INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH & DEVELOPMENT

ISSN: 2278 – 0211 (Online)

Fast Power Allocation In Wireless Multi Channel Systems

R.Rejin Manuel
Pg Student, Sri Balaji Chockalingam College Of Engineering, India

S.Boopathi
Lecturer, Sri Balaji Chockalingam College Of Engineering, India

Abstract :
Inmulti-channel communications, Water-Filling (WF) is a fundamental power allocation mechanismfor capacity maximization under a given total transmit power.In this letter, we propose simple and fast WF algorithms forachieving agile power allocation, such that the system can bettermatch thedynamic nature of the wireless channels for a betterperformance. Our algorithms remove the need of water-levelsearching. Numerical results show that they can run multipletimes faster than the existing ones and can converge to theoptimal solution in a few linear calculations.

1.Introduction

OFDM (Orthogonal Frequency Division Multiplexing)and MIMO (Multiple-Input Multiple-Output) have beenwidely considered as key enabling technologies for achievingthe next generation broadband wireless communications. Inparticular, OFDM supports multi-channels using orthogonalsub-carriers in frequency domain [1], and the single-carrierMIMO broadcast channel can be equivalently parallelized intomulti-channels using SVD (Singular Value Decomposition)[2].Under a given total transmit power constraint, power allocationis a critical task for maximizing the system capacity inthose multi-channel wireless systems. In discrete time wirelesscommunication systems, power allocation needs to be carriedout in each time slot to match the time-varying nature ofthe wireless channels. Therefore, it is important to have anagile power allocation algorithm, which can respond fast to the changing environment with low complexity.It is well known that Water-Filling (WF) [3-6] is the fundamentalpower allocation mechanism that can maximize systemcapacity over multi-channels. Conventional WF algorithmsneed to search for a water-level [3-5], which dominates thecomputational complexity. Besides, they do not consider anupper-bound on the allocated power in each sub-channel,which is generally necessary in engineering practice. In this letter, we propose simple and fast algorithms tocompute optimal WF solutions. Both scenarios, with andwithout an upper-bound on the allocated powers in the subchannels,are considered. Our algorithms require much lesscomputations than the existing ones by removing the waterlevelsearching process. They can converge to the optimalsolutions multiple times faster.

2.Preliminaries

2.1.Water-Filling Formulation And Optimal Solution

Consider a multi-channel system with Nindependent subchannels$N = \{1, 2, \cdots, N\}$ and a bandwidth B for each.Let γn be the SNR (Signal to Noise Ratio) of sub-channel$n \in N$, and PT be the total transmit power. The optimalpower allocation $p = \{pn \mid n \in N\}$ for maximizing the systemcapacity can be obtained by solving the following Water-Filling (WF)problem.

$$\max \sum_{n \in N} B \log_2 (1 + \gamma_n p_n) \quad (1)$$

s.t.

$$\sum_{n \in N} p_n \le P_T; \quad (2)$$

$$p_n \ge 0, \forall n \in N. \quad (3)$$

The optimal solution to the above classic WF problem is

$$p_n = \left[\frac{B}{\ln 2} \times \frac{1}{\mu} - \frac{1}{\gamma_n}\right]_0, \forall n \in N, \quad (4)$$

where[•]0 = max(•, 0) and μ is a positive Lagrange multiplier(or water-level). If there is an upper-bound Pmax on thepower allocated to each sub-channel, we can replace constraint(3) using the following constraint (5).

$$P_{\max} \ge p_n \ge 0, \forall n \in N. \quad (5)$$

$$p_n = \left[\frac{B}{\ln 2} \times \frac{1}{\mu} - \frac{1}{\gamma_n}\right]_0^{P_{\max}}, \forall n \in N, \quad (6)$$

