INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH & DEVELOPMENT

ISSN: 2278 – 0211 (Online)

Uncertainty Analysis For Ship Maneuvering In Model Scale And Full Scale Measurements

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Abstract:
In this paper the uncertainty analysis (UA) of the coefficients present in 4 DoF modular maneuvering mathematical model (Hull + Propeller + Rudder) has been done by considering error in data measurement and mathematical model. Asymmetric behaviour of rudder forces of starboard and port side rudder was analyzed. All relevant maneuvering coefficients were calculated from respective PMM tests and verified with available literature. There are sources of errors in data measurement, model structure, model algorithm, etc which are still to be analyzed. The DTMB 5415 was selected because of its added complexity due to twin-propeller twin-rudder configuration.

Ke words: *ship maneuvering, uncertainty analysis, roll motion, rudder asymmetry*

1.Introduction

The use of mathematical model to imitate dynamic behaviour of nonlinear systems requires frequent use of computer simulation. Using computer simulation to predict ship's maneuverability saves time and money. It is not feasible to make 100 percent accurate mathematical model, but simulation results should lie within acceptable limits. Therefore, uncertainty analysis of measurements, mathematical model, etc has become an important research topic. Uncertainty analysis is the analysis of the difference between the math-model output and the experimental output. Suitability of a mathematical model depends on several factors accuracy of experiment measurement, preciseness of mathematical model, influence of scale factors, etc. The captive maneuvering and free running model test reports for a twin rudder twin propeller surface combatant (DTMB 5415) ship is available through SIMMAN 2008 workshop held in Denmark. These data are being used as a benchmark for CFD code validation.

All linear, nonlinear and cross-coupled coefficients and acceleration derivatives can be determined accurately from the planar motion mechanism (PMM) captive model test. Uncertainty analysis (UA) for PMM benchmark data of model is carried out. Total uncertainties for multiple runs are estimated for static and dynamic PMM tests such as pure drift, static heel, drift + heel, pure sway, pure yaw, yaw + drift, etc. The existing modular maneuvering model is not sufficient to understand the hull-propeller-rudder interaction in twin-propeller-twin-rudder (TPTR) system. To make a wide range usable model, behaviour of the roll in maneuvering has analyzed. The asymmetric behavour of starboard and port rudder in drift and yaw condition, flow straightening coefficient, hull-rudder interaction coefficients are shown. Quantification and reduction of errors in full scale measurement are also important to predict ship maneuverability well.

2.Mathematical Model

For ships and other moving objects, it is convenient to describe their motion with respect to a coordinate system fixed in their body, instead of a coordinate system fixed in the earth. This is because, most of the motion sensors fitted on the ship measure the body dynamics in body fixed coordinate system. The coordinate system is fixed in the model. Origin for captive motion: intersection between the water line plane and the centre line plane at amidships. The x-axis is positive in the forward direction, the y-axis is positive towards starboard side and the z-axis is positive downwards. Angles, moments and directions of rotation follow the general right hand rule. The coordinate system for a twin-propeller-twin-rudder (TPTR) system is shown in Figure 1.

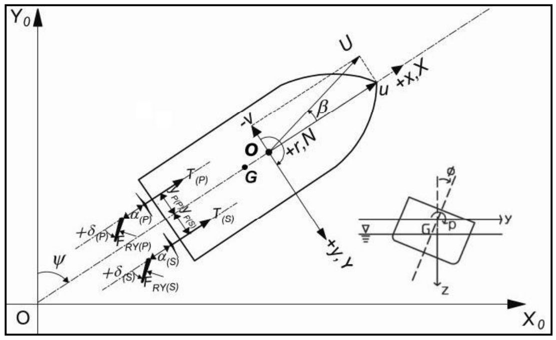

Figure 1: Coordinate system for TPTR ships

It is assumed that the heave and pitch motions are insignificant and are ignored. The 4 DoF maneuvering motion equations for the forces (surge, sway) and the moments (yaw, roll) has shown in Eq. 1.

$$\left.\begin{aligned} m\left(\dot{u}-rv-x_G r^2+z_G pr\right) &= X_H+X_P+X_R \\ m\left(\dot{v}+ur-z_G\dot{p}+x_G\dot{r}\right) &= Y_H+Y_P+Y_R \\ I_z\dot{r}+mx_G\left(\dot{v}+ur\right) &= N_H+N_P+N_R \\ I_x\dot{p}-mz_G\left(\dot{v}+ur\right) &= K_H+K_P+K_R \end{aligned}\right\} \quad (1)$$

The present 4DoF maneuvering mathematical model was developed based on an existing 4 DoF model [17].

