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Frequency Domain Filter In Digital Image Processing In Remote Sensing: An Overview

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Abstract:

Digital image processing involves the manipulation and interpretation of digital images with the aid of a computer [1]. Digital image processing is an extremely broad subject, and it often involves procedures that can be mathematically complex. Different digital filters have been developed in image processing for better interpretation and to improve the visual interpretability of an image by increasing the apparent distinction between the features in the scene. Filters are broadly classified into spatial domain filtering and frequency domain filtering .This paper reviews working of different frequency domain digital filters in image processing.

Keywords: *Frequency Domain Filters, Fourier Transform, Discrete Fourier Transform, Fast Fourier Transform.*

1.Introduction

The Fourier transform of an image is a breakdown of the image into its frequency or scale components. Such filters operate on the amplitude spectrum of an image and remove, attenuate, or amplify the amplitudes in specified wavebands. Any

function that periodically repeats itself can be expressed as a sum of sines and cosines of different frequencies each multiplied by a different coefficient is called a Fourier series.

A simple filter might set the amplitudes of all frequencies less than or more than a selected threshold to zero. If the amplitude spectrum information is converted back to the spatial domain by an inverse Fourier transform, the result is new improved

transformed image. Any wavelength or waveband can be operated upon in the frequency domain. The three general categories of filter are low-pass, high-pass and band-pass filters. The slowly varying background pattern in the image can be envisaged as a two-dimensional waveform with a long wavelength or low frequency; hence a filter that separates this slowly varying component from the

remainder of the information present in the image is called a low-pass filter. A filter that separate out the more rapidly varying detail like a two-dimensional waveform with a short wavelength or high frequency component is called a high-pass filter[2]. A band-pass filter removes both the high and low frequency components, but allows an intermediate range of frequencies to pass through the filter. Directional filters can also be developed, because the amplitude spectrum of an image contains information about the frequencies and orientations as well as the amplitudes of the scale components that are present in an image.

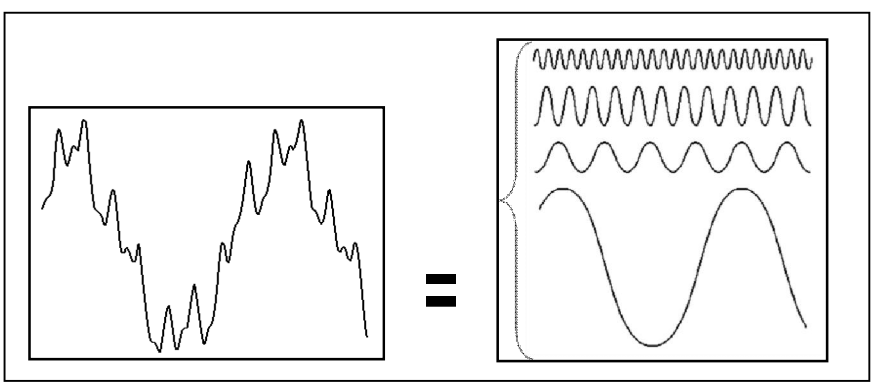

Figure 1: Original signal expressed as a sum of sines and cosines of different frequencies.

2.Sampling Theorem

The sampling theorem called "Shannons Sampling Theorem" states that a continuous signal must be discretely sampled

at least twice the frequency of the highest frequency in the signal[3].

More precisely, a continuous function f(t) is completely defined by samples every 1/fs (fs is the sample frequency) if the frequency

spectrum F(f) is zero for f > fs/2. fs/2 is called the Nyquist frequency and places the limit on the minimum sampling frequency when

digitizing a continuous signal.

If x(k) are the samples of f(t) every 1/fs then f(t) can be exactly reconstructed from these samples, if the sampling theorem

has been satisfied, by

$$f(t) = \sum_{k=-\infty}^{k=\infty} x(k)\ \mathrm{sinc}(t\ fs - k)$$

where

$$\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

Normally the signal to be digitized would be appropriately filtered before sampling to remove higher frequency components. If the

sampling frequency is not high enough the high frequency components will wrap around and appear in other locations in the

discrete spectrum, thus corrupting it.

The key features and consequences of sampling a continuous signal can be shown graphically as follows.

Consider a continuous signal in the time and frequency domain.

