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Magic Graphidal on Product graphs

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Abstract:

B.D.Acharya and E. Sampathkumar [1] defined Graphoidal cover as partition of edge set of G into internally disjoint paths (not necessarily open). The minimum cardinality of such cover is known as graphoidal covering number of G.

Let G = {V, E} be a graph and let ψ *be a graphoidal cover of G. Define* $f: V \cup E \rightarrow \{1, 2, \ldots, p+q\}$ *such that for every path* $P = (v_0v_1v_2 \ldots v_n)$ *in* ψ *with* $f^*(P) = f(v_0) + f(v_n) + \sum_{1}^{n} f(v_{i-1}v_i) = k$*, a constant, where* f^* *is the induced labeling on* ψ*. Then, we say that G admits* ψ *- magic graphoidal total labeling of G.*

A graph G is called magic graphoidal if there exists a minimum graphoidal cover ψ *of G such that G admits* ψ *- magic graphoidal total labeling.*

In this paper, we proved that Book $K_{1,n} \times K_2$*, Ladder* $P_n \times K_2$ *and* $C_n \times K_2$ *are magic graphoidal.*

Key words : *Graphoidal Cover, Magic Graphoidal, Graphoidal Constant.*

2000 Mathematics Subject Classification 05C78.

1.Introduction

By a graph, we mean a finite simple and undirected graph. The vertex set and edge set of a graph G denoted are by V(G) and E(G) respectively. $K_{1,n} \times K_2$ is a Book, $P_n \times K_2$ is Ladder and $C_n \times K_2$. Terms and notations not used here are as in [3].

2.Preliminaries

Let G = {V, E} be a graph with p vertices and q edges. A graphoidal cover ψ of G is a collection of (open) paths such that

(i) every edge is in exactly one path of ψ

(ii) every vertex is an interval vertex of atmost one path in ψ.

We define $\gamma(G) = \min_{\psi \in \zeta} |\psi|$,

where ζ is the collection of graphoidal covers ψ of G and

γ is graphoidal covering number of G.

Let ψ be a graphoidal cover of G. Then we say that G admits ψ - magic graphoidal total labeling of G if there exists a bijection f: $V \cup E \rightarrow \{1, 2, \ldots, p+ q\}$ such that for every path $P = (v_0 v_1 v_2 \ldots v_n)$ in ψ, then, $f^*(P) = f(v_0) + f(v_n) + \sum_{1}^{n} f(v_{i-1} v_i) = k$, a constant, where f^* is the induced labeling of ψ. A graph G is called magic graphoidal if there exists a minimum graphoidal cover ψ of G such that G admits ψ - magic graphoidal total labeling. In this paper, we proved that Book $K_{1,n} \times K_2$, Ladder $P_n \times K_2$ and $C_n \times K_2$ are magic graphoidal.

Result 2.1 [11] **:**

Let G = (p, q) be a simple graph. If every vertex of G is an internal vertex in ψ then $\gamma(G) = q - p$.

Result 2.2 [11] :

If every vertex v of a simple graph G, where degree is more than one ie $d(v) > 1$, is an internal vertex of ψ is minimum graphoidal cover of G and $\gamma(G) = q - p + n$ where n is the number of vertices having degree one.

Result 2.3 [11] :

Let G be (p, q) a simple graph then $\gamma(G) = q - p + t$ where t is the number of vertices which are not internal.

Result 2.4 [11] :

For any tree G, $\gamma(G) = \Delta$ where Δ is the maximum degree of a vertex in G.

Result 2.5 [11] :

For any k – regular graph G, $k \geq 3$, $\gamma(G) = q - p$.

Result 2.6 [11] :

For any graph G with $\delta \geq 3$, $\gamma(G) = q - p$.

3. Magic Graphoidal On Product Graphs

Theorem 3.1 :

Book $K_{1,n} \times K_2$ is magic graphoidal.

Proof:

Let $G = K_{1,n} \times K_2$

$$V(G) = \{u, v, u_i, v_i : 1 \leq i \leq n\}$$

$$E(G) = \{(uv) \cup [(uu_i) \cup (vv_i) \cup (u_iv_i) / 1 \leq i \leq n]\}$$

Define $f : V \cup E \rightarrow \{1, 2, 3, \ldots, p+q\}$ by

$$f(u) = p + q - 1$$

$$f(v) = p + q$$

$$f(u_i) = 4n + i, \qquad 1 \leq i \leq n$$

$$f(v_i) = f(vv_i) + 1 \qquad 1 \leq i \leq n$$

$$f(uu_i) = i \qquad 1 \leq i \leq n$$

$$f(u_iv_i) = n + i \qquad 1 \leq i \leq n$$

$$f(vv_{n+1-i}) = (2n - 1) + 2i \qquad 1 \leq i \leq n$$

$$f(uv) = p + q - 2$$

Let $\psi = \{(uv), (uu_iv_iv) / 1 \leq i \leq n\}$

$$f^*[(uv)] = f(u) + f(v) + f(uv)$$

$$= p + q - 1 + p + q + p + q - 2$$

$$= 15n + 6 \quad \textbf{------- (A)}$$

$f^*[(uu_iv_iu)] = f(u) + f(v) + f(uu_i) + f(u_iv_i) + f(v_iv)$

$= p + q – 1 + p + q + i + n + i + (2n-1) + 2(n+1-i)$

$= 15n + 6$ ------- (B)

