



Reliability Based Predictive Model For Estimating The Field Hydraulic Conductivity Of Compacted Cement Kiln Dust Treated Black Cotton Soil

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Abstract:

A predictive model of field hydraulic conductivity of compacted cement kiln dust treated black cotton soil as landfill liners was undertaken using FORTRAN-based First-Order Reliability Method (FORM). Reliability estimates of hydraulic conductivity was developed from data obtained from laboratory results for specimens compacted at the energy levels of British standard light (BSL) and West African standard (WAS) British standard heavy (BSH). Statistical analyses were carried out on data using Mini-tab R15 statistical software. Statistical Results obtained were incorporated into a FORM program and safety index values obtained. Generally, only BSH compactive effort produced satisfactory safety index value of 1.0 as specified for serviceability limit state design for hydraulic conductivity. Compositional factor such as water content relative to optimum failed to produce acceptable safety index value of 1.0 at the energy levels of BSL, WAS and BSH compactive efforts for coefficient of variation (COV) ranges of 10-100%. Observed trends indicate that the degree of saturation, plasticity Index, cement kiln dust content and percentage fines are greatly influenced by the COV and therefore must be strictly controlled in cement kiln dust-black cotton soil mixtures for use in covers and liners in waste containment facilities. Stochastically, only BSH compactive efforts can be used to model the hydraulic conductivity behavior of cement kiln dust treated black cotton soil for landfill liners at the variable ranges of 0-80% COV. Finally, care must be taken in ensuring that the compactive efforts required to produce successful safety index are carefully monitored during the construction of liners and covers in waste containment facilities.

Key words: *Compaction, compactive effort, Hydraulic conductivity, Cement kiln dust treated black cotton soil Reliability analysis, Reliability index, Soil composition, Soil liners*

1.Introduction

Uncertainties of many types pervade the practice of geotechnical engineering. Included are uncertainties due to the variable nature of soil and rock properties and other in situ conditions, uncertainties about the reliability of design and construction methods, and uncertainties about the costs and benefits of proposed design strategies. Probability theory is a mathematical tool that can be used to formally include such uncertainties in an engineering design and to assess their implications on performance. Its use in this context may result in a statement about the likelihood (probability) that a particular design will be successful or, conversely, about its likelihood of failure.

Compacted cement kiln dust treated black cotton soil have proven to be successful as an integral component of lining systems for municipal waste landfills (Oriola and Moses, 2011). The hydraulic conductivity of compacted liner material is required to be sufficiently low, usually less than 1×10^{-9} m/s (Daniel, 1993b; Benson et. al., 1994), as the primary purpose of such compacted cement kiln dust treated black cotton soil liner material is to impede the flow of fluids. Hydraulic conductivity is taken as the basic parameter for design and for characterizing liner performance and reliability (Bogardi et al., 1989). The variability in the properties of black cotton soil from one location to another is well documented in literatures (Moses, 2007; Stephen, 2010). As with other materials, this variability introduces uncertainties in engineering designs involving the use of black cotton soil.

Hydraulic conductivity is one of the material properties of soils that are significantly affected by this variability in composition. Engineering analyses and designs require the application of probabilistic methods as deterministic approaches do not rigorously account for these uncertainties. Probability theory has been widely accepted and used in engineering. The application of probability theory to engineering analysis requires the knowledge of some statistical attributes of the relevant random variables such as their mean values and standard deviations (Whitman, 2000; Christian and Baccher, 2001; Kaymaz et al., 1998). One of such probabilistic methods is reliability analysis which has been used in geoenvironmental engineering (e.g., McGuire, 1976; Tang, W.H. 1979; Duncan and Houston, 1983; Vick and Bromwell, 1989; Gilbert and Tang, 1995; Rowe and Fraser, 1995; Wu et al. 1987; Christian, et al, 1992; Nwaiwu et al, 2009).