2.1.Hull Forces And Moments

The hydrodynamic forces and moments acting on the hull are described as follows:

$$\left.\begin{aligned} X_H &= X_* + X_{\dot{u}}\dot{u} + X_{vv}v^2 + X_{rr}r^2 + X_{\phi\phi}\phi^2 + X_{vr}vr \\ &\quad + X_{|v||\phi|}|v||\phi| + X_{|v|}|v| + X_{|\phi|}|\phi| \\ Y_H &= Y_{\dot{v}}\dot{v} + Y_{\dot{\phi}}\dot{\phi} + Y_{\phi}\phi + Y_{\phi\phi\phi}\phi^3 + Y_v v + Y_{vvv}v^3 + Y_{\dot{r}}\dot{r} \\ &\quad + Y_r r + Y_{rrr}r^3 + Y_{vrr}vr^2 + Y_{rvv}rv^2 + Y_{v\phi\phi}v\phi^2 + Y_{\phi vv}\phi v^2 \\ &\quad + Y_{r\phi\phi}r\phi^2 + Y_{\phi rr}\phi r^2 \\ K_H &= K_{\dot{v}}\dot{v} + K_{\ddot{\phi}}\ddot{\phi} + K_{\dot{\phi}}\dot{\phi} + K_{\phi}\phi + K_{\phi\phi\phi}\phi^3 + K_v v + K_{vvv}v^3 \\ &\quad + K_{\dot{r}}\dot{r} + K_r r + K_{rrr}r^3 + K_{vrr}vr^2 + K_{rvv}rv^2 + K_{v\phi\phi}v\phi^2 \\ &\quad + K_{\phi vv}\phi v^2 + K_{r\phi\phi}r\phi^2 + K_{\phi rr}\phi r^2 - mgGM_T\phi - Y_H z_H \\ N_H &= N_{\dot{v}}\dot{v} + N_{\dot{r}}\dot{r} + N_{\dot{\phi}}\dot{\phi} + N_{\phi}\phi + N_{\phi\phi\phi}\phi^3 + N_v v + N_{vvv}v^3 \\ &\quad + N_r r + N_{rrr}r^3 + N_{vrr}vr^2 + N_{rvv}rv^2 + N_{v\phi\phi}v\phi^2 \\ &\quad + N_{\phi vv}\phi v^2 + N_{r\phi\phi}r\phi^2 + N_{\phi rr}\phi r^2 - Y_H x_G \end{aligned}\right\} \quad (2)$$

2.2.Propeller Forces

The hydrodynamic forces and moments due to a propeller can be written as follows:

$$
\left.
\begin{aligned}
&\mathrm{X}_P = (1-t_P)\,\rho\left(n_{P(s)}^2 D_{P(s)}^4 K_{T(s)} + n_{P(p)}^2 D_{P(p)}^4 K_{T(p)}\right)\\
&Y_P = 0\\
&N_P = (1-t_P)\,y_{P(s)}\rho\left(n_{P(p)}^2 D_{P(p)}^4 K_{T(p)} - n_{P(s)}^2 D_{P(s)}^4 K_{T(s)}\right)\\
&K_P = 0\\
&Q_P = -\rho\left(n_{P(s)}^2 D_{P(s)}^5 K_{Q(s)} + n_{P(p)}^2 D_{P(p)}^5 K_{Q(p)}\right)\\
&J_{P\binom{s}{p}} = u\left(1-w_{P\binom{s}{p}}\right) / \left(n_{P\binom{s}{p}} D_{P\binom{s}{p}}\right)
\end{aligned}
\right\} \quad (3)
$$

2.3.Rudder Forces And Moments

$$
\left.
\begin{aligned}
&\mathrm{X}_R = -(1-t_R)\left(F_{RY(s)}\sin\delta + F_{RX(s)}\cos\delta + F_{RY(p)}\sin\delta + F_{RX(p)}\cos\delta\right)\\
&\mathrm{Y}_R = -(1+a_H)\left(F_{RY(s)}\cos\delta - F_{RY(s)}\sin\delta + F_{RY(p)}\cos\delta - F_{RX(p)}\sin\delta\right)\\
&\mathrm{N}_R = -(x_R + a_H x_H)\left(F_{RY(s)}\cos\delta - F_{RX(s)}\sin\delta + F_{RY(p)}\cos\delta - F_{RX(p)}\sin\delta\right)\\
&\qquad + (1-t_R)\,y_{R(s)}\left(F_{RY(s)}\sin\delta + F_{RX(s)}\cos\delta - F_{RY(s)}\sin\delta - F_{RX(s)}\cos\delta\right)\\
&\mathrm{K}_R = -z_R \mathrm{Y}_R
\end{aligned}
\right\} \quad (4)
$$

$$
\begin{aligned}
&F_{\substack{RY(S)\\(P)}} = 0.5\rho A_R U_{\substack{R(S)\\(P)}}^2 f_\alpha \sin\alpha_{\substack{R(S)\\(P)}}\\
&f_\alpha = \frac{6.13\Lambda}{2.25+\Lambda}\\
&U_{\substack{R(S)\\(P)}} = \sqrt{u_{\substack{R(S)\\(P)}}^2 + v_{\substack{R(S)\\(P)}}^2}\\
&\alpha_{\substack{R(S)\\(P)}} = \delta - \delta_{\substack{0(S)\\(P)}} - \delta_{\substack{R(S)\\(P)}}\\
&\delta_{\substack{R(S)\\(P)}} = \tan^{-1}\left(\frac{v_{\substack{R(S)\\(P)}}}{u_{\substack{R(S)\\(P)}}}\right)
\end{aligned}
\quad (5)
$$

