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Modeling Of Permanent Magnet BLDC Motor Using State Space Analysis

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Abstract:

BLDC motor is one of motor type gaining popularity nowadays. We find that brushless DC (BLDC) motor modelling can be simply implemented using transfer function analysis. But it suffers from certain drawbacks such as transfer function is only defined under zero initial conditions, only applicable to linear time-invariant systems and so on. To overcome these drawbacks this paper presents a detailed state space modelling of permanent magnet brushless DC motor. In this paper, the motor is designed based on state space model to get information about the state of the system variables. By reading the instantaneous position of the rotor as an output, different variables of the motor can be controlled without the need of any external sensors or position detection techniques. With state space model representation, the motor performance can be analyzed for different motor parameters.

1.Introduction

Conventional dc motors are highly efficient and their characteristics make them suitable for use as servomotors. However, their only drawback is that they need a commutator and brushes which are subject to wear and require maintenance. When the functions of commutator and brushes were implemented by solid-state switches, maintenance-free motors were realised. These motors are now known as brushless dc motors.

With rapid developments in power electronics, power semiconductor technologies, modern control theory for motors and manufacturing technology for high performance magnetic materials, the brushless DC motors (BLDCM) have been widely used in many fields. Due to the advancement of small size, good performance, simple structure, high reliability and large output torque, BLDC motors have attracted increasing attention. However, the application of position sensor makes the motor body heavy, as well as lots of wires are needed, which in turn brings complication and interference in the design. Thus the position sensorless control technology attracts increasing research interest and currently becomes one of the most promising trends of BLDCM control system. The modeling for BLDCM depends on computer engineering and can effectively shorten development cycle of position sensorless BLDCM control system and evaluate rationality of the control algorithm imposed on the system. This provides a good foundation for system design and verify novel control strategy.

Though the transfer function model provides us with simple and powerful analysis and design techniques, it suffers from certain drawbacks such as transfer function is only defined under zero initial conditions. Further it has certain limitations due to the fact that the transfer function model is only applicable to linear time-invariant systems and there too it is generally restricted to single input single output systems. Another limitation of the transfer technique is that it reveals only the system output for a given input and provides no information regarding the internal state of the system.

In this paper, the motor is designed based on state space model to get information about the state of the system variables. By adopting this model, powerful processor requirement, large random access memory can be avoided with more design flexibility and faster results can be obtained.

2. Modeling Of Pm Brushless DC Motor

2.1.Assumptions

- The motor's stator is a star wound type
- The motor's three phases are symmetric, including their resistance, inductance and mutual inductances.
- There is no change in rotor reluctance with angle due to non-salient rotor.
- There is no misalignment between each magnet and the corresponding stator winding.

The coupled circuit equations of the stator windings in terms of motor electrical constants are

$$\begin{bmatrix} V_{as} \\ V_{bs} \\ V_{cs} \end{bmatrix} = \begin{pmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{pmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} + p \begin{pmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc} \end{pmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} + \begin{bmatrix} e_{as} \\ e_{bs} \\ e_{cs} \end{bmatrix}$$

(1)

Where R_s is the stator resistance per phase and it is assumed to be equal for all three phases. The induced emfs e_{as}, e_{bs} and e_{cs} are all assumed to be trapezoidal. E_p is the peak value derived as

$$E_p = (Blv)N = N(Blr\omega_m) = N\phi_a\omega_m = \lambda_p\omega_m \quad (2)$$

