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Computationally Efficient Design Of A Variable IIR Fractional Delay Filter

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Abstract:

In this paper, Taylor series expansion is used to design the variable IIR fractional delay filter. The proposed design is based upon collective response of two IIR filters having first-order. The Al-alaoui operator with α-approximation differentiator and reasonable modifications in this differentiator are caused to be designing these filters. The constructed Farrow structure is more computationally efficient than existing structures. In the end, to illustrate the effectiveness of proposed design approach magnitude response and group delay response are shown.

Keywords*: Al-alaoui operator, Fractional Delay Filter (FD), IIR, Taylor series expansion, Variable Fractional Delay filters (VFD).*

1. Introduction

In many areas of digital signal processing fields, a system which can provide fraction of unit delay is required. The Fractional delay can be classified into two categories, fixed fractional delay and variable fractional delay. The concept on design of VFD techniques comes in reality once Farrow structure was presented [1]. Variable fractional delay filter will give more flexibility and more interesting areas compare to fixed fractional delay. There are many applications include time delay estimation [2], speech coding [3], speech assisted video processing [4], beam steering of antenna array, modeling of music instruments, comb filter design and analog digital conversion etc. [5-15]. An excellent survey is presented in tutorial paper [12-13]. The desired frequency response of variable fractional delay filter is given by (1).

$$H_d(w, p) = e^{-jw(I+p)} \qquad (1)$$

Where delay "I" is an integer delay, and "p" is a variable or adjustable fractional delay in range of [-0.5 to 0.5]. The transfer function of FIR filters to approximate the frequency response given by Farrow as (2).

$$H(z, p) = \sum_{n=0}^{N-1} a_r(p) Z^{-r} \qquad (2)$$

Where $a_r(p)$ represents the coefficients of filter which depends upon two variables r, p & N is the order of the filter. However to design the IIR filters Al-alaoui operator with α-approximation transform is used as it attempts to obtain better approximation [16-19] comparing to bilinear transform impulse invariant technique. The bilinear transform introduces a warping effect due to its nonlinearity, whereas impulse invariant technique is

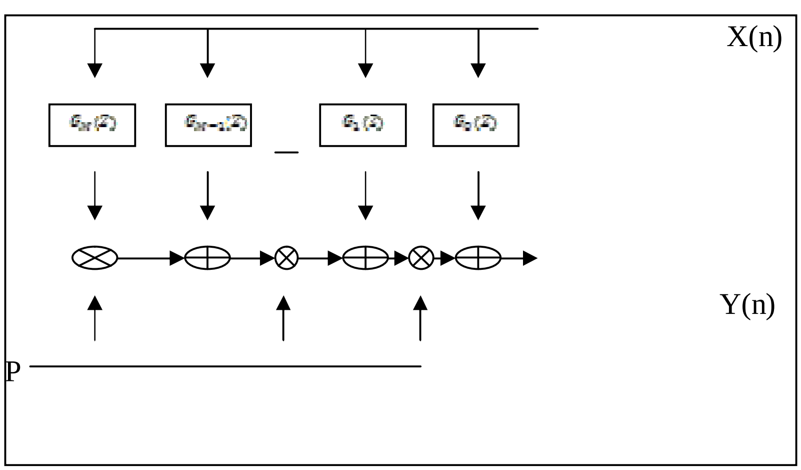

Figure 1: Farrow Structure for FD with adjustable delay ‘p’.

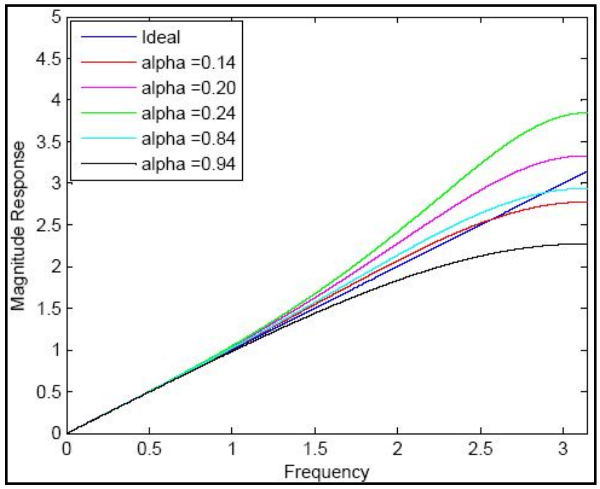

Figure 2: Comparison between Ideal Differentiator and Al-Alaoui Operator at different α-values.

