their noisy observations. In doing so, we are on a bridge between theory and applications. While merging theory and practice, from time to time we employ heuristics too.

To enhance the quality of medical images acquired by different sensors advanced denoising methods are required. Given a great variety of sensor technologies in medical electronics, and given that the same technologies appear in other application domains, it was decided to not to limit the proposed image denoising to any particular type of images.

In general, image denoising imposes a compromise between noise reduction and preserving significant image details. To achieve a good performance in this respect, a denoising algorithm has to adapt to image discontinuities. The wavelet representation naturally facilitates the construction of such spatially adaptive algorithms. It compresses the essential information in a signal into relatively few, large coefficients, which represent image details at different resolution scales.

S. Grace Chang [1] has proposed a spatially adaptive wavelet thresholding method based on context modeling. Here each wavelet coefficient is modeled as a random variable of a generalized Gaussian distribution with an unknown parameter. Context modeling is used to estimate the parameter for each coefficient, which is then used to adapt the thresholding strategy. Experimental results show that spatially adaptive wavelet thresholding yields significantly superior image quality and lower MSE.

M.I.H. Bhuiyan, M.O. Ahmad, M.N.S. Swamy [2] proposed a new spatially adaptive wavelet-based method in order to reduce the speckle noise from ultrasound images. Spatially adaptive threshold is introduced for denoising the coefficients of log-transformed ultrasound images. The threshold is obtained from a Bayesian maximum a Posteriori estimator. A simple and fast method is provided to estimate the parameters of the prior PDF from the neighbouring coefficients. They showed that the proposed method outperforms several existing techniques in terms of the signal-to-noise ratio, edge preservation index and structural similarity index and visual quality, and in addition, is able to maintain the diagnostically significant details of ultrasound images.

S. Grace Chang [3] proposed an adaptive, data-driven threshold for image denoising via wavelet soft-thresholding. The threshold is derived in a Bayesian framework, and the prior used on the wavelet coefficients is the generalized Gaussian distribution (GGD). The proposed threshold is simple and closed-form, and it is adaptive to each subband because it depends on data-driven estimates of the parameters. Experimental results show that the proposed method, called *BayesShrink*, is typically within 5% of the MSE. It also outperforms Donoho and Johnstone's *SureShrink* most of the time.

K. U. Barthel, H. L. Cycon [4] proposed a hybrid wavelet-fractal denoising method. Using a non-subsampled overcomplete wavelet transform the image was presented as a collection of translation invariant copies in different frequency subbands. Within this multiple representation fractal coding was done which tries to approximate a noise free image. The inverse wavelet transform of the fractal collage leads to the denoised image. The results were comparable to some of the most efficient known denoising methods.

The goal of the project is to estimate uncorrupted image from the distorted or noisy image which is referred to as image denoising. There are various methods to help restore an image from noisy distortions. In this project, some emerging wavelet domain Denoising methods are also considered. Selecting the appropriate method plays a major role in getting the desired image. In this project, a study is made on the various denoising algorithms and each is implemented in Matlab7.6 version

2.Theory Of Wavelet Based Noise Removal

Reducing noise from the medical images, a satellite image etc. is a challenge for the researchers in digital image processing. Several approaches are there for noise reduction. Generally speckle noise is commonly found in synthetic aperture radar images, satellite images and medical images. This paper proposes filtering techniques for the removal of speckle noise from the digital images. Quantitative measures are done by using signal to noise ration and noise level is measured by the standard deviation.

Medical images, Satellite images are usually degraded by noise during image acquisition and transmission process. The main purpose of the noise reduction technique is to remove speckle noise by retaining the important feature of the images. This section offers some idea about various noise reduction techniques. Synthetic Aperture Radar (SAR) imagery uses microwave radiation so that it can illuminate the earth surface. Synthetic Aperture Radar provides its own illumination. It is not affected by cloud cover

or radiation in solar illumination. ISUKF technique [5], which uses sampling to incorporate the Discontinuity adaptive Markov random field for the reduction of speckle noise. Context-based adaptive wavelet thresholding [6] method introduced a simple context-based method for the selection of adaptive threshold. Coherent filtering [7], is a speckle noise reduction technique of the ultrasound images. This technique is based on Coherent Anisotropic Diffusion for real time adaptive ultrasound Speckle noise reduction.

3.Problem Formulation

The basic idea behind this thesis is the estimation of the uncorrupted image from the distorted or noisy image, and is also referred to as image “denoising”. There are various methods to help restore an image from noisy distortions. Selecting the appropriate method plays a major role in getting the desired image. The denoising methods tend to be problem specific. For example, a method that is used to denoise satellite images may not be suitable for denoising medical images. In this project, a study is made on the various denoising algorithms and each is implemented in Matlab7.6 version.