In this case, the optimal solution iswhere the function [•]Pmax0 confines the range of pn as per (5).In either one of the above two scenarios, μ is the solution to

$$\sum_{n \in N} p_n = P_T.$$

2.2.Brief Summary Of Existing Algorithms

The existing works [3-5] solve the problem in (1)-(3) bysearching for a proper water-level μ before $p = \{| \forall n \in N\}$ can be calculated in (4). To this end, the iterative binarysearching technique in [6] can be used. In each iteration, thepossible range of μ is halved, and the half that can possiblycontain the value of μ is identified by checking the halvingpoint according to the total transmit power constraint (7).When the algorithm converges, the value of μ can be obtained.Another iterative μ searching process [4] uses the following(8)-(9) to find μ, where (8) defines the starting point of μ and(9) is used to update μ in the subsequent iterations.

$$\frac{B}{\ln 2} \times \frac{1}{\mu} = \frac{1}{N} \left(P_T + \sum_{n \in N} \frac{1}{\gamma_n}\right); \quad (8)$$

$$\frac{B}{\ln 2} \times \frac{1}{\mu} + \frac{\sigma}{N_{on}} \left(P_T - \sum_{n \in N_{on}} p_n\right) \to \frac{B}{\ln 2} \times \frac{1}{\mu}. \quad (9)$$

In (9), σ is a predefined step size for μ searching, and Nondenotes the set of Non = |Non| sub-channels in the currentiteration that have positive powers as per (4). With a similaridea as in [4], the algorithm in [5] sequentially discardssub-channels with small γn to determine Non in the finalsolution, based on which μ and the optimal solution can becalculated. The difference is that the water-level is estimatedby sequentially discarding sub-channels instead of using (9).

3.The Proposed Fast Wf Algorithms

We first focus on the classicWF problem in (1)-(3), and thenextend the result to the more general scenario where constraint(3) is replaced by (5). A. Observations on WFFor a given positive Δ (assume that PT ≥ NΔ), let p ={pn | ∀n ∈N} = WF(PT) and p_ = p_n | ∀n ∈N=WF(PT −NΔ) be two WF solutions under the total transmitpowers PT and PT −NΔ, respectively. We have the following

Lemma 1.

Lemma 1: If pn ≥ Δ for ∀n ∈N , then p_

n = pn − Δ for∀n ∈N.

$$\frac{B}{\ln 2} \times \frac{1}{\mu} = p_n + \frac{1}{\gamma_n}, \forall n \in N. \quad (10)$$

Because μ is a constant satisfying (7), we have

$$\frac{B}{\ln 2} \times \frac{1}{\mu} = \frac{1}{N} \sum_{n \in N} \left(p_n + \frac{1}{\gamma_n} \right) = \frac{1}{N} \left(P_T + \sum_{n \in N} \frac{1}{\gamma_n} \right). \quad (11)$$

Taking (10) into account, this leads to

$$p_n = \frac{1}{N} \left(P_T + \sum_{n \in N} \frac{1}{\gamma_n} \right) - \frac{1}{\gamma_n}, \forall n \in N. \quad (12)$$

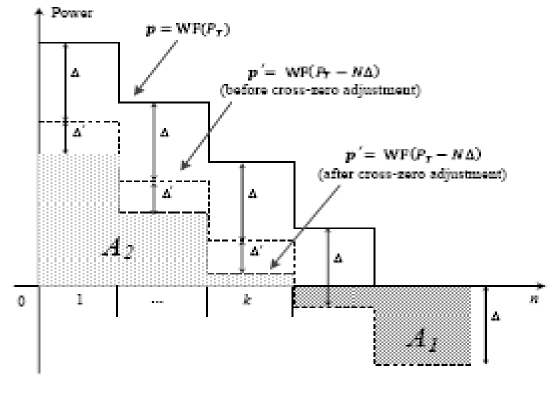


Fig. 1. Cross-zero adjustment in Water-Filling.

