INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH & DEVELOPMENT

ISSN: 2278 – 0211 (Online)

Image Compression Using Wavelets

Ms. Smita K Chaudhari
Assistant Professor, SSGBCOE&T, Bhusawal
Mr. Rathi Chandresh
Student, SSGBCOE&T, Bhusawal .
Mr. Pratik Patel
Student, SSGBCOE&T, Bhusawal
Mr. Hemant Ingale
Assistant Professor, SSGBCOE&T, Bhusawal
Mr. Viral Patel
Student, SSGBCOE&T, Bhusawal

Abstract:
Image require substantial storage and transmission resources, thus image compression is advantageous to reduce these requirements. The report covers some background of wavelet analysis, data compression and how wavelets have been and can be used for image compression. An investigation into the process and problems involved with image compression was made and the results of this investigation are discussed. It was discovered that thresholding was had an extremely important influence of compression results so suggested thresholding strategies are given along with further lines of research that could be undertaken.
Wavelets provide a powerful and remarkably flexible set of tools for handling fundamental problems in science and engineering, such as audio de-noising, signal compression, object detection and fingerprint compression, image de-noising, image enhancement, image recognition, diagnostic heart trouble and speech recognition, to name a few. Here, we are going to concentrate on wavelet application in the field of Image Compression so as to observe how wavelet is implemented to be applied to an image in the process of compression, and also how mathematical aspects of wavelet affect the compression process and the results of it. Wavelet image compression is performed with various known wavelets with different mathematical properties. We study the insights of how wavelets in mathematics are implemented in a way to fit the engineering model of image compression.

Key words: *Image Compression, Wavelet Transform, Haar Wavelet, DWT, Vector Quantization.*

1.Introduction

1.1.Compression – An Overview

Compression is a process of converting an input data stream into another data stream that has a smaller size. Compression is possible only because data is normally represented in the computer in a format that is longer than necessary i.e. the input data has some amount of redundancy associated with it. The main objective of compression systems is to eliminate this redundancy. When compression is used to reduce storage requirements, overall program execution time may be reduced. This is because reduction in storage will result in the reduction of disc access attempts. With respect to transmission of data, the data rate is reduced at the source by the compressor (coder),it is then passed through the communication channel and returned to the original rate by the expander(decoder) at the receiving end. The compression algorithms help to reduce the bandwidth requirements and also provide a level of security for the data being transmitted. A tandem pair of coder and decoder is usually referred to as codec.

2.Literature Survey

Khashman et al. in [1] proposed a technique for compressing the digital image using neural networks and Haar Wavelet transform. The aim of the work presented within the paper was to develop an optimum image compression system using haar wavelet transform and a neural network. With Wavelet transform based compression, the quality of compressed images is typically high, and the option of a perfect compression ratio is complicated to formulate as it varies depending on the content of the image. They proposed that neural networks can be trained to ascertain the non-linear relationship between the image intensity and its compression ratios in search for an optimum ratio.

A method of still image compression was put forth by Wilford Gillespie in [2]. The fundamental approach to image compression consists of a number of key steps. They are wavelet packet decomposition, quantization, organization of vectors, neural networks approximation or storage, and lossless encoding and reduction. As an initial stage of image compression, the image is put through several layers of wavelet packet decomposition. The results of the decomposition are then divide or processed in some way, depending on the method. Integer quantization is performed on all of the decomposed wavelet sections. The quantization value is the determining factor of quality. A quantization value of 1 is near lossless quality, although little to no compression is achieved. This is accomplished by taking each section and dividing it by a set value and rounding to the nearest integer.

3.Image Compression Using Wavelets

When retrieved from the Internet, digital images take a considerable amount of time to download and use a large amount of computer memory. The Haar wavelet transform that we will discuss in this application is one way of compressing digital images so they take less space when stored and transmitted. As we will see later, the word ``wavelet'' stands for an orthogonal basis of a certain vector space.