Each method is compared and classified in terms of its efficiency. In order to quantify the performance of the various denoising algorithms, a high quality image is taken and some known noise is added to it. This would then be given as input to the denoising algorithm, which produces an image close to the original high quality image. The performance of each algorithm is compared by computing Signal to Noise Ratio (SNR) besides the visual interpretation.

In case of image denoising methods, the characteristics of the degrading system and the noises are assumed to be known beforehand. The image $s(x,y)$ is blurred by a linear operation and noise $n(x,y)$ is added to form the degraded image $w(x,y)$. This is convolved with the restoration procedure $g(x,y)$ to produce the restored image $z(x,y)$.

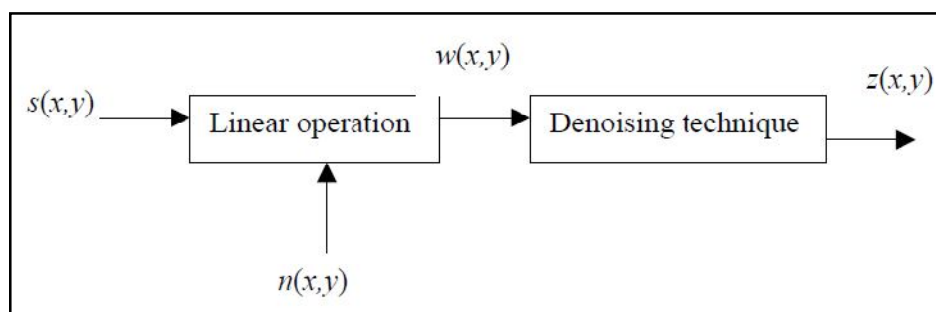


Figure 1: Denoising concept

The “Linear operation” shown in Figure 1 is the addition or multiplication of the noise $n(x,y)$ to the signal $s(x,y)$. Once the corrupted image $w(x,y)$ is obtained, it is subjected to the denoising technique to get the denoised image $z(x,y)$. The point of focus in this thesis is comparing and contrasting several “denoising techniques”.

Noise removal or noise reduction can be done on an image by filtering, by wavelet analysis, or by multifractal analysis. Each technique has its advantages and disadvantages. Denoising by wavelets and multifractal analysis are some of the recent approaches. Wavelet techniques consider thresholding while multifractal analysis is based on improving the Hölder regularity of the corrupted image.

4. Different Approaches For Thresholding

An image is often corrupted by noise in its acquisition or transmission. The underlying concept of denoising in images is similar to the 1D case. The goal is to remove the noise while retaining the important signal features as much as possible.

The noisy image is represented as a two-dimensional matrix $\{x_{ij}\}, i, j = 1, \dots, N$.

The noisy version of the image is modelled as

$$y_{i,j} = x_{ij} + n_{i,j} \quad i, j = 1, \dots, N$$

where $\{n_{ij}\}$ are *iid* as $N(0, \sigma^2)$. We can use the same principles of thresholding and shrinkage to achieve denoising as in 1-D signals. The problem again boils down to finding an optimal threshold such that the mean squared error between the signal and its estimate is minimized.

The wavelet decomposition of an image is done as follows: In the first level of decomposition, the image is split into 4 subbands, namely the HH, HL, LH and LL subbands. The HH subband gives the diagonal details of the image; the HL subband gives the horizontal features while the LH subband represent the vertical structures. The LL subband is the low resolution residual consisting of low frequency components and it is this subband which is further split at higher levels of decomposition.

The different methods for denoising we investigate differ only in the selection of the threshold. The basic procedure remains the same :

- Calculate the DWT of the image.
- Threshold the wavelet coefficients.
- Compute the IDWT to get the denoised estimate.

4.1. Hard And Soft Thresholding

Hard and soft thresholding with threshold λ , are defined as follows

The hard thresholding operator is defined as

$$D(U, \lambda) = \begin{cases} U & \text{for all } |U| > \lambda \\ 0 & \text{otherwise} \end{cases}$$

The soft thresholding operator on the other hand is defined as

$$D(U, \lambda) = \text{sgn}(U) \max(0, |U| - \lambda)$$

Hard threshold is a “keep or kill” procedure and is more intuitively appealing. It shrinks coefficients above the threshold in absolute value. While at first sight hard thresholding may seem to be natural, the continuity of soft thresholding has some advantages. It makes algorithms mathematically more tractable. Moreover, hard thresholding does not even work with some algorithms such as the GCV procedure. Sometimes, pure noise coefficients may pass the hard threshold and appear as annoying ‘blips’ in the output. Soft thresholding shrinks these false structures.