Where N is the number of conductors in series per phase

v is the velocity,m/s

l is the length of the conductor,m

r is the radius of stator bore,m

ω_m is the angular velocity,rad/s

B is the flux density of the field in which the conductors are placed

This flux density is solely due to the rotor magnets. The product (Blr),which is ϕ_a ,has the dimensions of flux and is directly proportional to the air-gap flux, ϕ_g , as

$$\phi_a = Blr = 1/\pi(B\pi lr) = 1/\pi(\phi_g) \quad (3)$$

If there is no change in the rotor reluctance with angle, and assuming symmetric three phases, the self-inductances of all phases are equal and the mutual inductance between phases are equal to one another and they are denoted as

$$L_{aa} = L_{bb} = L_{cc} = L \quad \text{and} \tag{4a}$$

$$L_{ab} = L_{ba} = L_{ac} = L_{ca} = L_{bc} = L_{cb} = M, H \tag{4b}$$

Substituting equations (4a) and (4b) in equation (1), The PMBDCM model is obtained as

$$\begin{bmatrix} V_{as} \\ V_{bs} \\ V_{cs} \end{bmatrix} = R_s \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} + \begin{pmatrix} L & M & M \\ M & L & M \\ M & M & L \end{pmatrix} p \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} + \begin{bmatrix} e_{as} \\ e_{bs} \\ e_{cs} \end{bmatrix} \tag{5}$$

The stator phase currents are constrained to be balanced ,i.e., $i_{as} + i_{bs} + i_{cs} = 0$, which leads to the simplification of inductance matrix in the model as

$$\begin{bmatrix} V_{as} \\ V_{bs} \\ V_{cs} \end{bmatrix} = \begin{pmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{pmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} + \begin{pmatrix} L-M & 0 & 0 \\ 0 & L-M & 0 \\ 0 & 0 & L-M \end{pmatrix} p \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} + \begin{bmatrix} e_{as} \\ e_{bs} \\ e_{cs} \end{bmatrix} \tag{6}$$

The generated electromagnetic torque is

$$T_e = e_{as} i_{as} + e_{bs} i_{bs} + e_{cs} i_{cs} / \omega_m \quad \text{Nm} \tag{7}$$

The instantaneous induced emfs can be written as

$$e_{as} = f_{as}(\theta_r) \lambda_p \omega_m \tag{8a}$$

$$e_{bs} = f_{bs}(\theta_r) \lambda_p \omega_m \tag{8b}$$

$$e_{cs} = f_{cs}(\theta_r) \lambda_p \omega_m \tag{8c}$$

Where the functions $f_{as}(\theta_r), f_{bs}(\theta_r), f_{cs}(\theta_r)$ have the same shape as e_{as}, e_{bs}, e_{cs} with a maximum magnitude of ± 1.

The electromagnetic torque can be written as

$$T_e = \lambda_p [f_{as}(\theta_r) i_{as} + f_{bs}(\theta_r) i_{bs} + f_{cs}(\theta_r) i_{cs}] \quad \text{Nm} \tag{9}$$

The equation of motion for a simple system with inertia J, friction coefficient B, and load torque T_l is

$$J d\omega_m / dt + B\omega = T_e - T_l \tag{10}$$

And electrical rotor speed and position are related by

$$d\theta_r / dt = P / 2 \omega_m \tag{11}$$

Where P is the number of poles

ω_m is the rotor speed in mechanical rad/s

θ_r is the rotor position in rad

From the above equation, the system state equation is re-written as

$$\dot{X}(t) = Ax(t)+Bu(t)$$

$$x(t) = \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \\ \omega_m \\ \theta_r \end{bmatrix}$$

$$u(t) = \begin{bmatrix} V_{as} \\ V_{bs} \\ V_{cs} \\ T_l \end{bmatrix}$$ and

$$dx/dt = \begin{bmatrix} di_{as}/dt \\ di_{bs}/dt \\ di_{cs}/dt \\ d\omega_m/dt \\ d\theta_r/dt \end{bmatrix}$$

