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Review: Assessment Of Working Of Spatial Domain Filters Versus Frequencydomain Filters

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Abstract:
Digital image processing involves the manipulation and interpretation of digital images with the aid of a computer [1]. Digital image processing is an extremely broad subject, and it often involves procedures that can be mathematically complex. Different digital filters have been developed in image processing for better interpretation and to improve the visual interpretability of an image by increasing the apparent distinction between the features in the scene. Filters are broadly classified into spatial domain filtering and frequency domain filtering .This paper reviews comparison between working of spatial domain filter and frequency domain digital filters in image processing.

Key words: Convolution, Digital Filters, Fast Fourier Transform, Frequency Domain Filters & Spatial Domain Filtering

1.Introduction

A digital filter in digital image processing has found wide applications in processing of digital images corrupted by noise. Filtering is one of the most important tasks in image processing. A variety of techniques has been developed to improve the visual interpretability of an image [1]. Although of great variety in the existing image noise filtering techniques, nearly all of them are based on spatial-domain processing of the distorted image. They generally process image data in the spatial domain to diminish the noise while preserve important image details such as edges and lines. Image noise filtering has been widely perceived as the spatial-domain estimation and processing.

On the other hand Fourier transform of an image is a breakdown of the image into its frequency or scale components. Such filters operate on the amplitude spectrum of an image and remove, attenuate, or amplify the amplitudes in specified wavebands. Any function that periodically repeats itself can be expressed as a sum of sines and cosines of different frequencies each multiplied by a different coefficient is called a Fourier series.

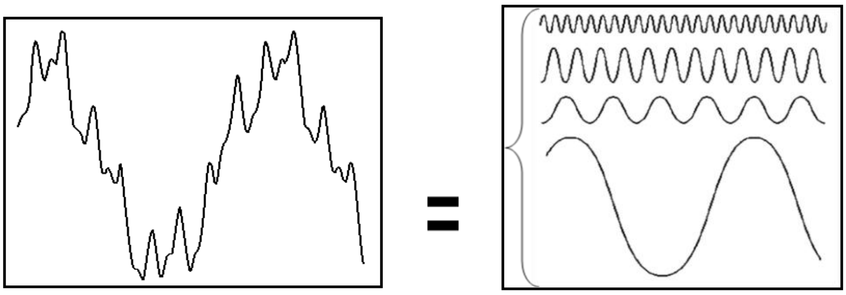

Figure 1: Original Signal Expressed As A Sum Of Sines And Cosines Of Different Frequencies

A simple filter might set the amplitudes of all frequencies less than or more than a selected threshold to zero. If the amplitude spectrum information is converted back to the spatial domain by an inverse Fourier transform, the result is new improved transformed image. Any wavelength or waveband can be operated upon in the frequency domain. The three general categories of filter are low-pass, high-pass and band-pass filters. The slowly varying background pattern in the image can be envisaged as a two-dimensional waveform with a long wavelength or low frequency; hence a filter that separates this slowly varying component from the remainder of the information present in the image is called a low-pass filter. A filter that separate out the more rapidly varying detail like a two-dimensional waveform with a short wavelength or high frequency component is called a high-pass

filter[2]. A band-pass filter removes both the high and low frequency components, but allows an intermediate range of frequencies to pass through the filter. Directional filters can also be developed, because the amplitude spectrum of an image contains information about the frequencies and orientations as well as the amplitudes of the scale components that are present in an image.
In this paper, a brief comparison between working of spatial domain filter and frequency domain digital filters in image processing on image noise filtering has been presented.

2.Principle Of Operation Of Spatial Domain Filter And Frequency Domain Filter

2.1.Spatial Domain Filtering

A spatial filter is an image operation where each pixel value is changed by a function of the intensities of pixels in a neighborhood. The process involved in spatial filter is convolution.
Convolution is the most important filtering technique used in image processing. Convolution of an image involves moving of a window (e.g. .size of 3*3,5*5,7*7 etc) ,that contains an array of coefficients or weighting factors referred as kernels , throughout the original image and output image is obtained [2]. To begin with, the window is placed in the top left corner of the image to be filtered .The digital number at the center of output window is obtained by multiplying each coefficient in the kernel by the corresponding digital number in the original image and adding all the resulting products.
Once the output value from the filter has been calculated, the window is moved one column (pixel) to the right and the operation is repeated. The window is moved rightwards and successive output values are computed until the right-hand edge of the filter window hits the right margin of the image. At this point, the filter window is moved down one row and back to the left-hand margin of the image. This procedure is repeated until the filter window reaches the bottom right-hand corner of the input image as shown in FIGURE 2.