Geomembrane liners are often used, either alone or overlying compacted clay, to minimize leakage of waste materials to the environment. During installation, geomembrane defects can result from inadequate seams between geomembrane panels or

from punctures and tears. In order to minimize defects in the final product, a CQA program is usually implemented. Gilbert and Tang (1993) described how probability theory can be used to obtain a consistent CQA program. Reliability analysis provides a frame work for establishing appropriate factors of safety and other design targets and leads to a better appreciation of the relative importance of uncertainties in different parameters (Christian and Baecher, 2001). The probability of survival is called reliability (Probability of failure = reliability). This study is aimed at estimating the field hydraulic conductivity of compacted cement kiln dust treated black cotton soil as barrier material in waste containment systems

2.Theoretical Background

2.1.Reliability

Reliability calculations provide a means of evaluating the combined effects of uncertainties, and a mean of distinguishing between conditions where uncertainties are particularly high or low. The numerical evaluation involved the use of the first-order reliability methods version 5 (FORM5 – Gollwitzer et al, 1988).

2.2.Probabilistic Methods in Geotechnical Engineering

Probabilistic methods employ the careful attention to evaluating uncertainties using statistical knowledge of random variables such as their mean values, and standard deviations; introducing them into the design plus some judgment as necessary to model errors. A wide range of probabilistic methods have been developed for use for different types of engineering projects such as landslides environmental problems, foundations for offshore structures etc. These include: Point estimate method; Monte Carlo simulation method; First order reliability method (FORM); Second Order Reliability method (SORM); Kriging, ; and, Search theory.

Soil liner reliability is defined as a probabilistic measure of assurance of post-construction performance characterized by hydraulic conductivity, k , and is represented as (Bogardi et al., 1989):

$$R = P(K_c < K_o) \quad (1)$$

where k_o = the specified hydraulic conductivity limit, such as 1×10^{-9} m/s. In this case, it will be reasonable to assume a condition of vertical porous media flow through compacted soil material with no macro pores or open flow paths (or flow channels).

Reliability assessment efforts are directed towards determining the probability that the compacted soil liner will not attain any of the known limit states likely to be violated throughout the useful life, or at least some specified design period, of the liner in the face of uncertainties as human error inputs, various environmental conditions, variation in material and engineering properties as well as prediction of future events. This probability of survival is given as:

$$P_s = 1 - P_f \quad (2)$$

where P_s = probability of survival and P_f = probability of failure.

2.3. Concept of First – Order Reliability Method (FORM)

The probabilistic and deterministic approaches to design differ in principle. Deterministic design is based on total ‘discounting’ of the contingency of failure. Design problems involve element of uncertainty; unpredictability of randomness. Probabilistic design is concerned with the probability that the structure will realize the functions assigned to it (Afolayan and Abubakar, 2003). The performance function of a soil liner can be modelled in terms of certain basic random variables x_1, x_2, \dots, x_n , which must operate within certain limits for the liner to function satisfactorily. Values of x_1, x_2, \dots, x_n , outside of these limits constitute the failure domain, and that surface within the n-dimensional space of basic variables x_i that divides values of these variables into the failure domain and the domain for adequate performance is called the failure surface (Ocholi, 2000). A mathematical representation of this surface is known as the limit state equation (Ditlevsen, 1981).

In the reliability analysis of compacted soil liners, *failure* may be defined as the event of a liner hydraulic conductivity equal to or greater than the specified regulatory maximum (that is, 1×10^{-9} m/s), over a given period of time such as the design life. The reliability problem is then formulated in terms of a limit state function $g(X)$, where X is a vector of random variables, and $g(X) < 0$ denotes the region in which the threshold value is met or exceeded. Adequate performance of the liner is established when $g(X) > 0$. The limit state surface is denoted as $g(X) = 0$. Thus, for a threshold hydraulic conductivity k_o , the limit state function can be formulated as

$$g(\underline{X}) = \ln k_o - \ln k_c \quad (3)$$

where k_e is the expected hydraulic conductivity.

If $X = x_1, x_2, \dots, x_n$, has a joint probability distribution function (pdf) given by:

$$F_{\mathbf{X}}(\mathbf{X}) = P\left[\prod_{i=1}^n (X_i \leq x_i)\right] \quad (4)$$

the probability of failure (that is, the probability that the threshold value is exceeded), $P(k_o < k_e)$ is obtained by integrating the joint pdf in the region where $g(x) < 0$:

$$P[g(\mathbf{X}) \leq 0] = \int_{g(\mathbf{x}) \leq 0} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \quad (5)$$

where $f_{\mathbf{X}}(\mathbf{x})$ is the joint probability density or distribution function (pdf) of X and the n -fold integral is over the unsafe region. In practice, a direct numerical evaluation of the multifold integral is virtually impossible (Jang et al., 1994) due to a lack of full probability information. Some approximate methods have been developed for evaluating the integral, including first - and second - order reliability methods (FORM and SORM), simulation methods, and hybrid methods combining simulation with FORM and/or SORM (Jang et al., 1994).

