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# Relation Between The Solutions Of BBGKY-Heirarchy Of Dynamic Equations And Particle Solution Of Vlasov Equation

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***Abstract:***

*In this paper it is presented the relation between the solution of BBGKY hierarchy of dynamic equations and the solution of vlasov equation obtained by particle method. It is also shown that the solution of BBGKY hierarchy of dynamic equation is obtained by the solution of vlasov equation which is derived by the particle method.*

**Key words:** *Dynamic equation, vlasov equation.*

**1.Introduction**

Consider a system of molecules of having only one atom. Suppose the molecules are interacting each other through a two body potential of Φ. In the analysis of statistical physics, we need to get solutions of BBGKY- dynamic equations.

$$\frac{\partial}{\partial t} f_n(t) = [H_n, f_n(t)] + \frac{1}{v}\int \sum_{i=1}^{n}[\Phi(q_i - q), f_{n-1}(t)]\, dx,$$

(1)

Where $f_n$ is the probability density of the gas ensemble of time‘t’$\in R_+$ at position$q_1 \in A$, $q_2 \in A$……..$q_n \in A$ with velocities $v_1 \in R_+^3 \ldots\ v_n \in R_+^3$ of particles. Therefore, f:$R_+ \times F \to R_+$ with the phase space F=$(A + R_+^3)^n$.

Here,$H_n = \sum_{1\le i\le n} T_i + \sum_{1\le i<j\le n} \emptyset(q_i - q_j)$

$$T_i = \frac{{P_i}^2}{2m}$$

Here we have consider the monatomic molecule and hence m=1 is the mass of the molecule, ‘p’ is the momentum of the molecule, n$\in$ N, where N is the number of molecules and Vis the volume of the system. As N→ ∞, V→ ∞ then $\frac{V}{N}$ is a constant value, which is a volume per unit molecule and [,] denotes the poison brackets.

**2.Notation Introduction**

$(Hf)_n$=$[H_n, f_n]$; $(D_x f)_n(x_1, \ldots\ldots x_n)$=$f_{n+1}(x_1, \ldots\ldots x_n, x)$;

$$(A_x f)_n = \frac{1}{v}\sum_{1\le i\le n}[\emptyset(q_i - q), f_n];$$

f (t)= $\{f_1(t_1 x_1), \ldots\ldots\ldots\ldots f_n(t, x_1, \ldots\ldots\ldots, x_n), \ldots\ldots\ldots\}$; where n=1,2,3,…..

(2)

We can write the equation in the following form:

$$\frac{\partial}{\partial t} f(t) = Hf(t) + \int A_x D_x f(t)\, dx, \quad \text{where } n=1,2,\ldots\ldots\ldots$$

(3)

**4.Evaluation Of Hierarchy Of Dynamic Equations For Correlation Functions**

Theorem1: The hierarchies of dynamic equations for the correlation function are in the form

$\frac{\partial}{\partial t}\Psi(t)= H_{\Psi}(t)+\frac{1}{2}W(\Psi(t),\Psi(t))+\int A_x D_x \Psi(t)dx+\int A_x\Psi(t)*D_x\Psi(t)dx,$

(4)

Where $f(t) = \Gamma\Psi(t) = 1+\Psi(t) +\frac{\Psi(t)*\Psi(t)}{2!}+\ldots\ldots (*\Psi(t))^n+\ldots\ldots$

$\Psi(t) = \{\Psi(t,x_1) \ldots\ldots \Psi(t,x_1\ldots\ldots x_n), \ldots\ldots\}$

(5)

$(\Psi*\Psi)(x) = \sum_{Y\subset X}\Psi(Y)\Psi(X\backslash Y);$ $1*\Psi=\Psi;$

(6)

$(*\Psi)^n= \Psi*\Psi*\Psi\ldots\ldots\ldots\ldots*\Psi$ up to 'n' times

$X= (x_1 \ldots\ldots x_n) = (x_{(n)});$ $Y=(x_{n'}), n^1 \in n$

(7)

Where $n^1$=1, 2, 3…

$(U\Psi_n) = [\sum_{1\le i<j\le n}\emptyset(q_i-q_j),\Psi_n]$

(8)

$W(\Psi,\Psi) = \sum_{Y\subset X}U(Y;X\backslash Y)\Psi(Y)\Psi(X\backslash Y)$

(9)

From the relation between the solutions of JDSGT, we have

$D_x\Gamma\Psi(t) = D_x\Psi(t)*\Gamma\Psi(t)$

(10)

$A_x\Gamma\Psi(t) = A_x\Psi(t)*\Gamma\Psi(t)$

(11)