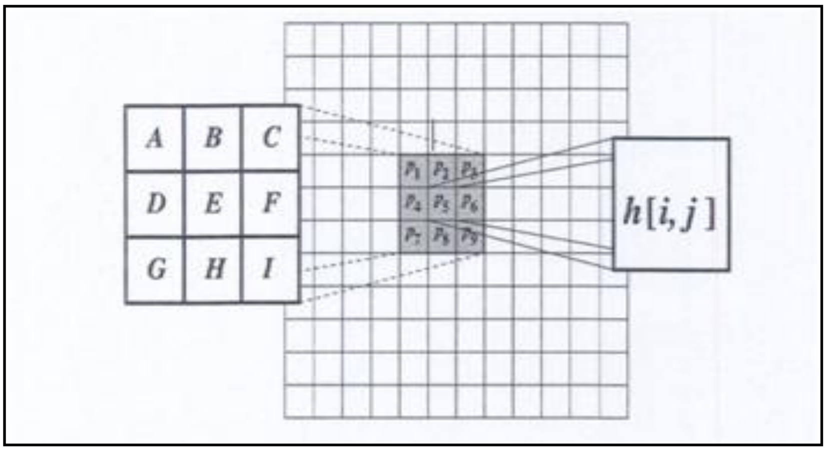

Figure 2: Shows Convolution Process

where,
$h[i,j]=Ap_1+B\ p_2+C\ p_3+D\ p_4+E\ p_5+F\ p_6+G\ p_7+H\ p_8+I\ p_9$
Moving Average Filter, Median Filter, Adaptive filters, Sigma Filter, Nagao-Matsuyama Filter, Derivative filters- Prewitt operator filter, Sobel Operator filter, Robert cross filter, Canny operator filter are spatial domain filters.

2.2.Frequency Domain Filtering (Fourier Transform

Another technique of filtering is in frequency–domain. There is generally a one-to one correspondence between spatial and frequency domain filters. Spatial domain filters are generally classed as both high- pass (sharpening) filters and filters in the frequency domain can be designed to suppress, attenuate, amplify or pass any group of spatial frequency.
The spatial features manipulations are implemented in the spatial domain the (x, y) co-ordinate space of image .An alternative co-ordinate space that can be used for image analysis is the frequency domain. In this approach, an image is separated into its various spatial frequency components through application of a mathematical operation known as the Fourier transform. A Fourier transform results from the calculation of the amplitude and phase for each possible spatial frequency in an image.
After an image is separated into its component spatial frequencies. When it is displayed these values in a two-dimensional scatter plot known as a Fourier spectrum. The lower frequency in the scene are plotted at the center of the spectrum and progressively higher frequency are plotted outward. If the Fourier spectrum of an image is known, it is possible to regenerate the original image through the application of an inverse Fourier transform.
Features trending horizontally in the original image result in vertical components in the Fourier spectrum; feature aligned vertically in the original image result in horizontal components in the Fourier spectrum.
Two-dimensional Fourier transforms are used for enhancement, compression, texture classification, quality assessment, cross-correlation etc.

Sampling theorem:
The sampling theorem called "Shannons Sampling Theorem" states that a continuous signal must be discretely sampled at least twice the frequency of the highest frequency in the signal [3]. More precisely, a continuous function f(t) is completely defined by samples every 1/fs (fs is the sample frequency) if the frequency spectrum F(f) is zero for f > fs/2. fs/2 is called the Nyquist frequency and places the limit on the minimum sampling frequency when digitizing a continuous signal.

If x(k) are the samples of f(t) every 1/fs then f(t) can be exactly reconstructed from these samples, if the sampling theorem has been satisfied, by

$$f(t) = \sum_{k=-\infty}^{k=\infty} x(k)\ \mathrm{sinc}(t\ fs - k)$$

Where ,

$$\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

Normally the signal to be digitized would be appropriately filtered before sampling to remove higher frequency components. If the sampling frequency is not high enough the high frequency components will wrap around and appear in other locations in the discrete spectrum, thus corrupting it. The key features and consequences of sampling a continuous signal can be shown graphically as follows. Consider a continuous signal in the time and frequency domain.

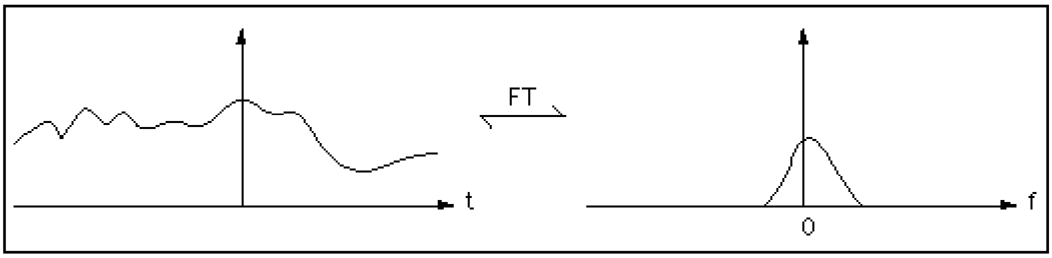

Figure : 3

Sample this signal with a sampling frequency fs, time between samples is 1/fs. This is equivalent to convolving in the frequency domain by delta function train with a spacing of fs.