$$
\left.
\begin{aligned}
&v_{\substack{R(S)\\(P)}} = v_{\substack{RP(S)\\(P)}} - \gamma_{\substack{R(S)\\(P)}}(v + L_R r)\\
&u_{\substack{R(S)\\(P)}} = \varepsilon_{\substack{(S)\\(P)}} u_{\substack{P(S)\\(P)}} \sqrt{\eta_{\substack{P(S)\\(P)}}\left\{1+\kappa\left(\sqrt{1+\frac{8K_{\substack{T(S)\\(P)}}}{\pi J_{\substack{P(S)\\(P)}}^2}}-1\right)\right\}^2 + 1 - \eta_{\substack{P(S)\\(P)}}}\\
&\varepsilon_{\substack{(S)\\(P)}} = \frac{\left(1-w_{\substack{R(S)\\(P)}}\right)}{\left(1-w_{\substack{P(S)\\(P)}}\right)}, \quad \kappa = \frac{K}{\varepsilon_{\substack{(S)\\(P)}}}, \quad \eta_{\substack{P(S)\\(P)}} = \frac{A_{\substack{R1(S)\\(P)}}}{A_{\substack{R(S)\\(P)}}} = \frac{D_{\substack{P(S)\\(P)}}}{h_{\substack{R(S)\\(P)}}}\\
&u_{\substack{P(S)\\(P)}} = \left(1-w_{\substack{P(S)\\(P)}}\right)\left(u - y_{\substack{P(S)\\(P)}} r\right)
\end{aligned}
\right\} \quad (6)
$$

2.4.Ship Particulars

Hull	**Ship**	**Model**
Lpp (m)	142.00	4.002
Lwl (m)	142.18	4.008
Bwl (m)	19.06	0.538
T (m)	6.15	0.172
CB	0.507	0.507
Rudders		
Geometric aspect ratio	1.26	1.26
Lat. area, each (m^2)	15.4	0.0122
Offset of rudder axis (m)	4.75	0.134
Propellers		
No. of blades	5	4
D (m)	6.15	0.171
P/D (0.7R)	0.87	1.40
Ae/A0	0.58	0.56
Rotation from top	Inward	inward
Hub ratio	0.16	0.28
Offset of shaft (m)	4.65	0.131

Table 1: Principal dimensions of DTMB 5415

3.Pmm Data And Its Uncertainty Analysis (UA)

Uncertainty methodology and procedure are based on the recommended procedure and guidelines of International Towing Tank Conference (ITTC 2005) Maneuvering Committee (MC) and AIAA/ASME standards [1]. The uncertainty or error is broadly classified into bias 'B' and precision 'P' error. Bias limits are estimated with consideration of elemental error sources for individual variables, whereas precision limits are usually estimated end to end for experimental results based on single or multiple tests at the same test conditions. Total uncertainties are estimated with a root sum square (RSS) and normalization with the average value or dynamic range of the variable. Precision errors often result from fluctuating experimental conditions. The precision limits are assessed through repeated tests with the repeated tests. The measurement of uncertainty [18] is taken as 95% confidence uncertainty U as below:

$$\left.\begin{aligned} U &= \sqrt{\left(B^2 + P^2\right)} \\ B &= \sqrt{\sum\left(\theta_i B_i\right)^2} \\ P &= \frac{2S}{\sqrt{M}} \end{aligned}\right\} \qquad (7)$$

S and M are the standard deviation of the measured values and degree of freedoms respectively. The UA was applied on nondimensional values including roll angle. The forces and moments are nondimensionalized by $0.5\rho LdU^2$ and $0.5\rho L^2dU^2$. The data reduction equations for hull forces of X, Y, N, and K are given below:

$$\left.\begin{aligned} X'_H &= \frac{F_{X_{measured}} + m\left(\dot{u} - rv - x_G r^2 + z_G pr\right)}{0.5\rho U^2 T_m L} \\ Y'_H &= \frac{F_{Y_{measured}} + m\left(\dot{v} + ur - z_G \dot{p} + x_G \dot{r}\right)}{0.5\rho U^2 T_m L} \\ N'_H &= \frac{M_{N_{measured}} + I_z \dot{r} + m x_G \left(\dot{v} + ur\right)}{0.5\rho U^2 T_m L^2} \\ K'_H &= \frac{M_{K_{measured}} + I_x \dot{p} - m z_G \left(\dot{v} + ur\right)}{0.5\rho U^2 T_m L^2} \end{aligned}\right\} \qquad (8)$$

The total uncertainty of forces and moments are combination of systematic (bias) and random (precision) errors.

$$\left.\begin{aligned} U_{X'_H} &= \sqrt{B^2_{X'_H} + P^2_{X'_H}}, U_{Y'_H} = \sqrt{B^2_{Y'_H} + P^2_{Y'_H}} \\ U_{N'_H} &= \sqrt{B^2_{N'_H} + P^2_{N'_H}}, U_{K'_H} = \sqrt{B^2_{K'_H} + P^2_{K'_H}} \end{aligned}\right\} \qquad (9)$$