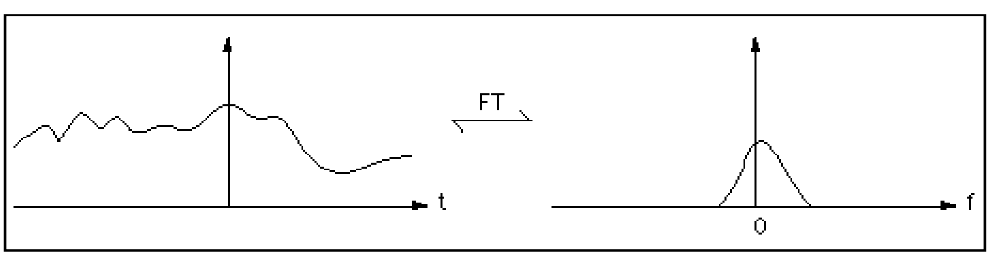

Figure 2

Sample this signal with a sampling frequency fs, time between samples is 1/fs. This is equivalent to convolving in the frequency
domain by delta function train with a spacing of fs.

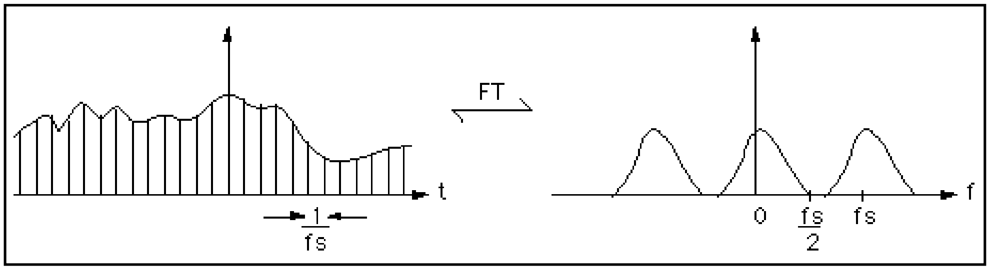

Figure 3

If the sampling frequency is too low the frequency spectrum overlaps, and become corrupted.

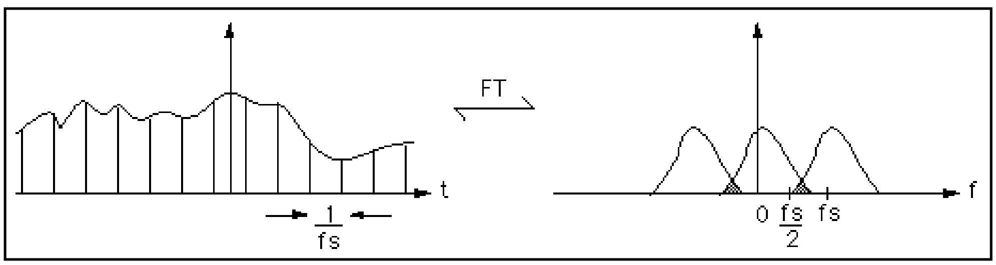

Figure 4

Another way to look at this is to consider a sine function sampled twice per period (Nyquist rate). There are other sinusoid functions
of higher frequencies that would give exactly the same samples and thus can't be distinguished from the frequency of the original
sinusoid[3].

2.1.Steps Involved To Filter An Image In The Frequency Domain Are As Follows

Firstly Compute F (u,v) the Fourier Transform of the image. Then to multiply F (u,v) by a filter function H (u,v).And finally compute its inverse Fourier transform of the result .