From (A) and (B), we conclude that ψ is minimum magic graphoidal cover. Hence, the Book $K_{1,n} \times K_2$ is magic graphoidal. For example, the magic graphoidal cover of the Book $K_{1,4} \times K_2$ is shown in figure 1.

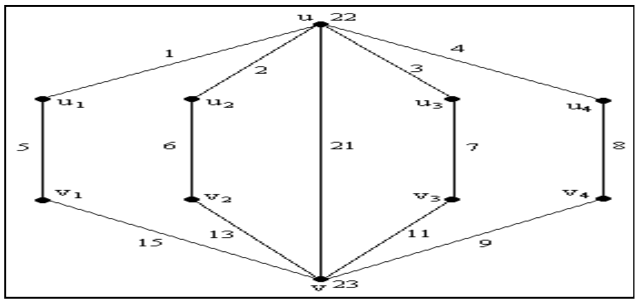

Figure 1: $K_{1,4} \times K_2$

$\psi = \{(uu_1v_1v), (uu_2v_2v), (uu_3v_3v), (uu_4v_4v), (uv)\}, \gamma = 5, K = 66$

Theorem 3.2

$P_n \times K_2$ (n - even) is magic graphoidal.

Proof :

Let $G = P_n \times K_2$

$V(G) = \{u_i, v_i : 1 \le i \le n\}$

$E(G) = \{[(u_iu_{i+1}) : 1 \le i \le n\text{-}1] \cup [(v_iv_{i+1}) : 1 \le i \le n\text{-}1] \cup [(u_iv_i) : 1 \le i \le n] \}$

Define $f : V \cup E \rightarrow \{1, 2, …., p+q\}$ by

$f(u_1) = p + q$

$f(v_1) = p + q -1$

$$f\left(u_{\frac{n+2}{2}}\right) = 4(n – 1)$$

$$f\left(v_{\frac{n+2}{2}}\right) = 4 (n –1) +1$$

$$f\left(u_{\frac{n+2}{2}+i}\right) = 4 (n –1) – 2i \qquad 1 \le i \le \frac{n}{2} -1$$

$$f\left(v_{\frac{n+2}{2}+i}\right) = 4\,(n-1) + i + 1 \qquad 1 \le i \le \frac{n}{2} - 1$$

$$f(u_{i+1}) = 4\,(n-1) + 1 - 2i \qquad 1 \le i \le \frac{n}{2} - 1$$

$$f\left(v_{\frac{n+2}{2}-i}\right) = 5\,(n-1) + 1 - i \qquad 1 \le i \le \frac{n}{2} - 1$$

$$f(u_i u_{i+1}) = i \qquad 1 \le i \le n - 1$$

$$f(v_i v_{i+1}) = 2(n-1) + i \qquad 1 \le i \le n-1$$

$$f(u_i v_i) = 2(n-1) + 1 - i \qquad 1 \le i \le n-1$$

$$f(u_n v_n) = 5(n-1) + 1$$

Let $\psi = \{ [(u_i u_{i-1} v_{i-1} v_i) : 2 \le i \le n] \cup (u_n v_n) \}$

$$f^*[(u_n v_n)] = f(u_n) + f(u_n) + f(u_n v_n)$$

$$= 4(n-1) - 2\left(\frac{n}{2} - 1\right) + 4(n-1) + 1 + \left(\frac{n}{2} - 1\right) + 5(n-1) + 1$$

$$= 13(n-1) - \left(\frac{n}{2}\right) + 3 \quad \ldots\ldots\ldots (A)$$

For $2 \le i \le \frac{n}{2} - 1$,

$$f^*[(u_i u_{i-1} v_{i-1} v_i)] = f(u_i) + f(v_i) + f(u_i u_{i-1}) + f(u_{i-1} v_{i-1}) + f(v_{i-1} v_i)$$

$$= 4(n-1) + 1 - 2(i-1) + 5(n-1) + 1 - \left(\frac{n}{2} - 1\right) - 2 + i + i -$$