$$\left.\begin{aligned} B^2_{X'_H} &= \theta^2_{X_{H_{measured}}} B^2_{X_{H_{measured}}} + \theta^2_\rho B^2_\rho + \theta^2_{T_m} B^2_{T_m} + \theta^2_U B^2_U + \theta^2_{L_{pp}} B^2_{L_{pp}} + \theta^2_m B^2_m \\ &\quad + \theta^2_{x_G} B^2_{x_G} + \theta^2_{z_G} B^2_{z_G} + \theta^2_{\dot{u}} B^2_{\dot{u}} + \theta^2_v B^2_v + \theta^2_r B^2_r + \theta^2_p B^2_p \\ B^2_{Y'_H} &= \theta^2_{Y'_{H_{measured}}} B^2_{Y'_{H_{measured}}} + \theta^2_\rho B^2_\rho + \theta^2_{T_m} B^2_{T_m} + \theta^2_U B^2_U + \theta^2_{L_{pp}} B^2_{L_{pp}} + \theta^2_m B^2_m \\ &\quad + \theta^2_{x_G} B^2_{x_G} + \theta^2_{z_G} B^2_{z_G} + \theta^2_u B^2_u + \theta^2_{\dot{v}} B^2_{\dot{v}} + \theta^2_r B^2_r + \theta^2_{\dot{r}} B^2_{\dot{r}} + \theta^2_{\dot{p}} B^2_{\dot{p}} \\ B^2_{N'_H} &= \theta^2_{N'_{H_{measured}}} B^2_{N'_{H_{measured}}} + \theta^2_\rho B^2_\rho + \theta^2_{T_m} B^2_{T_m} + \theta^2_U B^2_U + \theta^2_{L_{pp}} B^2_{L_{pp}} + \theta^2_m B^2_m \\ &\quad + \theta^2_{x_G} B^2_{x_G} + \theta^2_{I_z} B^2_{I_z} + \theta^2_u B^2_u + \theta^2_{\dot{v}} B^2_{\dot{v}} + \theta^2_r B^2_r + \theta^2_{\dot{r}} B^2_{\dot{r}} \\ B^2_{K'_H} &= \theta^2_{K'_{H_{measured}}} B^2_{K'_{H_{measured}}} + \theta^2_\rho B^2_\rho + \theta^2_{T_m} B^2_{T_m} + \theta^2_U B^2_U + \theta^2_{L_{pp}} B^2_{L_{pp}} + \theta^2_m B^2_m \\ &\quad + \theta^2_{z_G} B^2_{z_G} + \theta^2_{I_x} B^2_{I_x} + \theta^2_u B^2_u + \theta^2_{\dot{v}} B^2_{\dot{v}} + \theta^2_r B^2_r + \theta^2_{\dot{p}} B^2_{\dot{p}} \end{aligned}\right\} \qquad (10)$$

3.1.Hull Forces And Moments

Maneuvering tests are typical transient tests regarding PMM test from which the deterministic parameters are derived [19]. To generate an accurate 4Dof maneuvering mathematical model, one should know the uncertainties in the predicted modeling coefficients. These uncertainties will come from different sources of error: manufacturing of physical model and appendages, captive model test procedure and execution, curve fitting of the experimental scattered data, nonlinear structure of the mathematical model and its parameters, environmental factors, etc. All PMM, CMT experiment data and uncertainty sources of model were adopted from FORCE [18] and MARIN [22] at $f_n = 0.248, 0.28, 0.332$.

The acceptance of any mathematical model depends on the accuracy of its outputs. Although models can never be 100% accurate as reality, but better results can be predicted by knowing the sources of uncertainty, their sensitivity and reduced by additional research analysis. In maneuvering model Eq. (1-6) the coefficients can be found by using of data of the captive model tests such as Rotating Arm test and Planar Motion Mechanism (PMM) tests in calm water. In Static Drift tests the model is towed obliquely in towing tank and the sway velocity related hydrodynamic coefficients X'_{vv}, Y'_v, Y'_{vvv}, K'_v, K'_{vvv}, N'_v, N'_{vvv} are determined. The yaw rate related coefficients X'_{rr}, Y'_r, Y'_{rrr}, K'_r, K'_{rrr}, N'_r, N'_{rrr} are from pure yaw tests, the acceleration derivatives $Y'_{\dot{v}}$, $K'_{\dot{v}}$, $N'_{\dot{v}}$, $Y'_{\dot{r}}$, $K'_{\dot{r}}$, $N'_{\dot{r}}$ are from pure sway and pure yaw PMM tests, sway-yaw coupling coefficients Y'_{vrr}, Y'_{vvr}, K'_{vrr}, K'_{vvr}, N'_{vrr}, N'_{vvr} are calculated from "yaw + drift". In PMM test the static tests forces and moments are the mean values, which are obtained as an average of the instantaneously measured values from each run. For the dynamic tests the resulting time varying forces and moments for one motion period are obtained by fairing with Fourier series consisting of some harmonics. The fairing is based on the three to four periods, which are measured in each run [18]. Faired signal of one complete time period is taken for analysis. The math-models Eq. 2 are further simplified by substituting the simplified PMM periodic motions for sway and yaw velocity to leave terms for the variables of interest and to determine the hydrodynamic derivatives [9]. The different maneuvering coefficients are extracted from different PMM tests data by doing Fourier series analysis of the above modified math-model and validated with [18] in Table 3. In fact it is a validation of our understanding and derivative extraction methodology from PMM test data along with validation of mathematical model. During this harmonic analysis, if a coefficient present in both constant and oscillating part of equation, then coefficient value is more accurate if it is determined from oscillating part i.e. the value of coefficient from constant part of harmonic equation should be avoided [24]. The coefficients are calculated from respective PMM tests to get better accuracy. Filtering of raw data, curve fitting of static and time history data etc. can be the important sources of model uncertainty during extraction of model coefficients from experimental data. A linear regression gives us a best-fit line for a scatter plot of data. The standard error of estimate (SEE) is one of the metrics that tells us about the fit of the line to the data. The SEE is the standard deviation of the errors (or residuals). Intuitively and fundamentally, Sum of Squared Error (SSE) is similar to variance. It is the square of difference between two variables. Standard Error of Estimate (SEE) is similar to standard deviation. It measures the relative distance of variable from benchmark value. The bias error for model can be determined from SEE as below.