$A_xD_x\Gamma\Psi(t) = A_xD_x\Psi(t)*\Gamma\Psi(t) + A_x\Psi(t)*D_x\Psi(t)*\Gamma\Psi(t)$

(12)

$T\Gamma\Psi(t) = T\Psi(t)*\Gamma\Psi(t)$

(13)

$U\Gamma\Psi(t) = U\Psi(t)*\Gamma\Psi(t) + \frac{1}{2}W(\Psi(t), \Psi(t) * \Gamma\Psi(t))$

(14)

$\frac{\partial}{\partial t}\Gamma\Psi(t) = \frac{\partial}{\partial t}\Psi(t)*\Gamma\Psi(t)$

(15)

On substituting equations (6) → (10) in equation (5), and multiplying both sides by Γ (−Ψ (t)), we can get equation (3)

To find the considered system on the basis of arguments similar to those in equation (2), we can choose an expansion parameter v, setting

$\emptyset(q_i - q_j) = v\theta(q_i - q_j)$

(16)

On substituting equations (1)→ (4) in equation (8), we can get

$\Psi_n(t) = v^{n-1}\Psi_n(t)$

(17)

On the basis of equation (11), (12), the equation (3) for n takes the form

$$\frac{\partial}{\partial t}\Psi_n(t, X) = [\textstyle\sum_{1\le i\le n} T_i, \Psi_n(t)] + v\,(u\,\Psi(t))_n(x) + \frac{v}{2}(W\Psi(t), \Psi(t))_n(X)$$
$$+ v^2 \int (A_x D_x \Psi(t))_n(X)dx$$

(18)

To solve the equation (13), we need to apply perturbation theory and we have to see that the solution should be in the form of the series

$\Psi_n(t, X) = \sum_\mu v^\mu \Psi_n^\mu(t, X)$, n= 1, 2, 3…, μ=0, 1, 2…

(19)

On substituting equation (14) in equation (13) and equating the coefficients of equal powers of v, we get

$(\frac{\partial}{\partial t} + L_1)\, \Psi_1^0$ (t) =0, $(\frac{\partial}{\partial t} + L_1 + L_2)\, \Psi_2^0(t) = S_2^0 \ldots (\frac{\partial}{\partial t} + \sum_{i=1} L_i)\, \Psi_n^\mu(t) = S_n^\mu$,

Where we have introduced the notation

$$L_1(\Psi_1^0(t)) = v_1 \frac{\partial}{\partial q_1} \Psi_1^0(t, x_1) - \int \frac{\partial \theta(q_1 - q)}{\partial q_1} \frac{\partial \Psi_1^0(t_1 x)}{\partial p_1} \Psi_1^0(t, x) dx)$$

$$L_i(\Psi_n^\mu(t)) = v_1 \frac{\partial}{\partial q_1} \Psi_n^\mu(t, X) - v \int A_x \Psi^0(t)(x_i)(D_x \Psi^\mu)_{n-1}(t, X \backslash x_i) dx$$

$$S_n^\mu = (U\, \Psi^{\mu-1}(t))_n(X) + \frac{1}{2} \sum_{\delta_1+\delta_2=\mu} \left(W(\Psi^{\delta_1}(t), \Psi^{\delta_2}(t)\right)_n(X) + v \int (A_x B_x \Psi^{n-1}(t))_n(X)\, dx$$

$$+ V \int \sum_{\delta_1+\delta_2=\mu} (A_x \Psi^{\delta_1}(t) D_x \Psi^{\delta_2}(t))_n(X)\, dx$$

(20)

Thus, the solution of equation (13) reduces into the solution of the consistent, in-consistent and vlasov's equations for $\Psi_1^0$(t) as shown above.

Theorem2: The series (15), $\Psi_n$ (t, X) $= \sum_\mu v^\mu \Psi_n^\mu(t, X)$, where $\Psi_1^0$ is defined in accordance with solution of vlasov's equation and remaining $\Psi_n^\mu$ on the center of the formula

$$\Psi_n^\mu(t, X) = \int dx^1{}_1 \ldots\ldots \int dx_n^1 \int_{-\infty}^{t} dt^1\, S_n^\mu(t, x_1^1, \ldots x_n^1) \cap_{1 \le i \le n} G(t-t^1;\ x_i, x_i^1)$$

(21)

is a solution of equation (13), if 'G' satisfies the equation.

$$(\frac{\partial}{\partial t} + v_i \frac{\partial}{\partial q_1})\, G\,(t-t^1; x_i, x_i^1) - \frac{\partial \Psi(t, x_i)}{\partial v_i} \int \frac{\partial \theta(q_i - q)}{\partial q_i} G\,(t-t^1;\ x, xi1)\, dx$$

$$- \int \frac{\partial \theta(q_i - q)}{\partial q_i} \frac{\partial G(t, t^1; x_i, x_i^1)}{\partial v_i} \Psi\,(t, x) dx = 0$$ with initial conditions

$$G\,(0; x_i, x_i^1) = \delta\,(x_i - x_i^1).$$

Proof: We have considered equation (15) and (16), where (15) is the vlasov's equation. This system of coupled equations for single-molecule and two-molecule trepidations can serve to determine the successive approximations$\Psi_n^\mu(t)$. $\Psi_1^0$(t, X) is the solution of vlasov's equation.