$$1 + 2(n-1) + 1 - (i-1) + 2(n-1) + i - 1$$

$$= 13(n-1) - \left(\frac{n}{2}\right) + 3 \quad \ldots\ldots\ldots (B)$$

For $\frac{n}{2} < i \le n$,

$$f^*[(u_i u_{i-1} v_{i-1} v_i)] = f(u_i) + f(v_i) + f(u_i u_{i-1}) + f(u_{i-1} v_{i-1}) + f(v_{i-1} v_i)$$

$$= 4(n-1) - 2\left(i - \frac{n+2}{2}\right) + 4(n-1) + i - \left(\frac{n+2}{2}\right) + 1 + (i-1) +$$

$$2(n-1) + 1 - (i-1) + 2(n-1) + (i-1)$$

$$= 13(n-1) - \left(\frac{n}{2}\right) + 3 \quad \textbf{........ (C)}$$

From (A),(B) and (C), we conclude that ψ is minimum magic graphoidal cover. Hence, $P_n \times K_2$: (n -even) is magic graphoidal. For example, the magic graphoidal cover of the $P_6 \times K_2$ is shown in figure 2.

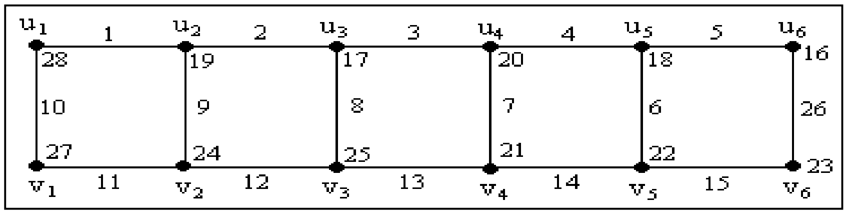

Figure 2: ($P_6 \times K_2$)

$\psi = \{(u_2u_1v_1v_2), (u_3u_2v_2v_3), (u_4u_3v_3v_4), (u_5u_4v_4v_5), (u_6u_5v_5v_6), (u_6v_6)\}, \gamma = 6, K = 65$

Theorem 3.3

$P_n \times K_2$ (n - odd, n ≤ 7) is magic graphoidal.

Proof :

When n = 3, the labeling is in figure 3

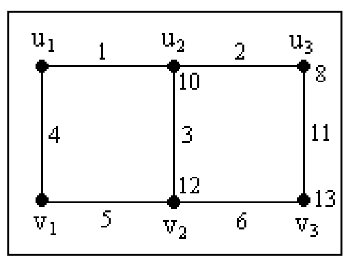

Figure 3: $P_3 \times K_2$

$\psi = \{(u_2u_1v_1v_2), (u_3u_2v_2v_3), (u_3v_3)\}, \gamma = 3, K = 32$

Let $G = P_n \times K_2$

Let $V(G) = \{u_i, v_i : 1 \le i \le n\}$

$E(G) = \{[(u_iu_{i+1}) : 1 \le i \le n-1] \cup [(v_iv_{i+1}) : 1 \le i \le n-1] \cup [(u_iv_i) : 1 \le i \le n]\}$

Define $f : V \cup E \rightarrow \{1, 2, ..., p+q\}$

When n = 5 or 7

$f(u_1) = p + q$

$f(v_1) = p + q - 1$

$f(u_iu_{i+1}) = n - i \qquad 1 \le i \le n-1$

$f(u_iv_i) = n - 1 + i \quad 1 \le i \le n-1$

$f(v_iv_{i+1}) = 2(n-1) + i \quad 1 \le i \le n-1$

$f(u_nv_n) = p + q - 2$

$$f\left(u_{\frac{n+1}{2}+i}\right) = 4(n-1) - 2(i-1) \quad 1 \le i \le \frac{n-1}{2}$$

$f(u_2) = 3n - 2$

$$f(u_{2+i}) = 4n - 3 - 2i \quad 1 \le i \le \frac{n-3}{2}$$

$$f\left(v_{\frac{n+1}{2}+i}\right) = 4(n-1) + i \quad 1 \le i \le \frac{n-1}{2}$$

$$f(v_{2+i}) = 9\left(\frac{n-1}{2}\right) + i$$

$$f(v_2) = 11\left(\frac{n-1}{2}\right)$$

For example, the magic graphoidal cover of the $P_7 \times K_2$ is shown in figure 4