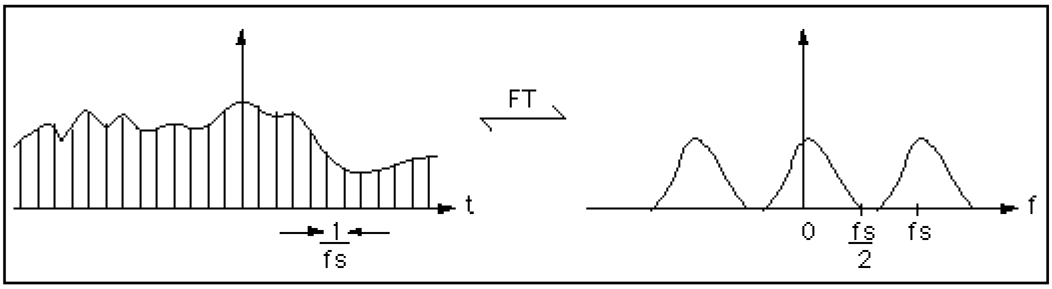

Figure : 4

If the sampling frequency is too low the frequency spectrum overlaps, and become corrupted.

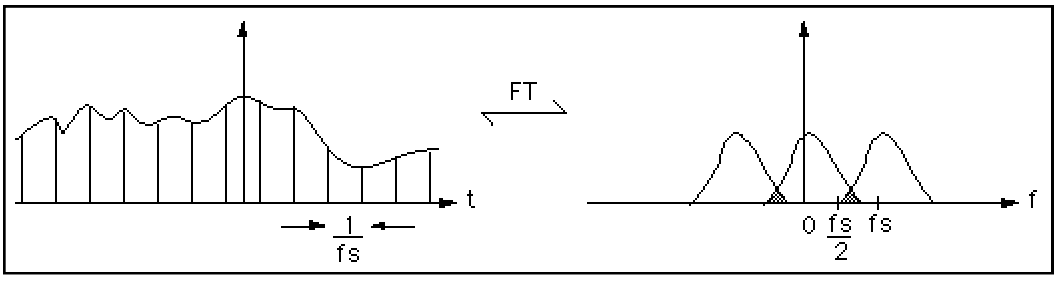

Figure: 5

Another way to look at this is to consider a sine function sampled twice per period (Nyquist rate). There are other sinusoid functions of higher frequencies that would give exactly the same samples and thus can't be distinguished from the frequency of the original sinusoid[3] .

2.2.1.<u>The Discrete Fourier Transform</u>
Discrete
Consider a complex series f(x, y) with N samples
Where f(x ,y) is a complex number
f(x, y) =f(x, y) real + j f(x , y) imag
Further, assume that that the series outside the range 0, N-1 is extended N-periodic
The FT of this series will be denoted F(u ,v) it will also have N samples. The forward transform will be defined as

$$F(u,v)=\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi(ux/M+vy/N)}$$

For u = 0, 1, 2…M-1 and v = 0, 1, 2…N-1.
The inverse DFT is given by

$$f(x,y)=\frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)e^{j2\pi(ux/M+vy/N)}$$

For x = 0, 1, 2…M-1 and y = 0, 1, 2…N-1
The functions are described as complex series, real valued series can be represented by setting the imaginary part to 0. In general, the transform into the frequency domain will be a complex valued function, that is, with magnitude and phase.
Magnitude= Square Root (f(x, y) real* f(x, y) real+ f(x , y) imag* f(x , y) imag)
Phase= tan-1(f(x , y) imag / f(x, y) real)
The DFT of a real series, i.e. imaginary part = 0, results in a symmetric series about the Nyquist frequency. The negative frequency samples are also the inverse of the positive frequency samples. The highest positive (or negative) frequency sample is called the Nyquist frequency. This is the highest frequency component that should exist in the input series for the DFT to yield "uncorrupted" results. More specifically if there are no frequencies above Nyquist the original signal can be exactly reconstructed from the samples[3].

2.2.2.<u>The Fast Fourier Transform</u>
Continuous
For a continuous function of one variable f(t), the Fourier Transform F(f) will be defined as[3]:

$$F(f)=\int_{-\infty}^{\infty} f(t)\, e^{-j2\pi ft}\, dt$$

and the inverse transform as

$$f(t)=\int_{-\infty}^{\infty} F(f)\, e^{j2\pi ft}\, df$$

Where j is the square root of -1 and e denotes the natural exponent

$$e^{j\phi}=\cos(\phi)+j\sin(\phi).$$

In place of DFT, an algorithm called the Fast Fourier Transform (FFT) is used of the FFT over the older method can be summarized by the fact that the number of operations required to evaluate the coefficients of the Fourier series using the older method is proportional to N2 where N is the number of sample points (length of the series) whereas the number of operations involved in the FFT is proportional to N log2N. Only the amplitude information is used[3].