$$\left.\begin{aligned} &Bias\ Error = 2 \times SEE \\ &SEE = \sqrt{\frac{\sum_{i=1}^{n}\left(O_{expt.} - O_{model}\right)^2}{n-k-1}} \end{aligned}\right\} \qquad (11)$$

Where, $(n-k-1)$ is the degree of freedom, which is the sample size (n) but reduced by the number of coefficients that need to be estimated. $O_{expt.}$ and O_{model} are the output of experiment and mathematical model respectively. It is assumed that the ship model movement is based on pure harmonic motions, however it is hardly correct, and the faired signals are obtained from noisy raw signals by Fast Fourier Transform (FFT) analysis where selection of exact frequency is uncertain. Unfortunately no raw signal is available regarding the model 5415, otherwise it could be demonstrated. The curve fitting error are taken into consideration were the SEE between the math-model output and corresponding experiment data of different PMM experiments. The mathematical modeling uncertainty for surge and sway forces and yaw moment are presented in By combining the experimental and modeling uncertainty value of forces and moment as per the uncertainty methodology Eq.7, the total uncertainty value is applied to the original PMM test data and so, two new set of hydrodynamic forces and moment are predicted at each data point at lower (-ve) and higher (+ve) value of uncertainty. Least square fit and Fourier series analysis are used on data signal to get two new set of model coefficients. In static tests the data acquired are less erroneous as compared to dynamic PMM tests. So the sway damping force derivatives $X_{vv}, Y_v, Y_{vvv}, N_v, N_{vvv}$ are more accurately extracted from pure drift test instead of pure sway test data. However, $Y_{\dot{v}}, N_{\dot{v}}$ from sway test, $Y_r, Y_{rrr}, N_r, N_{rrr}$ from pure yaw test and the sway-yaw coupling coefficients from "drift + yaw" test are sometime give more than 100 percentage error of the original value. Perhaps due to the stronger non-linearity in the pure sway Fourier series harmonics data [5], and/or due to improper selection of frequency during filtering of raw test data by spectrum analysis.

3.1.1.Maneuvering Roll Coefficients

The roll motions of a naval vessel can become significant during turning and zigzag maneuver at high speed. Rudder is also used for stabilizing roll in some naval vessels. Therefore the roll effect has to be considered in the horizontal equations of motion of the ship to predict the maneuverability well [21]. This ship model creates comparable roll motions during its turning [20]. The rolling moment has modeled by using the hydrodynamic pure roll, damping and added moment of inertia, the restoring moment due to hydrostatic and gravitational forces. A moment correction term has included due to the sway hydrodynamic force and its position vector of its acting point to the sensor location where rolling moment value are measured during test.

In Static Heel tests the model is towed with heel angle and the roll related hydrodynamic coefficients $X'_{\phi\phi}$, Y'_{ϕ}, $Y'_{\phi\phi\phi}$, K'_{ϕ}, $K'_{\phi\phi\phi}$, N'_{ϕ}, $N'_{\phi\phi\phi}$ are determined. The other coupling coefficients due to sway-roll $Y'_{v\phi\phi}$, $Y'_{vv\phi}$, $K'_{v\phi\phi}$, $K'_{vv\phi}$, $N'_{v\phi\phi}$, $N'_{vv\phi}$ and yaw-roll $Y'_{r\phi\phi}$, $Y'_{rr\phi}$, $K'_{r\phi\phi}$, $K'_{rr\phi}$, $N'_{r\phi\phi}$, $N'_{rr\phi}$ are calculated from "drift + heel", and "yaw + heel" respectively.