affected by the many to one mapping condition. In particular [16-17], the approach interpolates the rectangular integration rules and the trapezoidal integration rule. In [16] a fixed weighting of 0.75 was assigned to the rectangular rule and 0.25 for the trapezoidal rule, while in [17] the interpolation was parameterized with α-parameter. The operator developed in [17] may be designated as the parameterized Al-alaoui operator. In [19] the α-approximation is proposed. Hence, combining the Al-alaoui operator with α-approximation construct a better differentiator then existing transforms. In Figure 2 magnitude responses of Al-alaoui operator with different α-approximation values are compared with the Ideal differentiator, which helps in to getting best value α. A considerable change in differentiator give another IIR filter. Hence, using these filters proposed structure in Figure 3 is more computationally efficient than the Farrow structure in Figure 1. Finally, the related researches can be found in [21] and [22]. However, the idea of structuring of VFD filter using first-order IIR filters is novel.

2. DESIGN METHOD

In this section, design of the variable fractional delay filter using Taylor series expansion is described. The desired fractional delay response as shown in (1), where the e^{-jwp} can be expanded by Taylor series as a polynomial of ‘p’ as in (3).

$$e^{-jwp} = \sum_{k=0}^{\infty} \frac{(-jwp)^k}{k!} \qquad (3)$$

it can be rewritten as

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} (jw)^k p^k$$

$$= \sum_{k=0}^{M} \frac{(-1)^k}{k!} (jw)^k p^k + p^{M+1} \sum_{k=M+1}^{\infty} \frac{(-1)^k}{k!} (jw)^k p^{(k-M-1)}$$

$$= \sum_{k=0}^{M} \frac{(-1)^k}{k!} (jw)^k p^k + O(p^{M+1})$$

Because the fractional delay is in the range of [-0.5 to 0.5], the term $O(p^{M+1})$ approaches zero when M is very large. Thus, it can be approximated as (4).

$$e^{-jwp} \approx \sum_{k=0}^{M} \frac{(-1)^k}{k!} (jw)^k p^k \quad (4)$$

Now here, the term "jw" can be realized by any digital

differentiator filter G(z) as (5).

$$e^{-jwp} = \sum_{k=0}^{M} \frac{(-1)^k}{k!} (G(z))^k p^k \quad (5)$$

If integer delay is also incorporated in (5), then

$$e^{-jw(I+p)} = [\sum_{k=0}^{M} \frac{(-1)^k}{k!} (G(z))^k p^k] z^{-I} \quad (6)$$

But, here the 'I' is independent of k, so z^{-I} term (integer delay) can be taken inside the summation; finally the proposed filter is given by (7).

$$e^{-jw(I+p)} = \sum_{k=0}^{M} \frac{(-1)^k}{k!} (G(z))^k z^{-I} p^k \quad (7)$$

Here, the proposed filter $G(z)$ is the collective response of filters $G_1(z)$ & $G_2(z)$. Both the filters are IIR filters having order of only of (1, 1). The $G_1(z)$ filter is derived from the Al-alaoui operator with α-approximation factor as shown in (8).

$$G_1(z) = \frac{1}{T} \frac{z-1}{(1-\alpha)+\alpha z} \quad (8)$$

The exact value of α is decided by the Figure 2, in which magnitude responses at different values of α are shown.

According to that a suitable value of α is taken which fulfill the objective of designed filter. The $G_1(z)$ filter is workable in lower half of nyquist band as its magnitude and group delay responses are not corrupted with error in this range. Now, both magnitude and group delay responses to be lies at upper half of nyquist band another filter $G_2(z)$ has to be proposed. It is seen that some logical modifications in $G_1(z)$ give the idea of $G_2(z)$ as (9).

$$G_2(z) = \frac{1}{T}\frac{z+1}{(1-\alpha)-\alpha z} \qquad (9)$$

Hence at last, the collective responses of both filters span in complete nyquist band.

3. Results

In this section, to illustrate the effectiveness of performance of the proposed designed method magnitude response and group delay response are shown in Figure 4. To evaluate the performance parameters chosen to design the filters are filter order is (1, 1) and fractional delay ‘p’ lies in full range from -0.5 to 0.5 with integer delay ‘I’ of 6. It is clearly seen in Figure 4 that magnitude response is very near to ideal response which should be ‘1’ for complete nyquist band although it has some error below 0.2π and above 0.8π. The strength of the proposed method lies in the group delay response as it is very accurate to ideal response which should be sum of integer delay and fractional delay for full nyquist band. Now, three aspects of the efficiency are used to compare the Farrow structure in Figure 1 with the proposed structure in Figure 3.