The basic idea behind this method of compression is to treat a digital image as an array of numbers i.e., a matrix. Each image consists of a fairly large number of little squares called pixels (picture elements). The matrix corresponding to a digital image assigns a whole number to each pixel. For example, in the case of a 256x256 pixel gray scale image, the image is stored as a 256x256 matrix, with each element of the matrix being a whole number ranging from 0 (for black) to 225 (for white). The JPEG compression technique divides an image into 8x8 blocks and assigns a matrix to each block. One can use some linear algebra techniques to maximize compression of the image and maintain a suitable level of details.

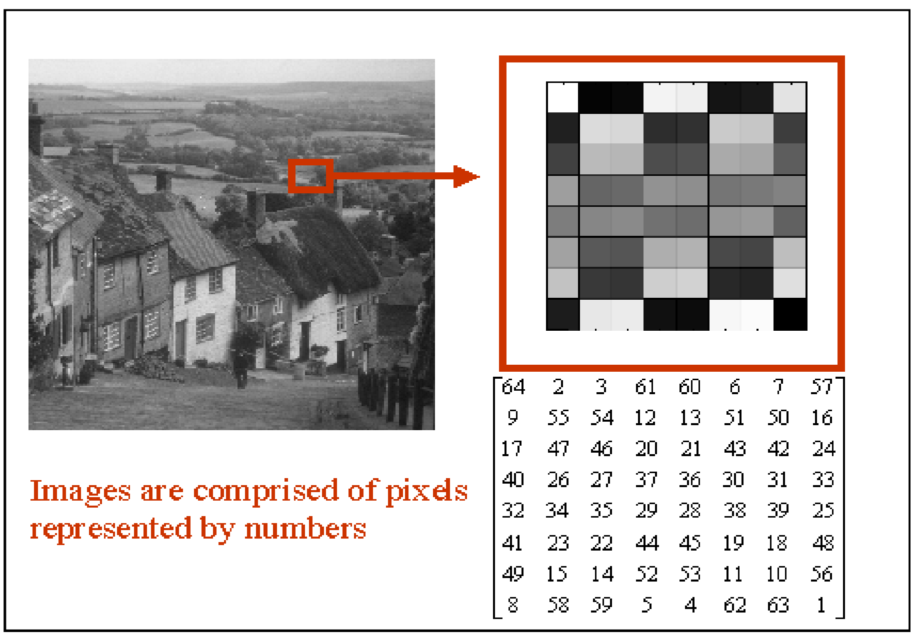

Figure 1:JPEG Compression Technique

3.1.Vector Transform Using Haar Wavelets

Before we explain the transform of a matrix, let us see how the wavelets transform vectors (rows of a matrix). Suppose

$$r = \begin{bmatrix}420 & 680 & 448 & 708 & 1250 & 1420 & 1600 & 1600\end{bmatrix}$$

is one row of an 8x8 image matrix. In general, if the data string has length equal to 2^k, then the transformation process will consist of *k* steps. In the above case, there will be 3 steps since $8=2^3$.

We perform the following operations on the entries of the vector *r:*

- Divide the entries of *r* into four pairs: (420, 680), (448, 708), (1260, 1410), (1600, 600).
- Form the average of each of these pairs:

$$\frac{420+680}{2} = 550, \quad \frac{448+708}{2} = 578, \quad \frac{1260+1420}{2} = 1340, \quad \frac{1600+1600}{2} = 1600$$

These will form the first four entries of the next step vector r_1.

3. Subtract each average from the first entry of the pair to get the numbers:

$$-130, \quad -130, \quad -75, \quad 0.$$

These will form the last four entries of the next step vector r_1.