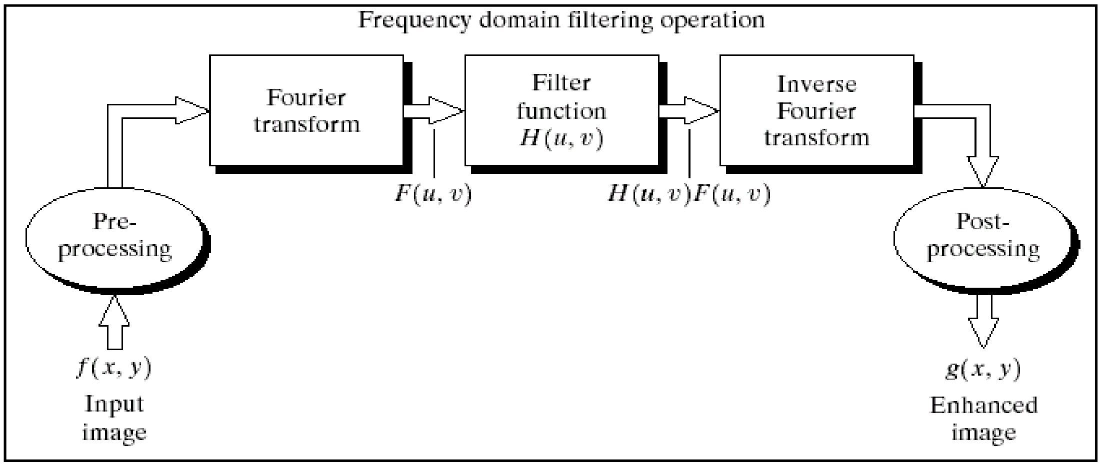

Figure 5: Various Steps Involved in Frequency Domain Filtering Operation

The Fourier transform operates on a grey scale image, not on a multispectral data set. Its purpose is to break down the image into its spatial scale components, which are defined to be sinusoidal waves with varying amplitudes, frequencies and directions. The idea underlying the Fourier transform is that the grey scale values forming a single-band image can be viewed as a three-dimensional intensity surface, with the rows and columns defining two axes and the grey level intensity value at each pixel giving the third (z) dimension[2]. The Fourier transform provides details of the frequency of each of the scale components (waveforms) fitted to the image and the proportion of information associated with each frequency component. It is really important to note that the Fourier transform is completely reversible.

2.2.The Discrete Fourier Transform

Consider a complex series f(x, y) with N samples

Where f(x ,y) is a complex number

$$f(x, y) = f(x, y)_{real} + j\, f(x, y)_{imag}$$

where imag means imaginary.

Further, assume that that the series outside the range 0, N-1 is extended N-periodic

The FT of this series will be denoted F(u ,v) it will also have N samples. The forward transform will be defined as

$$F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi(ux/M + vy/N)}$$

For u = 0, 1, 2…M-1 and v = 0, 1, 2…N-1.

The inverse DFT is given by

$$f(x,y) = \frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)e^{j2\pi(ux/M+vy/N)}$$

For x = 0, 1, 2…M-1 and y = 0, 1, 2…N-1

The functions are described as complex series, real valued series can be represented by setting the imaginary part to 0.

In general, the transform into the frequency domain will be a complex valued function, that is, with magnitude and phase.

- Magnitude= Square Root (f(x, y) real* f(x, y) real+ f(x , y) imag* f(x , y) imag)
- Phase= tan-1(f(x , y) imag / f(x, y) real)

The DFT of a real series, i.e.: imaginary part = 0, results in a symmetric series about the Nyquist frequency. The negative frequency samples are also the inverse of the positive frequency samples. The highest positive (or negative) frequency sample is called the Nyquist frequency. This is the highest frequency component that should exist in the input series for the DFT to yield "uncorrupted" results. More specifically if there are no frequencies above Nyquist the original signal can be exactly reconstructed from the samples.

2.3.The Fast Fourier Transform

For a continuous function of one variable f(t), the Fourier Transform F(f) will be defined as[3]:

$$F(f) = \int_{-\infty}^{\infty} f(t)\, e^{-j2\pi ft}\, dt$$

and the inverse transform as

$$f(t) = \int_{-\infty}^{\infty} F(f)\, e^{j2\pi ft}\, df$$

Where j is the square root of -1 and e denotes the natural exponent

$$e^{j\emptyset} = \cos(\emptyset) + j\sin(\emptyset).$$

In place of DFT, an algorithm called the Fast Fourier Transform (FFT) is used of the FFT over the older method can be summarized by the fact that the number of operations required to evaluate the coefficients of the Fourier series using the older method is proportional to N2 where N is the number of sample points (length of the series) whereas the number of operations involved in the FFT is proportional to N log2N. Only the amplitude information is used.

Figure 6:Original Image

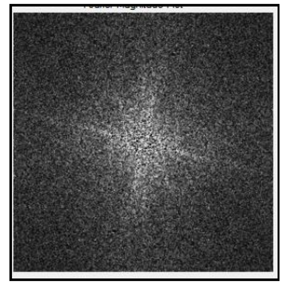

Figure 7: Fourier magnitude

Figure 8: New Transformed image after applying inverse transform

2.4.Ideal Frequency Domain Filter

2.4.1.Ideal low Pass Filter in Frequency Domain

The transfer function for the ideal low pass filter can [4]be given as:

$$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \le D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$$

where D(u,v) is given as:

Low Pass Filter means cut off all high frequency components that are a specified distance Do from the origin of the transform.