In FORM, the integral in Eq. (5) is evaluated in standard normal space by transforming the random variables X into a set of uncorrelated standard normal variates $U = U(X)$, having the pdf (Jang et al., 1994);

$$\varphi_n(\mathbf{u}) = (2\pi)^{-n/2} \exp\left(-\frac{1}{2}|\mathbf{u}|^2\right) \quad (6)$$

where n is the number of random variables. Eq. (5) can then be written as:

$$P[G(\mathbf{u}) \leq 0] = \int_{G(\mathbf{u}) \leq 0} \varphi_n(\mathbf{u}) d\mathbf{u} \quad (7)$$

where $G(u) = g(x(u))$ is the limit state function in the transformed space. The limit state surface is replaced by a tangent point u^* and the distance from the origin, known as the reliability index, β , is given by the inner product

$$\beta = \alpha^* \cdot u^* \quad (8)$$

where α^* is the unit normal at the design point directed toward the failure region. The first-order approximation of the probability of exceeding the regulatory maximum is (Jang et al., 1994):

$$P_f = \varphi(-\beta) \quad (9)$$

where $j(\cdot)$ is the standard cumulative normal probability and β is also known as (geometric) safety index

3. Materials and Methods

3.1. Database and Statistical Analysis

A database was compiled by extracting data on black cotton soil from laboratory test results of published literature (e.g Oriola and Moses, 2011). The statistical characteristics of the material composition and compaction variables for black cotton soil are shown in Table 1.

3.2. Statistical Distributions of Variables

Hydraulic conductivity is usually assumed to be lognormally distributed (Harrop - Williams, 1985; Bogardi et al., 1989; Benson, 1993). Although alternative distributions have been proposed by Harrop - Williams (1985) and Benson (1993), the two parameter lognormal distribution type was adopted here for hydraulic conductivity.

A Kolmogorov-Smirnov (K-S) goodness-of-fit test on the black cotton soil-cement kiln dust mixtures properties as well as on the compactor weight (sheeps foot rollers only from Benson et al., 1994) indicates that degree of initial saturation are lognormally distributed, while the compactor weight has a Weibull (extreme value Type III) distribution. The distribution type for each variable as stated above was used in this study, although there are also some alternative distribution types.

3.3. Set-up of Numerical Experiments Reliability Analysis

The results of all laboratory experiments on hydraulic conductivity and the parameters associated with hydraulic conductivity were measured during the laboratory work. The various parameters measured include the following hydraulic conductivity (k), water content with respect to optimum (WRO), initial degree of saturation (S_i), plasticity index (PI), cement kiln dust content, percentage fines (PF) and compactive effort index (E). Fundamentally, hydraulic conductivity, water content with respect to optimum, initial degree of saturation (S_i) and cement kiln dust content is normally assumed to have a lognormal distribution (Benson, 1993; Harrop-Williams, 1985; Eberemu, 2008; Stephen, 2010; Benson and Daniel, 1994a, 1994b; Borgadi, *et al.*, 1989; Gui *et al.*, 2000; Nwaiwu *et al.*, 2009). While percentage fines (PF) and cement kiln dust content has a normal distribution (Eberemu, 2008; Stephen, 2010). The compactive effort index is an integer categorical variable describing compactive effort. It was assigned -1.0 and 1 for British

Standard light, West African Standard and British Standard heavy compactive efforts, respectively. These results were used to run a regression model for predicting laboratory hydraulic conductivity results. The statistical analyses were carried out using the tools of analysis Mini-tab R15 software.

Reliability analysis can be used to assess the suitability of compacted cement kiln dust treated black cotton soil for use as liners and covers in waste containment structure. This becomes necessary due to the variability that might exist from black cotton soils obtained from one foundry to another. The statistical characteristics of the relevant black cotton soil-cement kiln dust mixtures as well as physical properties of their probability distribution functions types were established. The relevant statistical properties for black cotton soil-cement kiln dust mixtures were then incorporated into FORTRAN program for a field based predictive model in order to evaluate reliability levels and to predict hydraulic conductivities using the 'first order reliability methods' version 5.0 (FORM 5) (Gollwitzer *et al.*, 1988). The input data for the reliability analysis from the laboratory hydraulic conductivity results are shown in Table. 1.