4. Form the new vector:

$$r_1 = \begin{bmatrix}550 & 578 & 1340 & 1600 & -130 & -130 & -80 & 0\end{bmatrix}.$$

Note that the vector r_1 can be obtained from *r* by multiplying *r* on the right by the matrix:

$$W_1 = \begin{bmatrix}
1/2 & 0 & 0 & 0 & 1/2 & 0 & 0 & 0 \\
1/2 & 0 & 0 & 0 & -1/2 & 0 & 0 & 0 \\
0 & 1/2 & 0 & 0 & 0 & 1/2 & 0 & 0 \\
0 & 1/2 & 0 & 0 & 0 & -1/2 & 0 & 0 \\
0 & 0 & 1/2 & 0 & 0 & 0 & 1/2 & 0 \\
0 & 0 & 1/2 & 0 & 0 & 0 & -1/2 & 0 \\
0 & 0 & 0 & 1/2 & 0 & 0 & 0 & 1/2 \\
0 & 0 & 0 & 1/2 & 0 & 0 & 0 & -1/2
\end{bmatrix}$$

The first four coefficients of r_1 are called the **a**pproximation coefficients and the last four entries are called the detail coefficients**.**

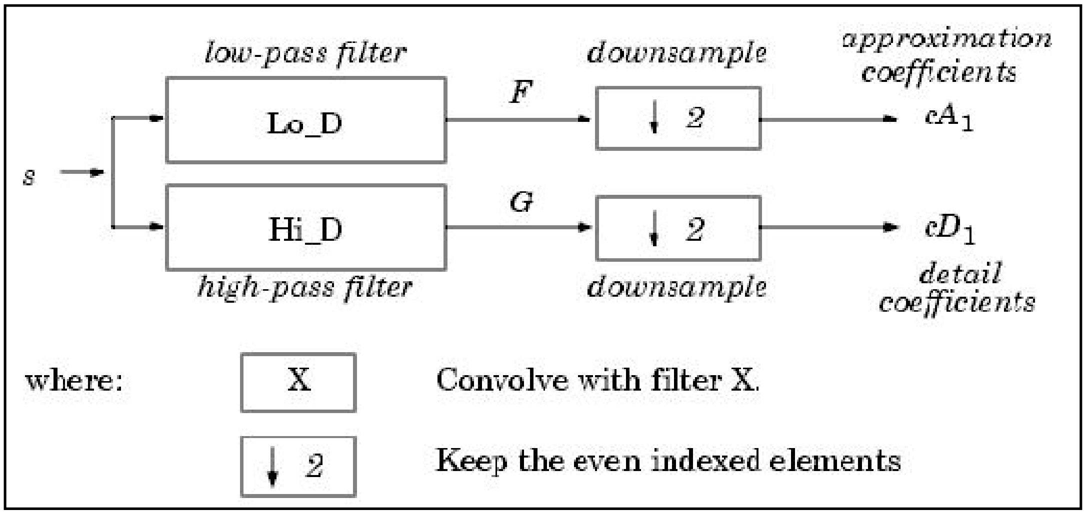

Figure 2: Filtering operation of the DWT

For our next step, we look at the first four entries of r_1 as two pairs that we take their averages as in step 1 above. This gives the first two entries: 564, 1470 of the new vector r_2. These are our new approximation coefficients. The third and the fourth entries of r_2 are obtained by subtracting these averages from the first element of each pair. This results in the new detail coefficients: -14, -130. The last four entries of r_2 are the same as the detail coefficients of r_1:

$$r_2 = \begin{bmatrix} 564 & 1470 & -14 & -130 & -130 & -130 & -80 & 0 \end{bmatrix}$$

Here the vector r_2 can be obtained from r_1 by multiplying r_1 on the right by the matrix:

$$W_2 = \begin{bmatrix} 1/2 & 0 & 1/2 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & -1/2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 1/2 & 0 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & -1/2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

For the last step, average the first two entries of r_2, and as before subtract the answer from the first entry. This results in the following vector:

$$r_3 = \begin{bmatrix} 1017 & -453 & -14 & -130 & -130 & -130 & -80 & 0 \end{bmatrix}$$

As before, r_3 can be obtained from r_1 by multiplying r_2 on the wright by the matrix:

$$W_3 = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & -1/2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