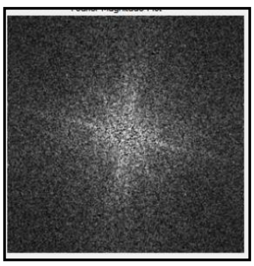


Figure 6: Original Image & Figure 7: Fourier Magnitude &
Figure 8: New Transformed Image After Applying Inverse Fourier Transform

Steps involved to filter an image in the frequency domain are as follows:
Firstly Compute F (u,v) the Fourier Transform of the image. Then to multiply F (u,v) by a filter function H (u,v).And finally compute its inverse Fourier transform of the result .

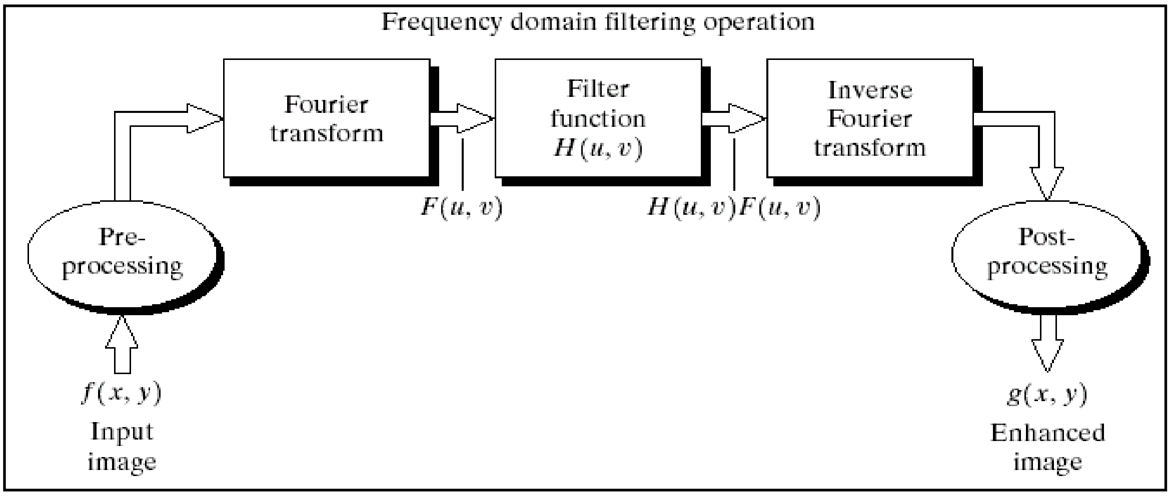

Figure : 9

The Fourier transform operates on a grey scale image, not on a multispectral data set. Its purpose is to break down the image into its spatial scale components, which are defined to be sinusoidal waves with varying amplitudes, frequencies and directions. The idea underlying the Fourier transform is that the grey scale values forming a single-band image can be viewed as a three-dimensional intensity surface, with the rows and columns defining two axes and the grey level intensity value at each pixel giving the third (z) dimension. The Fourier transform provides details of the frequency of each of the scale components (waveforms) fitted to the image and the proportion of information associated with each frequency component. It is really important to note that the Fourier transform is completely reversible. Butterworth filter and Gaussian filter are frequency domain filter

3.Conclusion

This paper serves as comparison of working of the spatial domain digital filters versus frequency domain digital filter. Filtering in the frequency domain is much faster especially for large images. The reason that Fourier based techniques have become so popular is the development of the Fast Fourier Transform (FFT). The field of image processing are constantly evolving in the last decade, but still improvements in filtering methods are needed This paper serves as a review of comparison of working of both spatial domain filter and frequency domain filter and also highlights the working of digital filters in image processing. Filtering in the spatial domain can be easier to understand but is time consuming process. Filtering in the frequency domain can be much faster –especially for large images Whilst the FFT technique is valuable technique for determining the harmonic content of waveforms, it's value lies where attempting to provide a spectrum analysis of waveforms.

4.References

1. T. M. Lillesand , R.W. Kiefer, and J. Chipman, Rem. Sens. andImage Interpretation, J. Wiley & Sons, NY, 2008.
2. Paul M. Mather ,John Wiley & Sons 2004 Computer Processing of Remotely-Sensed Images: An Introduction, Third Edition. ISBNs: 0-470-84918-5 (HB); 0-470-84919-3 (PB).
3. http://paulbourke.net/miscellaneous/dft.