Sensitivity analysis for each of the independent variables that affect hydraulic conductivity was performed by varying the assumed values of coefficient of variation (COV) ranging from 10-100% to obtain reliability indices (safety indices or β -values). The safety indices for the five independent variables evaluated that affect hydraulic conductivity are: water content relative to optimum ((WRO), degree of saturation, cement kiln dust content and percentage fines) at compaction energy levels of British Standard light (BSL), West African Standard (WAS) and British Standard heavy (BSH) were obtained.

S/No	Variables	Distribution type	Mean E(x)	Standard Deviation S(x)	Coefficient of Variation COV (%)
1.	Maximum Hydraulic Conductivity (K)	Log normal	1.55E-08	2.24E-08	1443.23
1b.	Lnk	Log normal (=3)	-21.0	1.66	-5.56
2.	Water Content Relative to Optimum (WRO)	Log normal (=3)	1	2.27	227.0
3.	Degree of Saturation (Sr)	Log normal (=3)	80.40	13.37	16.62
4.	Plasticity index	Log normal (=3)	47.2	5.96	12.61
5.	Percentage Fines (PF)	Normal (=2)	27.7	1.71	6.16
6.	Cement kiln dust Content	Normal (=2)	4	2.85	71.3
7.	Compactive Effort Index (E)	Deterministic Parameter	-1,0,1	-	-

Table 1: Input data for reliability based design for five independent variable using FORM 5 from laboratory measured hydraulic conductivity

4.Results and Discussion

4.1.Effect of Hydraulic Conductivity on Reliability Index

The effect of hydraulic conductivity on reliability index for various coefficients of variation is shown in Fig.1. Generally, higher compactive energy levels produced higher safety index as COV increased in the range 10-100% which indicates that the better hydraulic conductivity values of liner material will be obtained at higher compactive efforts. Hydraulic conductivity varied non-linearly as the coefficient of variation

increased in the range 10-100%, while reliability or safety index recorded decrease in value, It was observed that β values decreased in the ranges 0.47-0.039, 0.906-0.48 and 1.34-0.916 for BSL, WAS and BSH compactions, respectively..

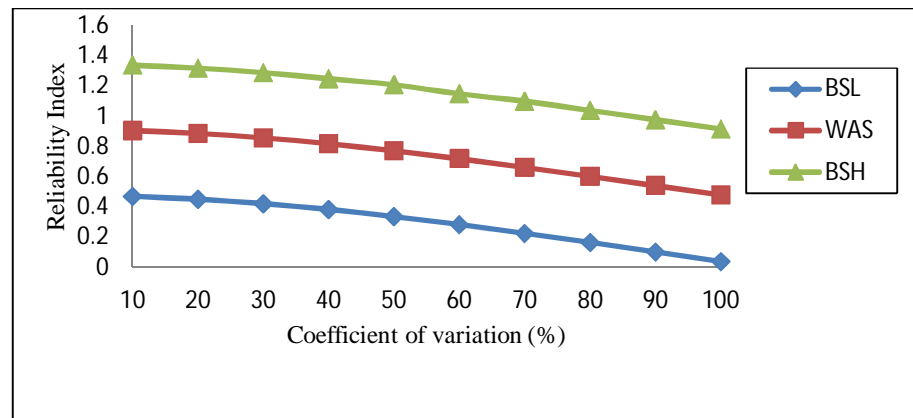


Figure1: Variation of reliability index with coefficient of variation for hydraulic conductivity

4.2.Effect of Water Content Relative to Optimum on Reliability Index

The effect of water content relative to optimum on reliability index as the coefficient of variation is varied is shown in Fig.2. Higher safety indices were recorded for higher compaction energies. Water content relative to optimum produced a linear relationship with coefficient of variation the range 10-100%, while reliability or safety index varied slightly. This is an indication that variability of WRO has no drastic influence on the safety index similar results were reported by Stephen (2009) and Oriola et al. (2012). As COV increased from 10-100%, β value increased from -0.082 to -0.157, 0.365 to 0.286 and 0.805 to 0.723 for BSL, WAS and BSH compactions, respectively.