Substituting equation (3) in (8), we have

$\Psi_2^0\,(t, x_1, x_2) = \int dx_1^1 \int dx_2^1 \int_{-\infty}^{t} dt^1\, s_2^0(t;\ x_1^1, x_2^1)$

(22)

G $(t\text{-}t^1;x_1, x_1^1)$ G $(t\text{-}t^1;x_2, x_2^1)$ in equation (16), we observe that equation (20) is a solution of (16) if $S_2^0$ $(t,x_1, x_2)$ = $[\theta(q_1 - q_2), \Psi_1^0(t;x_1)\Psi_1^0\,(t, x_2)]$ $+\int_{i=1}^{2}[\theta(q_1 - q), \Psi_1^0(t;x_1)\Psi_1^0\,(t;x)]$dx and if G satisfies the equation

$(\frac{\partial}{\partial t} + v_1 \frac{\partial}{\partial q_1})$ G $(t\text{-}t^1;x_1, x_1^1) - \frac{\partial \Psi(t,x_1)}{\partial v_1} \int \frac{\partial \theta(q_1 - q)}{\partial q_1}$G $(t\text{-}t^1;x_1, x_1^1)$ dx

$-\int \frac{\partial \theta(q_1-q)}{\partial q_1} \frac{\partial G\,(t-t^1;x_1,x_1^1)}{\partial v_1}\Psi$ (t, x) dx = 0 with initial conditions.

(25) $G(0;x_1, x_1') = \delta(x_1 - x_1')$.

The recursive system of equation (18) can, with allowance for the establishment structure of the solutions, serve to determine the successive approximations $\psi_n^{\mu}(t)$ and, therefore, formula (15). Indeed substituting again (20) directly in (18), we can see that (20) directly in (18), we can see that (20) is a solution of (18) if$S_n^{\mu}$ is defined in accordance with (19) and if G satisfies equation (22) with the initial condition (23)

Existence and uniqueness of the solution of the following Vlasov equation is studied in [5-7] by the particle method:

$$\partial_t \psi_1^0(t;x_1) = -v_1 \nabla_x \psi_1^0(t;x_1) + \frac{e_s}{m_s} \nabla_x A^{k-1} \nabla_{v_1} \psi_1^0(t_1;x_1), \quad (26)$$

$$\psi_1^0(T_k) = f_1^{k-1}(T_k) \quad (27)$$

$$-\Delta_x U^k = \frac{1}{\epsilon_0} \sum_s \int_{\Gamma_2} e_1 f_1^k ds \qquad T = T_k,$$

Where $T_k = \frac{k}{n} T, k = 1 \ldots, n, n \in N$ of size $\frac{1}{n} T, U^0$, solution of (25)with $f^0(0, P) = f^0(P); \theta(|q_i - q_j|)$ is Coulomb potential; U-potential by E=-ΔU satisfies Poisson's equation . In [5, 11], it is shown that $\psi_1^0(t_1, x_1\grave{}, v_1) = (\psi^0 \phi_0. t)(x_1, v_1)$ is solution of the

Vlasov equation. Here, we assume that E is Lipschitz continuous, $\phi_{t,r}: F \to F$ is a measure –preserving group homomorphism [6] and $\psi^0$ is continuous initial conditions.

A numerical scheme for the Vlasov equation is as follows [11]: For every time step $t_k = k\Delta t, k = 0{,}1, \ldots$.

$$v_i^N(t_{k-1}) = v_i^N(t_k) + \Delta t E\left(q_i^N(t_k)\right)$$

$$q_i^N(t_{k-1}) = q_i^N(t_k) + \Delta t v_i^N(t_{k+1})$$

$$\alpha_i^N(t_{k+1}) = \alpha_i^N(t_k)$$

Solutions (20) of two equations (16), (17) of hierarchy are in good agreement with results of [3] for plasma physics and this method is opening possibilities to calculate the solutions of the complex kinetic equations of BBGKY hierarchy.

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