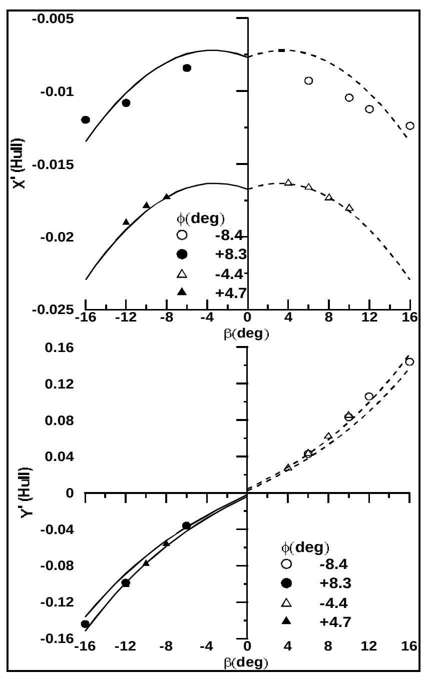

Figure 2a

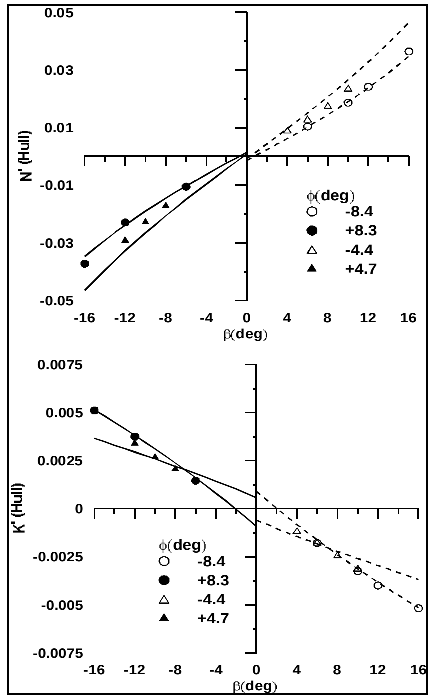

Figure 2b

Figure 2a, 2b: Reconstruction of nondimensional X, Y , N and K values by using maneuvering model (Eqs. 1-6) at ± 4° $f_n = 0.332$ *, ± 8°* $f_n = 0.248$ *at a range of drift angle. Dotted and Solid lines are modeling values and symbols are experimental values.*

The coefficients of the model were calculated at $f_n = 0.28$ used for reconstruction and it fits well. The offset of the model data fit line from the origin specifies the effect of roll in X, Y, N and K at ± 4° ± 8° even at straight run of vessel which can satisfy the reason of adding roll related derivatives in mathematical model Eqs. (1-6).

3.2.Propeller Forces

It is found that a polynomial equation can be established between wake fraction $1 - w_{P0}$ and thrust deduction $1 - t_{P0}$ versus advance ratio J . From open water test data thrust coefficient was calculated which is given below:

$$\left.\begin{array}{l} K_T = 0.6745 - 0.3448J - 0.0928J^2 \\ \left(1 - w_{P0\binom{s}{p}}\right) = \begin{pmatrix} 1.862 \\ 1.591 \end{pmatrix} + \begin{pmatrix} -1.803 \\ -1.226 \end{pmatrix} J + \begin{pmatrix} 0.919 \\ 0.623 \end{pmatrix} J^2 \\ \left(1 - t_{P0\binom{s}{p}}\right) = \begin{pmatrix} 0.874 \\ 0.807 \end{pmatrix} + \begin{pmatrix} 0.339 \\ 0.460 \end{pmatrix} J_P + \begin{pmatrix} -0.380 \\ -0.410 \end{pmatrix} J_P^2 \\ where\ J = u/(nD), J_P = u_P/(nD), u_P = u(1 - \omega_P) \end{array}\right\} \quad (12)$$

Propeller wake variation during maneuvering is asymmetric and significant. For ease of analysis we consider:

$w_{P0\binom{s}{p}} - w_{P\binom{s}{p}}$

This need to be investigated further. The coefficients of this wake model are determined using Hill-climbing method from the PMM pure-sway test data of MARIN. The nominal wake for model in straight run is very less; it is coming on third decimal point. The variations in coefficients are very less in maneuvering wake model.

3.3.Rudder Forces

With the huge demand for improving maneuvering ability of high speed vessels, twin propeller twin rudder (TPTR) systems have been adopted throughout this decade. However, it is still required to modify the modular maneuvering model by considering the asymmetric behavior of both the propellers and rudders during port and starboard turning. The flow straightening coefficients show a certain recognizable pattern in Figure 3 at less than 20° rudder angle with a wide range of drift angle variation. For starboard turning the starboard rudder (inboard rudder) wouldn't get straight flow stream like port rudder (outboard rudder), so $\gamma_{R(s)}$ is –ve & $\gamma_{R(p)}$ is +ve, and vice versa for port turning. PMM experiment data has taken from 'straight-run-rudder' test of MARIN [22]. Here drift angle, yaw rate are zero whereas rudder is only moving. The interaction coefficients a_H, x'_H are calculated by least square fit of experimental data and their value with uncertainty are shown in Table 3. The rudder normal forces of starboard and port side are reconstructed in Figure 4. Due to the value of flow straightening coefficient is not certain and L'_R has taken a constant value, rudder normal force model data fit line with experiment data is not so much accurate. Rudder needs a changes in mathematical model regarding sway and yaw velocity with introduction of higher nonlinear terms.

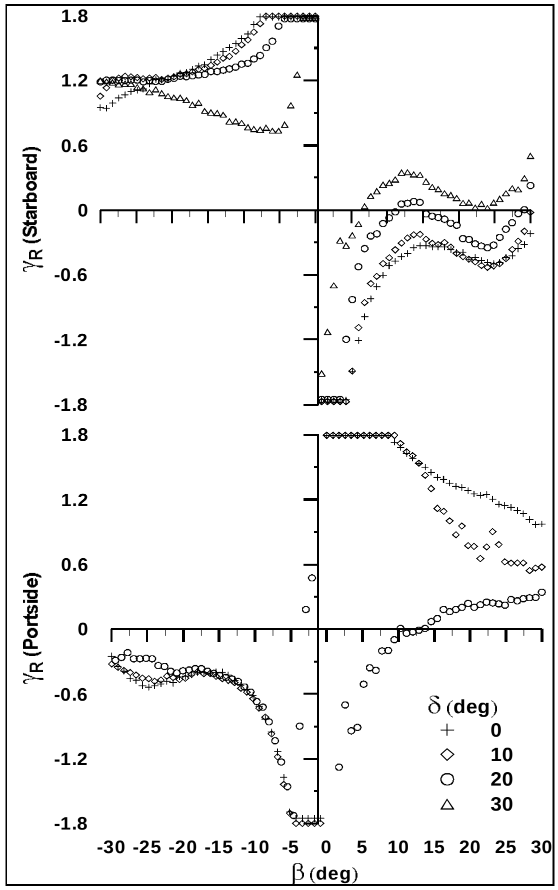

Figure 3: Variation of flow straightening coefficient related to sway γ_R for TPTR system. Starboard rudder (top), port rudder (bottom).