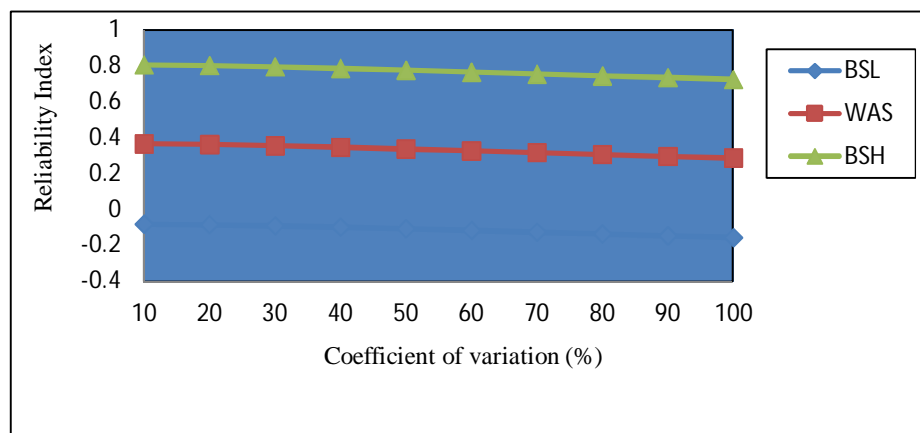


Figure 2: Variation of reliability index with coefficient of variation for water content relative to optimum

4.3. Effect of Degree of Saturation on Reliability Index

The effect of degree of saturation on reliability index as the coefficient of variation is varied is shown in Fig.3. The degree of saturation varied non-linearly with the coefficient of variation in the range 10-100%, while, reliability or safety index increased. Generally, higher compactive efforts produced higher safety indices. The trends recorded are consistent with the findings of Benson et al., (1994; 1999). As COV increased from 10-100%, β value increased in the corresponding ranges -0.248-0.042, 0.20-0.413 and 0.643-0.756 for BSL, WAS and BSH compactions, respectively. The changes in safety index as COV is varied an indication that hydraulic conductivity is significantly influenced by changes in initial degree of saturation. A similar trend was reported by Eberemu (2007) and Oriola et al. (2012). It implies that the initial degree of saturation must be carefully controlled when compacting cement kiln dust treated black cotton soil for use as covers and liners in waste containment facilities.

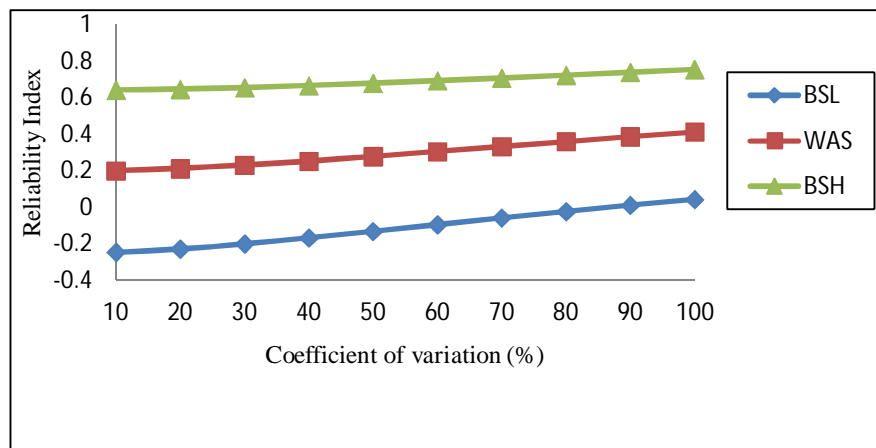


Figure 3: Variation of reliability index with coefficient of variation for degree of saturation

4.4. Effect of Plasticity Index on Reliability Index

The effect of plasticity index on the safety index as the coefficient of variation is varied is shown in Fig.4. Plasticity index produced a non-linear relationship as the coefficient of variation increased in the range 10-100%. Increased reliability or safety index was recorded for BSL and WAS compactions, in the range -0.269-0.338, and 0.207-0.450 respectively. While, BSH compaction recorded a decrease in β value from 0.681-0.552. It was generally observed that higher compaction energies recorded higher safety index. The change in safety index as COV is varied from 10-100% from the results obtained is an indication that plasticity index must be carefully controlled when compacting cement

kiln dust treated black cotton soil for use as covers and liners in waste containment facilities.