As a consequence, one gets r_3 immediately from r using the following equation

$$r_3 = W_1 W_2 W_3 r$$

Let

$$W = W_1 W_2 W_3 = \begin{bmatrix} 1/8 & 1/8 & 1/4 & 0 & 1/2 & 0 & 0 & 0 \\ 1/8 & 1/8 & 1/4 & 0 & -1/2 & 0 & 0 & 0 \\ 1/8 & 1/8 & -1/4 & 0 & 0 & 1/2 & 0 & 0 \\ 1/8 & 1/8 & -1/4 & 0 & 0 & -1/2 & 0 & 0 \\ 1/8 & -1/8 & 0 & 1/4 & 0 & 0 & 1/2 & 0 \\ 1/8 & -1/8 & 0 & 1/4 & 0 & 0 & -1/2 & 0 \\ 1/8 & -1/8 & 0 & -1/4 & 0 & 0 & 0 & 1/2 \\ 1/8 & -1/8 & 0 & -1/4 & 0 & 0 & 0 & -1/2 \end{bmatrix}.$$

Note the following:

- The columns of the matrix W_1 form an orthogonal subset of R^8 (the vector space of dimension 8 over R); that is these columns are pair wise orthogonal (try their dot products). Therefore, they form a basis of R^8. As a consequence, W_1 is invertible. The same is true for W_2 and W_3.
- As a product of invertible matrices, W is also invertible and its columns form an orthogonal basis of R^8. The inverse of W is given by:

$$W^{-1} = W_3^{-1} W_2^{-1} W_1^{-1}$$

The fact the W is invertible allows us to retrieve our image from the compressed form using the relation

$$r = W^{-1} r_3.$$

Suppose that A is the matrix corresponding to a certain image. The Haar transform is carried out by performing the above operations on each row of the matrix $A$and then by repeating the same operations on the columns of the resulting matrix. The row-transformed matrix is AW. Transforming the columns of AW is obtained by multiplying AW on the left by the matrix W^T (the transpose of W). Thus, the Haar transform takes the matrix A and stores it as W^TAW.

Let S denote the transformed matrix

$$S = W^T AW.$$

Using the properties of inverse matrix, we can retrieve our original matrix:

$$A = (W^T)^{-1} SW^{-1} = (W^{-1})^T SW^{-1}.$$

This allows us to see the original image (decompressing the compressed image).

The point of doing Haar wavelet transform is that areas of the original matrix that contain little variation will end up as zero elements in the transformed matrix. A matrix is considered sparse if it has a "high proportion of zero entries". Sparse matrices take much less memory to store. Since we cannot expect the transformed matrices always to be sparse, we decide on a non-negative threshold value known as ε, and then we let any entry in the transformed matrix whose absolute value is less than ε to be reset to zero. This will leave us with a kind of sparse matrix. If ε is zero, we will not modify any of the elements. Every time you click on an image to download it from the Internet, the source computer recalls the Haar transformed matrix from its memory. It first sends the overall approximation coefficients and larger detail coefficients and a bit later the smaller detail coefficients. As your computer receives the information, it begins reconstructing in progressively greater detail until the original image is fully reconstructed.

3.2.Linear Algebra Can Make The Compression Process Faster, More Efficient

Let us first recall that an nxn square matrix A is called orthogonal if its columns form an orthonormal basis of R^n, that is the columns of A are pairwise orthogonal and the length of each column vector is 1. Equivalently, A is orthogonal if its inverse is equal to its transpose. That latter property makes retrieving the transformed image via the equation

$$A = (W^T)^{-1} SW^{-1} = (W^{-1})^T SW^{-1} = WSW^T$$

much faster.

Another powerful property of orthogonal matrices is that they preserve magnitude. In other words, if v is a vector of R^n and A is an orthogonal matrix, then $||Av||=||v||$. Here is how it works

$$
\begin{aligned}
\| Av\|^2 &= (Av)^T (Av) \\
&= v^T A^T Av \\
&= v^T Iv \\
&= v^T v \\
&= \| v |^2
\end{aligned}
$$