Similarly, if the total transmit power decreases to $P_T - N\Delta$, for $p' = \left\{ p_n' \mid \forall n \in N \right\} = \mathrm{WF}(P_T - N\Delta)$ we have

$$p_n' = \frac{1}{N} \left(P_T - N\Delta + \sum_{n \in N} \frac{1}{\gamma_n} \right) - \frac{1}{\gamma_n}, \forall n \in N. \quad (13)$$

Figure 1

Proof: Since pn ≥ Δ for ∀n ∈N, from (4) we haveFrom (12)-(13), we have p_n = pn − Δ for ∀n ∈N.Lemma 1 shows that if the total transmit power PT isdecreased by NΔ, then the transmit power in each sub-channelwill decrease by the same amount of Δ.However, Lemma 1 is only for a special case where pn ≥ Δis assumed for ∀n ∈N, which cannot be ensured in the general case with a given Δ. Fig. 1 shows an example. Thehorizontal axis denotes the sub-channel index and the verticalaxis denotes the allocated power in each sub-channel. Letp=WF(PT) be denoted by the solid curve in Fig. 1. Withoutloss of generality, we assume that the power values allocatedto the sub-channels are decreasingly ordered as n increases.If the total transmit power is decreased from PT to PT −NΔ,without considering the infeasibility of negative powers insome sub-channels, Lemma 1 can be extended such that thepower value in each sub-channel will be decreased by thesame amount of Δ (see the dashed curve in Fig. 1). This maylead to negative powers in some sub-channels, as illustratedby those after the kth sub-channel in Fig. 1. In practice,those negative transmit powers (denoted by the shadowed areaA1) should be set to zero according to (4). Meanwhile, thosepositive transmit powers on the dashed curve should be furtherdecreased accordingly as a compensation to keep the totaltransmit power PT − NΔ unchanged. We define this processas cross-zero adjustment.Lemma 2: If pn ≥ Δ+Δ_ for n ∈ {1, ・・・, k} and pn <Δfor n ∈ {k + 1, ・・・,N}, then p_
n = pn − Δ − Δ_ for n ∈{1, ・・・, k} and p_n = 0 for n ∈ {k + 1, ・・・,N}.Proof : If [•]0 = max(•, 0) in (4) is not considered, equation(4) is equivalent to (10). When the total transmit power is decreased from PT to PT − NΔ, with similar analysis as in

Lemma 1, we can see that $p\_n = pn - \Delta$ should hold (in a pure mathematical sense) for $\forall n \in N$, though

WF(P_T)

1. Initialization:

1.1) Initial calculation

Calculate $\{p_n | \forall n \in N\}$ using (12).

1.2) Sorting

Sort $\{p_n | \forall n \in N\}$ in a descending order of p_n and save the result in an ordered set $\{P_1, P_2, \ldots, P_N\}$.

1.3) Parameter initialization

$N \rightarrow x$.

2. Cross-zero adjustment:

```
while ( P_x < 0 )
{
    Set 0→S.
    for (n = x; n ≥ 1, P_n ≤ 0; n--)
    {
        −P_n + S→S;
        0 → P_n.
    }
    S/n → Δ;
    n → x;
    for (n=1; n ≤ x; n++) P_n − Δ→P_n.
}
```

3. Finish:

Save $\{P_1, P_2, \ldots, P_N\}$ back to the ordered set $\{p_n | \forall n \in N\}$.

Fig. 2. Fast water-filling without sub-channel power upper-bound.

Figure 2

Assume $p_n \geq \Delta$ for $n \in \{1, \cdots, k\}$ and $p_n < \Delta$ for $n \in \{k+1, \cdots, N\}$. Define

$$\Delta' = \frac{1}{k} \sum_{n=k+1}^{N} (\Delta - p_n). \qquad (14)$$

We have the following Lemma 2 for cross-zero adjustment.