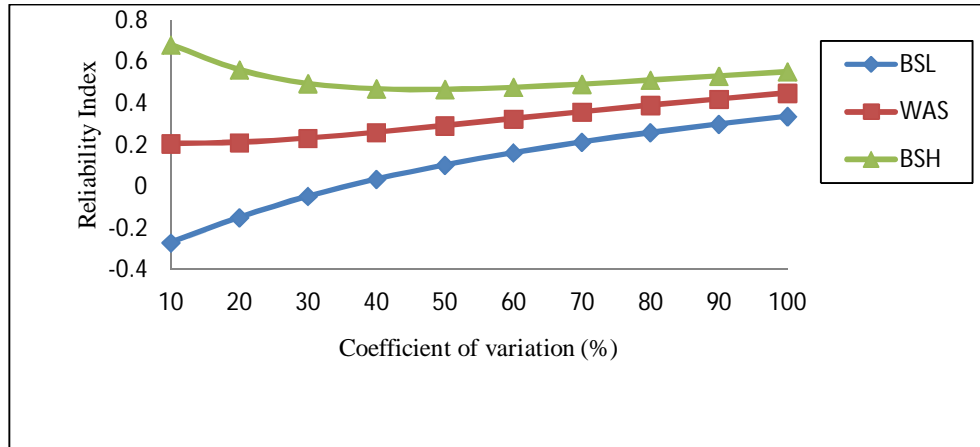


Figure 4: Variation of reliability index with coefficient of variation for plasticity index

4.5. Effect of Cement kiln dust Content on Reliability Index

The effect of cement kiln dust content on safety index as the coefficient of variation is varied is shown in Fig.5. Cement kiln dust content varied linearly, the coefficient of variation increased from 10-100. Increased reliability or safety index was recorded for BSL compactions, in the range -0.368 to -0.167. For WAS and BSH compactions reliability or safety index recorded a decrease in the ranges of 0.316-0.458, and 0.956-0.458, respectively. It was generally observed that higher compaction energies recorded higher safety index. From the results attained it shows the cement kiln dust content must be strictly controlled when compacting cement kiln dust treated black cotton soil for use as covers and liners in waste containment facilities. The change in safety index as COV is varied from 10-100% is an indication that hydraulic conductivity is significantly influenced by changes in cement kiln dust content contrary to the finding of Stephen (2007). Therefore, when compacting cement kiln dust-black cotton soil mixtures for use in covers and liners in waste containment facilities the cement kiln dust content must be strictly controlled.

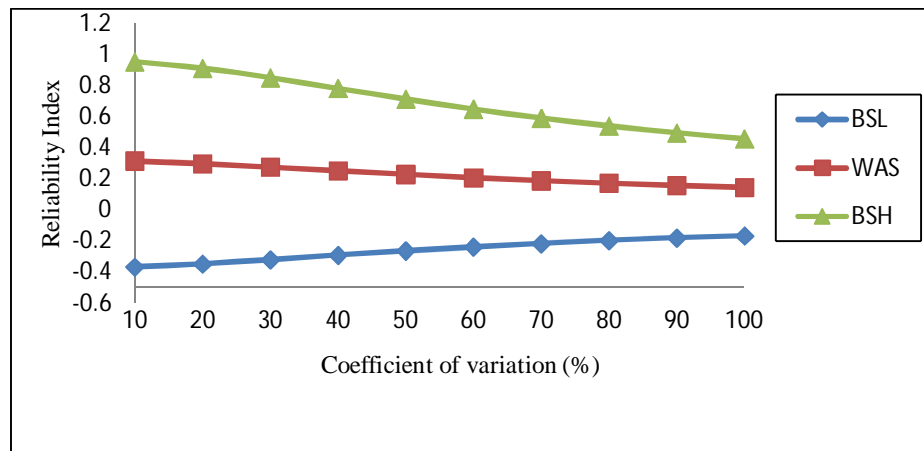


Figure 5: Variation of reliability index with coefficient of variation for cement kiln dust content

4.6. Effect of Percentage Fines on Reliability Index

The effect of percentage fines on the safety index as the coefficient of variation is varied is shown in Fig.6. Increased reliability or safety index was recorded for BSL compactions, in the range -0.232 to -0.101. While, WAS and BSH compactions recorded a decrease in reliability or safety index in the ranges of 0.203-0.088, and 0.633-0.278, respectively. It was generally observed that higher compaction energies recorded higher safety index. From the results obtained it shows that the percentage fines content must be strictly controlled when compacting cement kiln dust treated black cotton soil for use as covers and liners in waste containment facilities.