This in turns shows that $||Av||=||v||$. Also, the angle is preserved when the transformation is by orthogonal matrices: recall that the cosine of the angle between two vectors *u* and *v* is given by:

$$\cos \phi = \frac{u.v}{\|u\| \|v\|}$$

so, if *A* is an orthogonal matrix, ψ is the angle between the two vectors *Au* and *Av*, then

$$
\begin{aligned}
\cos \psi &= \frac{(Au).(Av)}{\| Au \| \| Av \|} \\
&= \frac{(Au)^T (Av)}{\| u \| \| v \|} \\
&= \frac{u^T A^T Av}{\| u \| \| v \|} \\
&= \frac{u^T v}{\| u \| \| v \|} \\
&= \frac{u.v}{\| u \| \| v \|} \\
&= \cos \phi
\end{aligned}
$$

Since both magnitude and angle are preserved, there is significantly less distortion produced in the rebuilt image when an orthogonal matrix is used. Since the transformation matrix *W* is the product of three other matrices, one can normalize *W* by normalizing each of the three matrices.

The normalized version of *W* is

$$
W = \begin{bmatrix}
\sqrt{8}/64 & \sqrt{8}/64 & 1/2 & 0 & \sqrt{2}/4 & 0 & 0 & 0 \\
\sqrt{8}/64 & \sqrt{8}/64 & 1/2 & 0 & -\sqrt{2}/4 & 0 & 0 & 0 \\
\sqrt{8}/64 & \sqrt{8}/64 & -1/2 & 0 & 0 & \sqrt{2}/4 & 0 & 0 \\
\sqrt{8}/64 & \sqrt{8}/64 & -1/2 & 0 & 0 & -\sqrt{2}/4 & 0 & 0 \\
\sqrt{8}/64 & \sqrt{8}/64 & 0 & 1/2 & 0 & 0 & \sqrt{2}/4 & 0 \\
\sqrt{8}/64 & -\sqrt{8}/64 & 0 & 1/2 & 0 & 0 & -\sqrt{2}/4 & 0 \\
\sqrt{8}/64 & \sqrt{8}/64 & 0 & 1/2 & 0 & 0 & 0 & \sqrt{2}/4 \\
\sqrt{8}/64 & -\sqrt{8}/64 & 0 & -1/2 & 0 & 0 & 0 & -\sqrt{2}/4
\end{bmatrix}
$$

3.2.1.Compression Ratio

If we choose our threshold value ε to be positive (i.e. greater than zero), then some entries of the transformed matrix will be reset to zero and therefore some detail will be lost when the image is decompressed. The key issue is then to choose ε wisely so that the compression is done effectively with a minimum "damage" to the picture. Note that the compression ratio is defined as the ratio of nonzero entries in the transformed matrix ($S=W^TAW$) to the number of nonzero entries in the compressed matrix obtained from S by applying the threshold ε.

3.3.Experimental Results

Figure 3

Name	**Compression Ratio**	**Energy Retained**	**PSNR Of Decompressed Image**
Cameraman.tiff	27.314	99.9724	58.942
Moon.tiff	31.546	99.9482	57.348
Lena.jpeg	28. 489	96.4874	58.342

Table 1: Comparison Of Experimental Results

3.4 .Conclusion

The wavelet transform we have discussed is the simplest, and crudest, member of a large class of possibilities. These Haar methods have been known and used in image processing for many years. As we have seen, some quite acceptable results can be achieved with this modest little transform. The performance of the wavelet scheme in terms of compression scores and signal quality is incomparable with other good techniques such as MP3 codecs; however the implemented scheme performs reasonably well with an average fidelity and with much less computational burden. In addition, using wavelets, the compression ratio can be easily varied, while most other compression techniques have fixed compression ratios. Wavelets provide an alternative to classical Fourier methods for both one- and multi-dimensional data analysis and synthesis, and have numerous applications both within mathematics (e.g., to partial differential operators) and in areas as diverse as physics, seismology, medical imaging, digital image processing, signal processing, and computer graphics and video. Unlike their Fourier cousins, wavelet methods make no assumptions concerning periodicity of the data at hand. As a result, wavelets are particularly suitable for studying data exhibiting sharp changes or even discontinuities. Wavelet-based compression is claimed to be more efficient at low bit rates but are actually less successful than discrete cosine transform (DCT) -based systems in achieving good efficiency at near-transparent compression ratios.

4.Reference

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