4.2. Visushrink

Visushrink is thresholding by applying the Universal threshold proposed by Donoho and Johnstone. This threshold is given by $\sigma \sqrt{2 \cdot \log(M)}$ where σ is the noise variance and M is the number of pixels in the image. It is proved that the maximum of any M values *iid* as $N(0, \sigma^2)$ will be smaller than the universal threshold with high probability, with the probability approaching 1 as M increases. Thus, with high probability, a pure noise signal is estimated as being identically zero.

However, for denoising images, Visushrink is found to yield an overly smoothed estimate. This is because the universal threshold (UT) is derived under the constraint that with high probability the estimate should be at least as smooth as the signal. So the UT tends to be high for large values of M , killing many signal coefficients along with the noise. Thus, the threshold does not adapt well to discontinuities in the signal.

4.3. Sureshrink

Let $\mu = (\mu_i : i = 1, \dots, d)$ be a length- d vector, and let $x = \{x_i\}$ (with x_i distributed as $N(\mu_i, 1)$) be multivariate normal observations with mean vector μ . Let $\hat{\mu} = \hat{\mu}(x)$ be an fixed estimate of μ based on the observations x .

SURE (Stein's unbiased Risk Estimator) is a method for estimating the loss $\|\hat{\mu} - \mu\|^2$ in an unbiased fashion.

In our case $\hat{\mu}$ is the soft threshold estimator $\hat{\mu}_i^{(t)}(x) = \eta_t(x_i)$.

We apply Stein's result to get an unbiased estimate of the risk $E\|\hat{\mu}_i^{(t)}(x) - \mu\|^2$:

$$SURE(t; x) = d - 2 \cdot \#\{i : |x_i| < t\} + \sum_{i=1}^d \min(|x_i|, t)^2.$$

For an observed vector x (in our problem, x is the set of noisy wavelet coefficients in a subband),

we want to find the threshold t^S that minimizes $SURE(t; x)$, i.e.

$$t^S = \operatorname{argmin}_t SURE(t; x).$$

The above optimization problem is computationally straightforward. Without loss of generality, we can reorder x in order of increasing $|x_i|$. Then on intervals of t that lie between two values of $|x_i|$, $SURE(t)$ is strictly increasing. Therefore the minimum value of t^S is one of the data values $|x_i|$. There are only d values and the threshold can be obtained using $O(d \log(d))$ computations.

4.4. BayesShrink

In *BayesShrink* we determine the threshold for each subband assuming a *Generalized Gaussian*

Distribution (GGD). The GGD is given by

$$GG_{\sigma_X, \beta}(x) = C(\sigma_X, \beta) \exp[-\alpha(\sigma_X, \beta)|x|^\beta]$$

$-\infty < x < \infty, \beta > 0$, where

$$\alpha(\sigma_X, \beta) = \sigma_X^{-1} \left[\frac{\Gamma(3/\beta)}{\Gamma(1/\beta)} \right]^{1/2}$$

and

$$C(\sigma_X, \beta) = \frac{\beta \cdot \alpha(\sigma_X, \beta)}{2\Gamma(1/\beta)}$$

and $\Gamma(t) = \int_0^\infty e^{-u} u^{t-1} du$.

The parameter σ_X is the standard deviation and β is the shape parameter. It has been observed that with a shape parameter β ranging from 0.5 to 1, we can describe the distribution of coefficients in a subband for a large set of natural images. Assuming such

a distribution for the wavelet coefficients, we empirically estimate β and σ_x for each subband and try to find the threshold T which minimizes the *Bayesian Risk*, i.e, the expected value of the mean square error.

Where $\hat{X} = \eta_T(Y), Y | X \sim N(x, \sigma^2)$ and $\hat{X} \sim GG_{x,\beta}$.

The optimal threshold T^* is then given by

$$\tau(T) = E(\hat{X} - X)^2 = E_x E_{Y|X}(\hat{X} - X)^2.$$

This is a function of the parameters σ_x and β . Since there is no closed form solution for T^* , numerical calculation is used to find its value.

It is observed that the threshold value set by

$$T_B(\sigma_x) = \frac{\sigma^2}{\sigma_x}$$

is very close to T^* .

The estimated threshold $T_B = \frac{\sigma^2}{\sigma_x}$ is not only nearly optimal but also has an intuitive appeal.