The polynomial equation GZ in full scale w.r.t roll angle in radian is

$$\overline{GZ} = 2.1956\phi - 1.09173\phi^3 \qquad (13)$$

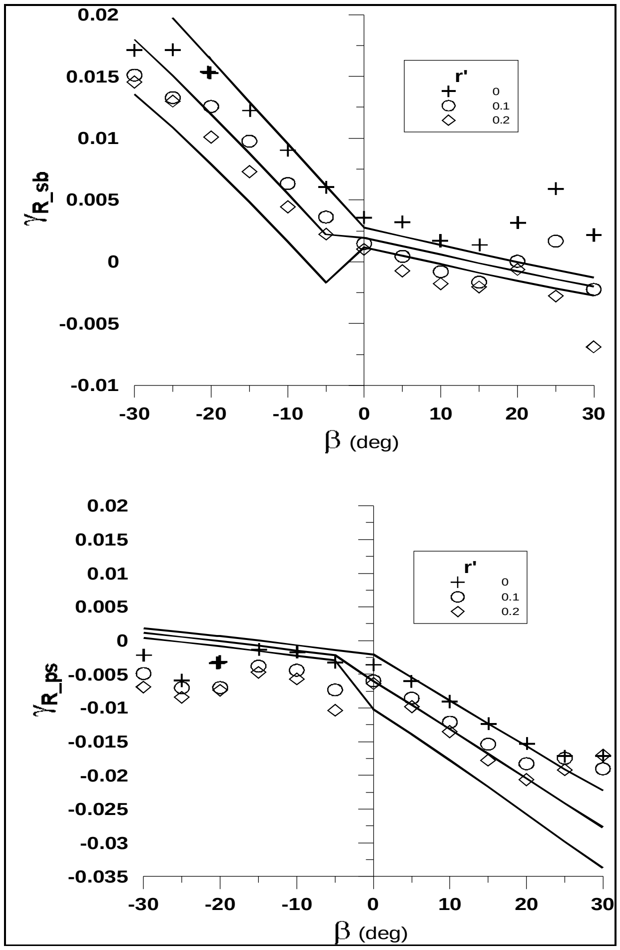

Figure 4: Variation of Rudder normal forces at different yaw rate at wide range of drift angle for starboard rudder (top), port rudder (bottom).

4. Uncertainties In Full Scale Measurements Eading

The uncertain maneuvering coefficients in model scale and input data errors in full scale are not independent. They can interact in various ways during simulation of ship scale maneuvering behaviours. Herein, some uncertainty in full scale parameters has quantified by Schulten [23] as shown in Table 4.

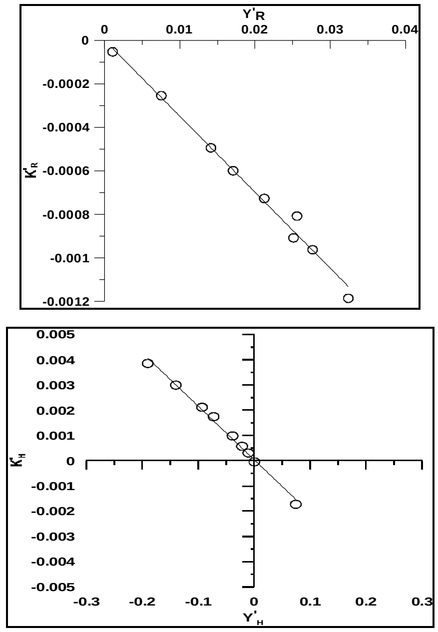

Figure5: Relation between rudder roll moment K'_R and sway force Y'_R on straight run (Top), between hull roll moment K'_H and sway force Y'_H under oblique run test with heel angle (bottom).

5.Conclusions

A 4 DoF maneuvering mathematical model for a TPTR model ship is developed using experiment data. The following are the main conclusions:

- Mathematical modeling uncertainty for sway-yaw-roll coefficients is more as compared to the experimental uncertainty. This indicates that the coefficients need to be determined with additional experiment data.
- Hull-rudder interaction coefficients show peculiar behaviour at below 5° and above 30° rudder angle. Besides above, the asymmetric behaviour of rudder for port and starboard maneuvers is significant. These characteristics have been modeled in this paper.
- There is 9% variation in thrust and 3% variation in torque of propeller during maneuvering motions. These need to be incorporated in the mathematical model.

The above factors need to be considered to investigate uncertainty in motion stability analysis of the vessel during high speed maneuvering motions.