3.1.Storage requirement

For implementation of the proposed structure, only coefficients of a first order IIR filters need to be stored in memory.

3.2.Restriction free FD value

In Farrow structure, the integer delay is fixed whereas the proposed structure has a universal response for all value of integer delay and fractional delay.

3.3.Number Of Sub-Filters

The Farrow structure has M+ 1 sub-filters $G(z)$, but in our structure number of sub-filters M is very less (only-3).

Thus, the proposed structure is more computationally efficient than the existing design.

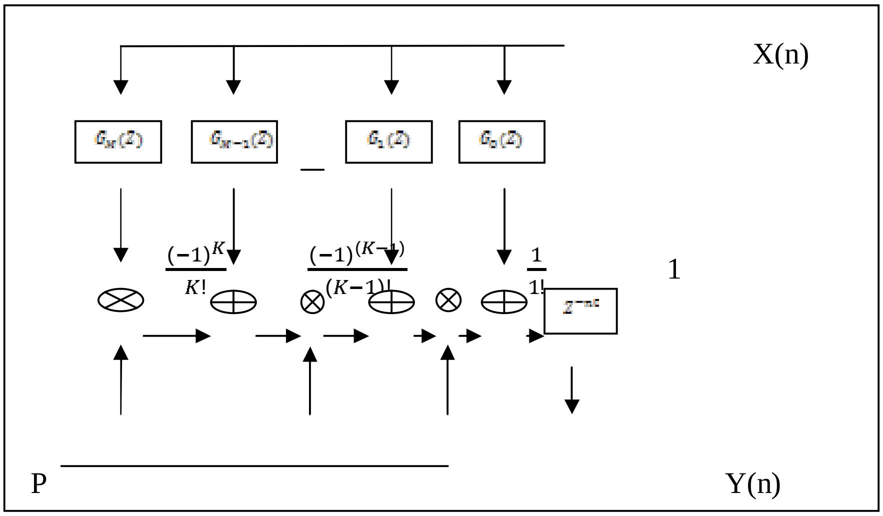

Figure 3:Proposed Farrow structure based on designed VFD filter

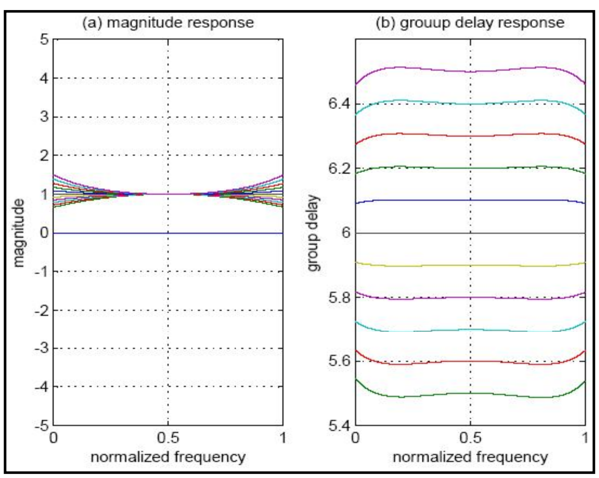

Figure 4

4. Conclusion

In this paper, variable fractional delay IIR filter is designed with the help of Taylor series. Proposed method based structure is more computationally efficient than the existing design as coefficient storage requirement is very less for first-order only. However, magnitude response of design method has some deviation below 0.2π and above 0.8π frequency bands. So there is possibility of design a filter such that which magnitude response should lie for complete nyquist band with the help of Taylor Series or other such expansion series.