From eqn (6)

$$V_{as} = R_s i_{as} + d/dt(L-M)i_{as} + e_{as}$$

$$di_{as}/dt = (V_{as}/L-M) - (R_s i_{as}/L-M) - (e_{as}/L-M)$$

Let $L_1 = L - M$

$$di_{as}/dt = (V_{as}/L_1) - (R_s i_{as}/L_1) - (e_{as}/L_1)$$

$$V_{bs} = R_s i_{bs} + d/dt(L-M)i_{bs} + e_{bs} \qquad (12)$$

$$di_{bs}/dt = (V_{bs}/L-M) - (R_s i_{bs}/L-M) - (e_{bs}/L-M)$$

$$= (V_{bs}/L_1) - (R_s i_{bs}/L_1) - (e_{bs}/L_1) \qquad (13)$$

$$V_{cs} = R_s i_{cs} + d/dt(L-M)i_{cs} + e_{cs}$$

$$di_{cs}/dt = (V_{cs}/L-M) - (R_s i_{cs}/L-M) - (e_{cs}/L-M)$$
$$= (V_{cs}/L_1) - (R_s i_{cs}/L_1) - (e_{cs}/L_1) \quad (14)$$

From equation (10)

$$d\omega_m/dt = (T_e - T_l)/\mathrm{J} - \mathrm{B}\,\omega_m/\mathrm{J}$$
$$= T_e/J - T_l/J - B\omega_m/J$$
$$= (e_{as}i_{as} + e_{bs}i_{bs} + e_{cs}i_{cs}/\omega_m J) - (T_l/J) - (B\omega_m/J)$$
$$= f_{as}(\theta_r)\lambda_p I_a + f_{bs}(\theta_r)\lambda_p I_b + f_{cs}(\theta_r)\lambda_p I_c/\mathrm{J} - (T_l/J) - (B\omega_m/J)$$

$$(15)$$

$$\begin{bmatrix} di_{as}/dt \\ di_{bs}/dt \\ di_{cs}/dt \\ d\omega_m/dt \\ d\theta_r/dt \end{bmatrix} = \begin{bmatrix} -R_s/L_1 & 0 & 0 & f_{as}(\theta_r)\lambda_p/L_1 & 0 \\ 0 & -R_s/L_1 & 0 & f_{bs}(\theta_r)\lambda_p/L_1 & 0 \\ 0 & 0 & -R_s/L_1 & f_{cs}(\theta_r)\lambda_p/L_1 & 0 \\ f_{as}(\theta_r)\lambda_p/J & f_{bs}(\theta_r)\lambda_p/J & f_{cs}(\theta_r)\lambda_p/J & -B/J & 0 \\ 0 & 0 & 0 & P/2 & 0 \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \\ \omega_m \\ \theta_r \end{bmatrix}$$

$$+ \begin{bmatrix} 1/L_1 & 0 & 0 & 0 & 0 \\ 0 & 1/L_1 & 0 & 0 & 0 \\ 0 & 0 & 1/L_1 & 0 & 0 \\ 0 & 0 & 0 & -1/\mathrm{J} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_{as} \\ V_{bs} \\ V_{cs} \\ T_l \\ 0 \end{bmatrix} \quad (16)$$

$$A = \begin{bmatrix} -R_s/L_1 & 0 & 0 & f_{as}(\theta_r)\lambda_p/L_1 & 0 \\ 0 & -R_s/L_1 & 0 & f_{bs}(\theta_r)\lambda_p/L_1 & 0 \\ 0 & 0 & -R_s/L_1 & f_{cs}(\theta_r)\lambda_p/L_1 & 0 \\ f_{as}(\theta_r)\lambda_p/J & f_{bs}(\theta_r)\lambda_p/J & f_{cs}(\theta_r)\lambda_p/J & -B/J & 0 \\ 0 & 0 & 0 & P/2 & 0 \end{bmatrix}$$

$$(17)$$

$$B = \begin{bmatrix} 1/L_1 & 0 & 0 & 0 & 0 \\ 0 & 1/L_1 & 0 & 0 & 0 \\ 0 & 0 & 1/L_1 & 0 & 0 \\ 0 & 0 & 0 & -1/\mathrm{J} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (18)$$

3.Conclusion

BLDC motor analysis based on state space model has been done. This model has many advantages over transfer function model. Further using state space model, the performance characteristics of the BLDC motor can be evaluated for different machine parameters, which can be easily varied in the simulation study and useful information can be obtained. The method proposed in this paper provides a novel and effective tool for analyzing and designing the control system of brushless DC motor.

4.Reference

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