PMM tests	**Type**	U_X %	U_Y %	U_N %	U_K %
Static drift	Expt.	6.53	2.17	1.07	4.93
	Modelling	7.31	9.11	10.31	8.23
Static heel	Expt.	11.68	17.27	7.11	20.34
	Modelling	17.19	6.85	16.473	9.1
Heel + drift	Expt.	-	-	-	-
	Modelling	32.11	8.57	15.97	28.22
Pure sway	Expt.	2.79	1.66	1.49	2.13
	Modelling	8.09	24.46	25.29	-
Pure yaw	Expt.	7.47	1.78	1.04	5.46
	Modelling	8.02	7.37	7.82	6.23
Yaw + drift	Expt.	6.28	2.41	6.10	3.002
	Modelling	197.05	59.06	83.03	30
Yaw + heel	Expt.	-	-	-	-
	Modelling	130	11.68	16.5	29.91

Table 2: Uncertainty values of X, Y, N, and K in different PMM tests

items	$U\%$	items	$U\%$
X'_{*}	10	N'_{r}	8
X'_{vv}	10	N'_{rrr}	8
$X'_{\|v\|}$	10	$N'_{\dot{r}}$	8
X'_{rr}	11	N'_{vrr}	83
$X'_{\phi\phi}$	20 9	N'_{vvr}	256
X'_{vr}	19 8	N'_{ϕ}	18
Y'_{v}	9	$N'_{v\phi\phi}$	18
Y'_{vvv}	9	$N'_{vv\phi}$	90
$Y'_{\dot{v}}$	25	K'_{v}	40
Y'_{r}	8	K'_{vvv}	7
Y'_{rrr}	8	K'_{r}	11
$Y'_{\dot{r}}$	8	K'_{rrr}	50
Y'_{vrr}	60	K'_{ϕ}	0
Y'_{rvv}	14 5	$K'_{v\phi\phi}$	15
Y'_{ϕ}	19	$K'_{vv\phi}$	21
$Y'_{\phi\phi\phi}$	27	L'_{R}	
$Y'_{v\phi\phi}$	34	a_{H}	8
$Y'_{vv\phi}$	11 3	x'_{H}	20
N'_{v}	10	z'_{H}	3
N'_{vvv}	10	z'_{R}	6.5
$N'_{\dot{v}}$	26	$(1-\omega_{P})$	-
$(1-t_{P})$	-	ϕ	0.15°

Table 3: Uncertainty values of maneuvering coefficients

Items	**Symbols**	$U\%$
Lengthbetween perpendiculars	L_{pp}	0.7
Beam	B	0.6
Draft	d	3
Water density	ρ	2
Vertical centre of gravity	KG	4.3
Radius of gyration	K_{zz}	8
Propeller diameter	D_P	0.2
Propeller thrust coefficient	K_T	3.5
Propeller torque coefficients	K_Q	3.5
Propeller effective wake fraction	$(1-\omega_P)$	5
Longitudinal position of propeller	x_P	1
Lateral position of propeller	y_P	4.3
Longitudinal position of Rudder stock	x_R	1
Lateral position of rudder stock	y_R	4.3
Vertical position of rudder C.G	Z_R	7
Angle of the rudder in XY plane	θ_{xy}	2.6
Rudder span	S_R	2.5
Rudder chord	C_R	3.4
Surge velocity	u	2.9
Sway velocity	v	2.8
Yaw velocity	r	2.7

Table 4: Uncertainty In Full-Scale Parameters

- **Nomenclature**

a_H = Interaction force coefficient induced on ship hull by rudder normal force

d = Ship's mean draft (m)

$D_{P(S)(P)}$ = Diameter of starboard and port propeller (m)

$F_{RY(S)(P)}$ = Normal force for starboard and port rudder (N)

$F_{RX(S)(P)}$ = Axial force for starboard and port rudder (N)

h_R = Rudder height for starboard and port rudder (m)

I_{zz} = Yaw moment of inertia (kg.m^2)

J_{zz} = Added yaw moment of inertia (kg.m^2)

$K_{T(S)(P)}$ = Thrust coefficient for starboard and port propeller

$L_{R(S)(P)}$ = Flow straightening coefficient of yaw rate for starboard and port rudder (m)

r = Yaw rate of ship (deg/sec)

$\dot{r}$ = Yaw acceleration of ship (deg/sec^2)

u = Ship's surge velocity (m/s)

$\dot{u}$ = Ship's surge acceleration (m/s^2)

v = Ship's way velocity (m/s)

$\dot{v}$ = Ship's sway acceleration (m/s^2)

x_H = Location of acting point of interaction force induced on ship hull by rudder normal force (m)

$y_{P(S)(P)}$ = Location of propeller in Y-axis for starboard and port propeller (m)

$\alpha_{R(S)(P)}$ = Effective inflow angle to starboard and port rudder (deg)

β = Drift angle of ship (deg)

$\gamma_{R(S)(P)}$ = Flow straightening coefficient of sway velocity for starboard and port rudder

$\delta_{\substack{(S)\\(P)}}$ = Angle of starboard and port rudder (deg)

$\delta_{0\substack{(S)\\(P)}}$ = Hydrodynamic neutral angle for starboard and port rudder (deg)

$\varepsilon_{\substack{(S)\\(P)}}$ = Ratio of effective wake fraction in way of propeller and rudder for starboard and port rudder

$\kappa_{\substack{(S)\\(P)}}$ = Propeller race amplification factor for starboard and port rudder

ρ = Water density (kg/m^3)

The subscript ‘s’ and ‘p’ refer to starboard and port twin–propeller and twin-rudder respectively.

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