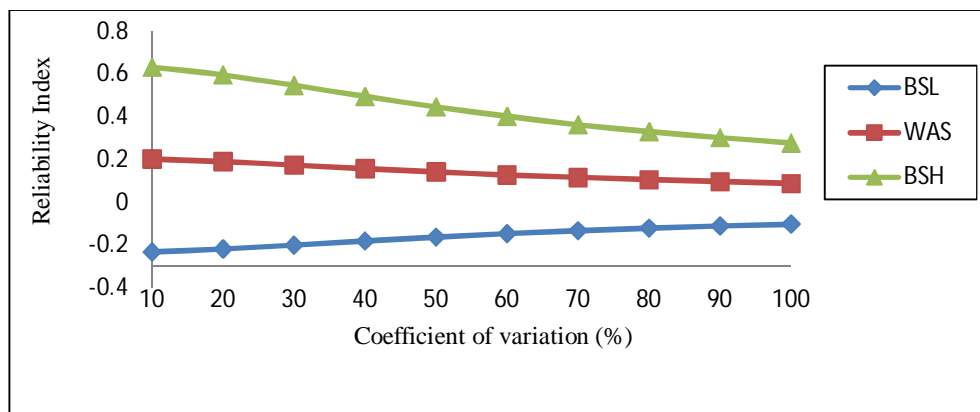


Figure 6: Variation of reliability index with coefficient of variation for percentage fines

4.7. Stochastic Model Assessment

The safety indices obtained for the three compactive efforts BSL, WAS, and BSH are in the ranges 0.47-0.039, 0.906-0.48 and 1.34-0.916 respectively. NKB Report (1978) specified a safety index value of 1.0 as the lowest value for serviceability limit state design (model 1) of structural components.

Variables Factors	Beta Values			Acceptable Ranges of COV (%)		
	BSL	WAS	BSH	BSL	WAS	BSH
Hydraulic conductivity (k)	0.47-0.039	0.906-0.48	1.34- 0.916	Nil	Nil	10- 80
Water content relative to optimum	-0.082-0.157	0.365- 0.286	0.805- 0.723	Nil	Nil	Nil
Initial degree of saturation	-0.248-0.042	0.20-0.413	0.643- 0.756	Nil	Nil	Nil
Plasticity Index	0.269-0.338	0.207-0.450	0.681- 0.552	Nil	Nil	Nil
Cement kiln dust content	-0.368- -0.167	0.316-0.146	0.956- 0.458	Nil	Nil	Nil
Percentage fines	-0.232- -0.101	0.203-0.088	0.633- 0.278	Nil	Nil	Nil

Table 3: Stochastic Model Assessment of acceptable safety index

5. Conclusion

Reliability estimates of field hydraulic conductivity of compacted cement kiln dust treated black cotton soil as landfill liners was under taken by incorporating a predictive model for hydraulic conductivity, which was developed from data obtained from laboratory results for specimens compacted at the energy levels of British standard light (BSL), West African standard (WAS) and British standard heavy (BSH). Results were incorporated into a FORTRAN-based first-order reliability program and safety index values obtained. Generally, the safety index produced satisfactory beta value of 1.0 as specified for serviceability limit state design for hydraulic conductivity at BSH

compactive effort only. Compositional factor such as water content relative to optimum failed to produced acceptable safety index value of 1.0 at the energy levels of BSL, WAS and BSH compactive efforts at COV ranges of 10-100%. Observed trends indicate that the degree of saturation, plasticity Index, cement kiln dust content and percentage fines is greatly influenced by the COV and therefore must be strictly controlled in cement kiln dust-black cotton soil mixtures for use in covers and liners in waste containment facilities.

Stochastically, only BSH compactive efforts can be used to model the hydraulic conductivity behavior of cement kiln dust treated black cotton soil for landfill liners at the variable ranges of 0-80% COV. Finally, care must be taken in ensuring that the compactive efforts required to produce successful safety index are carefully monitored during the construction of liners and covers in waste containment facilities.

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