p_k+1, p_k+2, ・・・, $p_N$are negative by assuming $pn < \Delta$ for$n \in \{k + 1,$ ・・・$,N\}$.If $[\bullet]0 = \max(\bullet, 0)$ in (4) is considered, the set of negativepowers p_k+1, p_k+2, ・・・, $p_N$should be set to zero. To keepthe total transmit power $PT - N\Delta$ unchanged, those positivepowers p_1, p_2, ・・・, $p_k$must be decreased for a total amountof $Nn=k+1(\Delta - pn)$ as a compensation. From Lemma 1,each p_n, $n \in \{1,$ ・・・, $k\}$ should be decreased further for anamount of $\Delta\_$ as formulated in (14). Consequently, we have$p_n = pn - \Delta - \Delta_ \geq 0$ for $n \in \{1,$ ・・・, $k\}$ and $p\_n = 0$ for$n \in \{k + 1,$ ・・・$,N\}$.Since we assume $pn \geq \Delta + \Delta\_$ for $n \in \{1,$ ・・・, $k\}$ inLemma 2, we have $p\_n = pn-\Delta-\Delta\_ \geq 0$ for $n \in \{1,$ ・・・, $k\}$ after the cross-zero adjustment. Generally, this may not bethe case, and some p_n, $n \in \{1,$ ・・・, $k\}$ may become negativewhen $\Delta$is subtracted. So, the cross-zero adjustment shouldbe an iterative process until all allocated powers becomenonnegative.

B. Fast WF Algorithms Without μ Searching

Based on the above analysis, we can propose a fast waterfillingalgorithm without μ searching for solving the WFproblem in (1)-(3), as in Fig. 2. Fig. 3 gives another algorithmwhich replaces constraint (3) using (5) to impose an upper- bound Pmaxon the allocated power in each sub-channel. Thealgorithm in Fig. 3 is a direct extension of that in Fig. 2.

$\text{WF}^*(P_T)$

1. Initialization:

1.1) Initial calculation

If $P_T \geq NP_{max}$, set $\{P_{max} \rightarrow p_n | \forall n \in \boldsymbol{N}\}$ and go to Step 3; otherwise calculate $\{p_n | \forall n \in \boldsymbol{N}\}$ using (12).

1.2) Sorting

Sort $\{p_n | \forall n \in \boldsymbol{N}\}$ in a descending order of p_n and save the result in an ordered set $\{P_1, P_2, ..., P_N\}$.

1.3) Parameter initialization

Set $\{P_n \rightarrow \mathcal{P}_n | \forall n \in \boldsymbol{N}\}$, $N \rightarrow x$ and $1 \rightarrow y$.

2. Sub-channel power adjustment:

2.1) Cross-zero adjustment

```
while ( P_x < 0)
{
    Set 0→S.
    for (n = x; n ≥ y, P_n ≤ 0; n--)
    {
        −P_n + S→S;
        0 → P_n.
    }
    n → x;
    S/(x − y + 1) → Δ;
    for (n=y; n ≤ x; n++)  P_n − Δ→P_n.
}
```

2.2) Upper-bound checking

If $P_y \leq P_{max}$, save $\{P_1, P_2, ..., P_N\}$ back to the ordered set $\{p_n | \forall n \in \boldsymbol{N}\}$ and go to Step3; otherwise continue.

2.3) Cross-P_{max} adjustment

```
Set 0→S.
for (n=y; n≤N, P_n ≥ P_max; n++)
{
    𝒫_n − P_max + S→S;
    P_max → P_n.
}
n → y;
S/(N − y + 1) → Δ;
for (n=y; n≤N; n++)
{
    𝒫_n + Δ→𝒫_n,
    𝒫_n → P_n.
}
Set N → x and go to Step 2.1.
```

3. Finish:

$\boldsymbol{p} = \{p_n | \forall n \in \boldsymbol{N}\}$ is the optimal solution.

Fig. 3. Fast water-filling with sub-channel power upper-bound.

Figure 3

4.Numerical Results

Simulations are run in MATLAB 7.11 on a 2.66 GHzcomputer with 4 GB memory. γnsare i.i.d chi-square randomvariables with a degree of freedom of one. Definespectrum efficiency as the capacity in unit bandwidth. Fig.4 shows that the WF in Fig. 2 achieves the same spectrumefficiency as those in [4-6]. For the WF∗in Fig. 3, similarresults can be obtained.Figs. 5-6 compare the running time of our water-fillingalgorithms with others, where PT is set to 2N for each N.Fig. 5 shows that the WF in Fig. 2 runs 4-5 times faster than[4], 3-22 times faster than [5] and 5-7 times faster than WFwith binary μ searching [6]. Since the upper-bound considered in [4-5], in Fig. 6 our proposed WF∗in Fig. 3 isonly compared with the binary μ searching based algorithm,where it runs about 6 times faster.considered in [4-5], in Fig. 6 our proposed WF∗in Fig. 3 is only compared with the binary μ searching based algorithm, where it runs about 6 times faster.