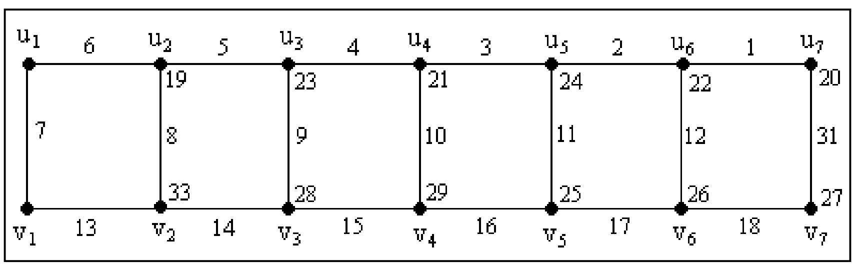

Figure4: $P_7 \times K_2$

$\psi = \{(u_2u_1v_1v_2), (u_3u_2v_2v_3), (u_4u_3v_3v_4), (u_5u_4v_4v_5), (u_6u_5v_5v_6), (u_7u_6v_6v_7), (u_7v_7)\}$, $\gamma = 7$, $K = 78$

Theorem 3.4

$C_m \times K_2$, (m - odd) is magic graphoidal.

Proof:

Let $G = C_m \times K_2$

Let $V(G) = \{u_i, v_i : \ 1 \le i \le m\}$

$E(G) = \{[(u_iu_{i+1}) \cup (v_iv_{i+1}): 1 \le i \le m-1] \cup (u_1u_m) \cup (v_1v_m) \cup [(u_iv_i): 1 \le i \le m]\}$

Define $f : V \cup E \rightarrow \{1, 2, \ldots, p+q\}$ by

$f(u_i) = 3m + i \qquad 1\leq i \leq m$

$f(v_i) = 5m + 1 - i \qquad 1\leq i \leq m$

$f(u_i\, u_{i+1}) = i \qquad 1\leq i \leq m-1$

$f(u_1 u_m) = m \qquad 1\leq i \leq m-1$

$f(u_i v_i) = \frac{3m+1}{2} + i \qquad 1\leq i \leq \frac{m-1}{2}$

$f\left(u_{\frac{m-1}{2}+i}\, v_{\frac{m-1}{2}+i}\right) = m + i \qquad 1\leq i \leq \frac{m+1}{2}$

$f(v_i v_{i+1}) = 3m+1-2i \qquad 1\leq i \leq \frac{m-1}{2}$

$f\left(v_{\frac{m-1}{2}+i}\, v_{\frac{m+1}{2}+i}\right) = 3m - 2(i-1) \qquad 1\leq i \leq \frac{m+1}{2}$ where $v_{m+1} = v_1$

Let $\psi = \{(u_{i+1}u_iv_iv_{i+1}) : 1\leq i \leq m$ where $u_{m+1} = u_1, v_{m+1} = v_1\}$

For $1\leq i \leq \frac{m-1}{2}$

$$f^*[(u_{i+1}u_iv_iv_{i+1})] = f(u_{i+1}) + f(v_{i+1}) + f(u_{i+1}u_i) + f(u_iv_i)+f(v_iv_{i+1})$$

$$= 3m + i +1 + 5m + 1 - (i+1) + i + \frac{3m+1}{2} + i + 3m +1-2i$$

$$= 11m + \frac{3m+1}{2} +2 \quad \text{----- (A)}$$

For $\frac{m+1}{2} \leq i \leq m$,

$$f^*[(u_{i+1}u_iv_iv_{i+1})] = f(u_{i+1}) + f(v_{i+1}) + f(u_{i+1}u_i) + f(u_iv_i)+f(v_iv_{i+1})$$

$$= 3m + i +1 + 5m + 1 - (i+1) + i + m + i - \left(\frac{m-1}{2}\right) + 3m$$

$$- 2\left(i - \frac{m-1}{2} - 1\right)$$

$$= 11m + \frac{3m+1}{2} +2 \quad \text{----- (B)}$$

From (A) and (B), we conclude that ψ is minimum magic graphoidal cover. Hence, $C_m \times K_2$: (m -odd) is magic graphoidal. For example, the magic graphoidal cover of the $C_5 \times K_2$ is shown in figure 4.

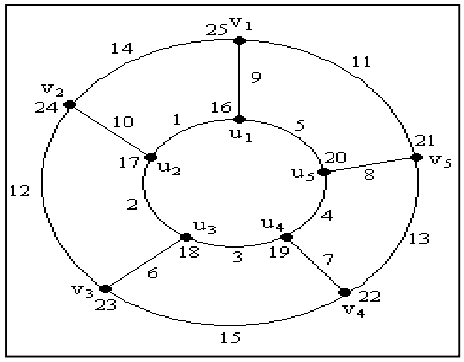

Figure 4: $C_5 \times K_2$

$\psi = \{(u_2u_1v_1v_2), (u_3u_2v_2v_3), (u_4u_3v_3v_4), (u_5u_4v_4v_5), (u_1u_5v_5v_1)\}, \gamma = 5, K = 65$

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