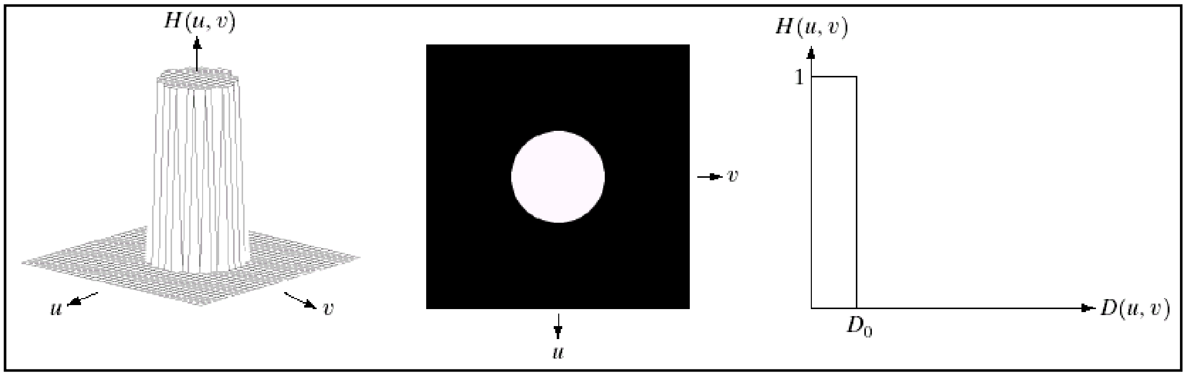

Figure 9: Ideal Low Pass Filter

2.4.2.<u>Ideal High Pass Filter</u>

The ideal high pass filter is given as:

$$H(u,v) = \begin{cases} 0 & \text{if } D(u,v) \leq D_0 \\ 1 & \text{if } D(u,v) > D_0 \end{cases}$$

where Do is the cut off distance as before

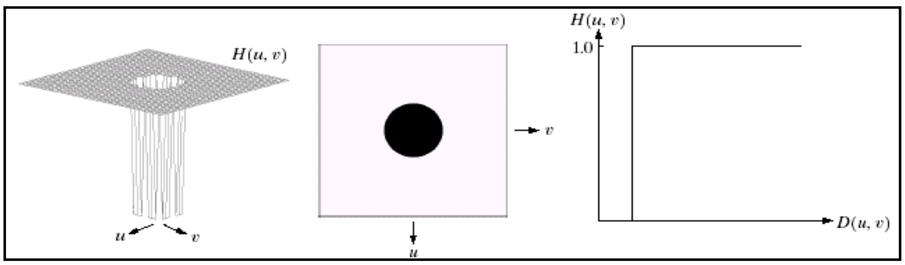

Figure 10: Ideal High Pass Filter

2.5.Butterworth Filter

2.5.1.<u>Butterworth Low Pass Filter</u>

The transfer function of a Butterworth low pass filter of order n with cut off frequency at distance Do from the origin is defined as[4]:

$$H(u,v) = \frac{1}{1+[D(u,v)/D_0]^{2n}}$$

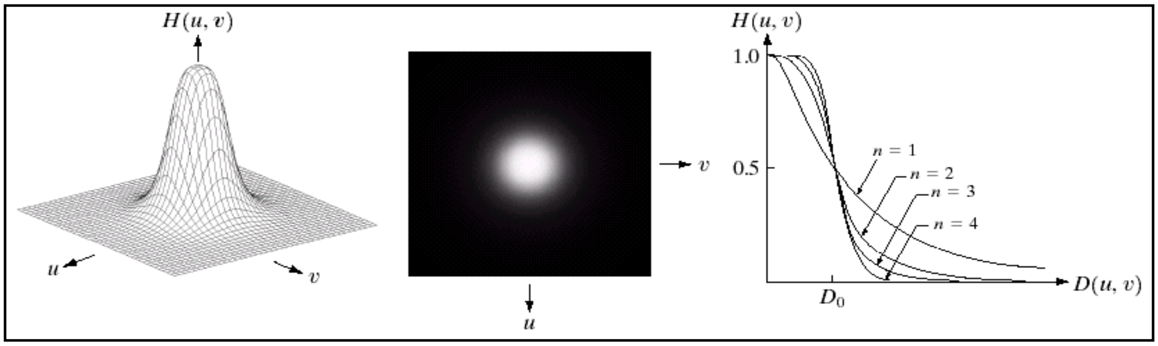

Figure 11: Butterworth Low Pass Filter

2.5.2.Butterworth High Pass Filter

The Butterworth high pass filter is given as[4]:

$$H(u,v) = \frac{1}{1+[D_0 / D(u,v)]^{2n}}$$

where n is the order and Do is the cut off distance as before

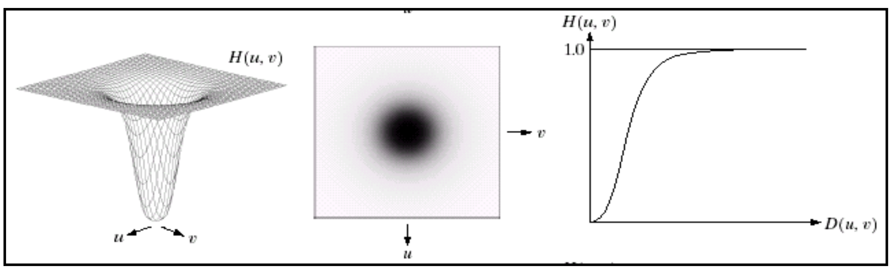

Figure 12: Butterworth High Pass Filter

2.6.Gaussian Filter

2.6.1.Gaussian Low Pass Filter

The transfer function of a Gaussian low pass filter is defined as[4]:

$$H(u,v) = e^{-D^2(u,v)/2D_0^2}$$

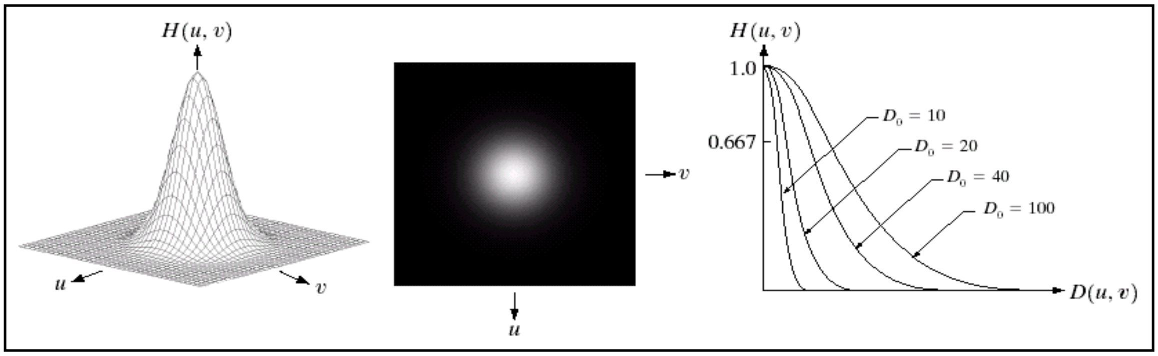

Figure 13: Gaussian Low Pass Filter

2.6.2.Gaussian High Pass Filter

The Gaussian high pass filter is given as:

where Do is the cut off distance as before

$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$

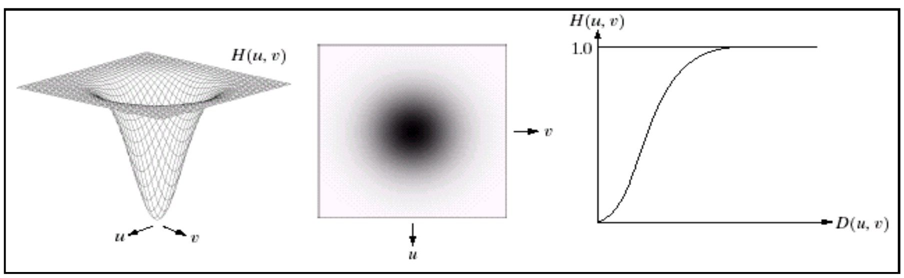

Figure 14: Gaussian High Pass Filter

3.Conclusion

This paper serves as assessment of working of the different digital filters in frequency domain . Filtering in the frequency domain is much faster especially for large images . The reason that Fourier based techniques have become so popular is the development of the Fast Fourier Transform (FFT). Fast Fourier Transforms are faster than Discrete Fourier Transforms. The field of image processing are constantly evolving in the last decade, but still improvements in filtering methods are needed .This paper serves as a review of different frequency domain filtering and highlights the working of each digital filters image processing in Remote sensing.

4.Reference

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