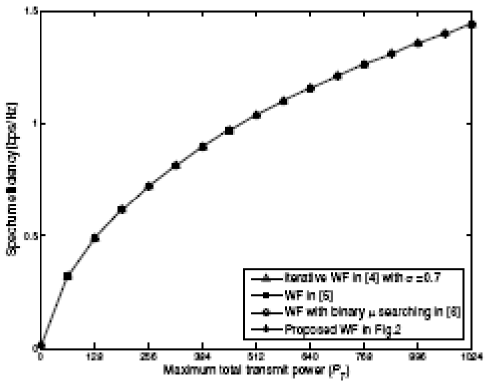


Fig. 4. Spectrum efficiency of the WF in Fig.2 (N=512).

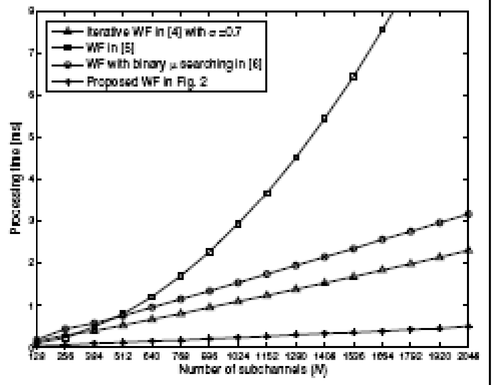
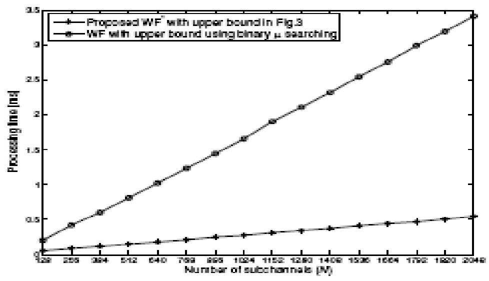


Fig. 5. Running time comparison for the WF in Fig.2.

Figure 4 & Figure 5

5.Conclusion

We proposed simple and fast Water-Filling algorithms tofind the optimal power allocation for capacity maximization inmulti-channel wireless communications. Both scenarios, withand without an upper-bound on the allocated power in eachsub-channel, are considered. Our algorithms remove the needof Lagrange multiplier or water-level searching. They can runmultiple times faster than the existing ones and converge tothe optimal solutions in a few linear calculations.



ig. 6. Running time comparison for the WF* in Fig.3.

Figure 6

6.References

1. M.-O. Pun, M. Morelli, and C.-C. J. Kuo, Multi-Carrier Techniques forBroadband Wireless Communications: A Signal Processing Perspective.Imperial College Press, 2007.
2. Q. H. Spencer, A. L. Swindlehurst, and M. Haardt, “Zero-forcing methodsfor downlink spatial multiplexing in multiuser MIMO channels,” IEEETrans. Signal Process., vol. 52, no. 2, pp. 461–471, 2004.
3. W. Yu, W. Rhee, S. Boyd, and J. M. Cioffi, “Iterative water-filling forGaussian vector multiple-access channels,” IEEE Trans. Inf. Theory, vol.50, no. 1, pp. 145–152, 2004.

4. J. Jang, K. B. Lee, and Y.-H.Lee, “Transmit power and bit allocationsfor OFDM systems in a fading channel,” in Proc. 2003 IEEE GlobeCom,vol. 2, pp. 858–862.
5. D. P. Palomar and J. R. Fonollosa, “Practical algorithms for a family ofwaterfilling solutions,” IEEE Trans. Signal Process., vol. 53, no. 2, pp.686–695, 2005.
6. K. Son, B. C. Jung, S. Chong, and D. K. Sung, “Power allocation forOFDM-based cognitive radio systems under outage constraints,” in Proc.2010 IEEE ICC.