The normalized threshold, $\frac{T_B}{\sigma}$, is inversely proportional to σ , the standard deviation of

X, and proportional to σ_x , the noise standard deviation. When $\frac{\sigma}{\sigma_x} \ll 1$, the signal is

much stronger than the noise, $\frac{T_B}{\sigma}$ is chosen to be small in order to preserve most of the

signal and remove some of the noise; when $\frac{\sigma}{\sigma_x} \gg 1$, the noise dominates and the

normalized threshold is chosen to be large to remove the noise which has overwhelmed the signal. Thus, this threshold choice adapts to both the signal and the noise characteristics as reflected in the parameters σ and σ_x .

5.Results

The experimental evaluation is performed on three grey scale images 256×256 pixels at different noise levels. The wavelet transform employs Daubechies's least asymmetric compactly supported wavelet with eight vanishing moments at five levels of decomposition. The objective quality of the reconstructed image is measured by:

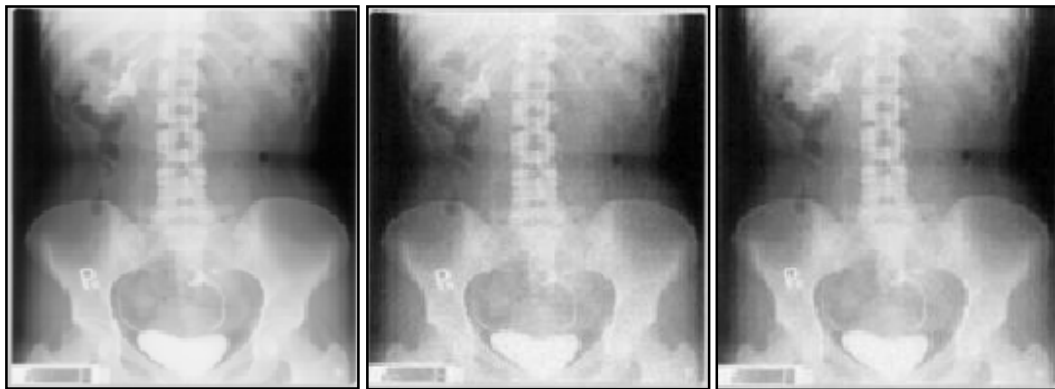
$$PSNR = 10 \log_{10} \frac{255^2}{mse} \text{ dB}$$

Where mse is the mean square error between the original and the denoised image with size $I \times J$:

$$mse = \frac{1}{I \times J} \sum_{i=1}^I \sum_{j=1}^J [x(i, j) - x(\hat{i}, \hat{j})]^2$$

Here the original image is corrupted with noise and then the image is recovered. MSE between the original image and enhanced image is calculated and is used in the calculation of PSNR. Thus enhancement in image quality is quantified using values of PSNR calculated for all output images enhanced through different algorithms.

Results are shown when the noise variance is 40 and wavelet decomposition is 5



(a)

2 (b)

(c)



(d)

(e)

(f)



(g)

Figure 2:a)original image b)noisy image when noise variance 40 c)soft thresholding d)hard thesholdinge)Sureshrink f)bayesshrink g)visushrink

Methods	MSE	PSNR	Time taken for execution
Soft thresholding	29.107655	33.490731	0.123431
Hard thresholding	16.169735	36.043775	0.156305
Sureshrink	11.484705	37.529605	4.628975
Bayeshrink	11.895056	37.377139	44.512027
Visushrink	32.371046	33.029236	48.245183

Table 1

6. Conclusion

The wavelet transform is a simple and elegant tool that can be used for many digital signal processing applications. It overcomes some of the limitations of the Fourier transform with its ability to represent a function simultaneously in the frequency and time domains using a single prototype function (or wavelet) and its scales and shifts. Use of FFT in filtering has been restricted due to its limitations in providing sparse representation of data. Wavelet Transform is the best suited for performance because of its properties like sparsity, multiresolution and multiscale nature. In addition to performance, issues of computational complexity must also be considered. Thresholding techniques used with the Discrete Wavelet Transform are the simplest to implement. When using Wavelet Transform, Nason emphasized that issue such as choice of primary resolution (the scale level at which to begin thresholding) and choice of analyzing wavelet also have a large influence on the success of the shrinkage procedure. When comparing algorithms, it is very important that researchers do not omit these comparison details. Several papers did not specify the wavelet used neither the level of decomposition of the wavelet transform was mentioned.

It is expected that the future research will focus on building robust statistical models of non-orthogonal wavelet coefficients based on their intra scale and inter scale correlations. Such models can be effectively used for image denoising and compression.

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