5. Refrence

1. C. W. Farrow, “A continuously variable digital delay element”, in Proc. IEEE Int. Symp. Circuits System (ISCAS-88), vol-3 Espoo, Finland, June 6-9, 1988, pp 2641-2645.
2. J. O. Smith and B. Friedlander, “Adaptive interpolated time delay estimation”, IEEE Trans Aerospace and Electronic System, vol. AES- 21, no-3, March 1985, pp 180-199.
3. P. Kroom and B. S Atal, “Pitch prediction and temporal resolution” in Proc. IEEE Intr. conf. Acoustic Speech Signal Processing (ICASSP-90), vol-2, Albuquerque, New Mexico, April 3-6, 1990, pp 661-664.
4. T. Chen, H. P Graf and K. Wang, “Speech-assisted video processing: interpolation and low bit rate coding”, in Proc. 28th Annual Asilomar Conf. On Signals, System and Computers, Pacific Grove, California, oct-1994, pp 975-979.
5. J K. Rajamani, Y.S. Lai and C.W. Farrow, “An efficient algorithm for sample rate conversion from CD to DAT”, IEEE Signal Processing Lett., vol. 7, Oct. 2000, pp. 288- 290.
6. C. C. Tseng, “Eigen filter approach for the design of variable fractional delay FIR and all-pass filters”, Proc. Inst. Elect. Eng. Vis. Image Signal Process. vol. 149, no. 5, Oct. 2002, pp. 297–303.
7. H. Johansson and P. Löwenborg, “On the design of adjustable fractional delay FIR filters”, IEEE Trans. Circuits Syst. II, Analog Digital Signal Process, vol. 50, no. 4, Apr. 2003, pp. 164–169.
8. T. B. Deng, “No iterative WLS design of all pass variable fractional delay digital filters”, IEEE Trans. Circuits Syst. I, Reg. Papers, vol. 53, no. 2, Feb. 2006, pp. 358–371.
9. H. Zhao and J. Yu, “A simple and efficient design of variable fractional delay FIR filters”, IEEE Trans. Circuits Syst. II, Exp. Briefs, vol. 53, no. 2, Feb. 2006, pp. 157–160.
10. K. M. Tsui, S. C. Chan, and H. K. Kwan, “A new method for designing causal stable IIR variable fractional delay digital filters”, IEEE Trans. Circuits Syst. II, Exp. Briefs, vol. 54, no. 11, Nov. 2007, pp. 999–1003.

11. S. Basu and Y. Bresler, "An empirical study of minimax-optimal fractional delays for low-pass Signals", IEEE Trans. Circuits Syst. 11, vol. 49, Apr. 2002, pp. 288-292.
12. T.I. Laakso, V. Valimaki, M. Karjalainen and U.K.Laine, "Splitting the unit delay: Tools for fractional delay filter design", IEEE Signal Processing Mag., vol.13, Jan. 1996, pp. 30-60.
13. V. Valimaki and T.I. Laakso, "Principle of fractional delay filters", Int. Conf. Acoust. Speech Signal Processing, May 2000, pp. 3870-3873.
14. T.-B. Deng, "'Variable 2-D FIR digital filter design and parallel implementation", IEEE Trans. Circuits Syst. II, vol. 46, May 1999, pp. 631-635.
15. Z. Wang, "Interpolation using the discrete cosine transform reconsideration", Electronics Letters, vol.29, Jan. 1993, pp.198-200.
16. M. A. Al-alaoui, "Novel digital integrator and differentiator", IEEE Electronics Letters, vol. 29, no. 4, Feb.18, 1993, pp. 376–378.
17. M. A. Al-alaoui, "Filling the gap between the bilinear and the backward difference transforms: An interactive design approach", International Journal of Electrical Engineering Education, vol. 34, no. 4, Oct. 1997, pp. 331–337.
18. M. A. Al-alaoui, "Novel stable higher order s-to-z transforms", IEEE Trans. on Circuits and Systems I: Fundamental Theory and Applications, vol. 48, no. 11, Nov. 2001, pp. 1326 –1329,
19. T. B. Sekara and M. R. Stojic, Application of the a-approximation for disretization of analogue systems", Facta Universitatis, Ser.: Elec. and Energ., no. 3, Dec. 2005, pp. 571– 586.
20. F. Harris, "Performance and design considerations of the Farrow filter when used for arbitrary resampling of sampled time series", in Proc. 31st Asilomar Conf. Signals, Systems, and Computers, vol. 2, Nov. 1997, pp. 1745–1749.
21. Soo-Chang Pei, Chien-Cheng Tseng, " An Efficient Design of a Variable Fractional Delay Filter Using a First-Order Differentiator", IEEE signal processing letters, vol. 10, no. 10, Oct. 2003,pp. 307-310.
22. R. Sudhakar, R. C. Agarwal, and S. C. Dutta Roy, "Time domain interpolation using differentiators", IEEE Trans. Acoust. Speech Signal Process. Vol. ASSP-30, Dec. 1982, pp. 992–997.