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An Optimisation-Based Approach To The Disbursing Of Loan Portfolio

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Abstract:
The paper aim at contributing to the body of knowledge that exists in the area of optimal loan portfolio by developing a model to allocate funds to prospective loan seekers in order to maximize profits using secondary data from Atweaban Rural Bank at Duayaw Nkwanta in the Tano North District of the Brong Ahafo Region, Ghana. The Simplex method was used for the algorithm and software called management scientist was also for the analysis. The results indicated that the funds allocated to the type of loans should be in increasing order of farm loan, suso loan, commercial loan, salary loan and funeral loan in order for the bank to maximize profit. Future research should increase the number of constraints and also vary the coefficients of the constraints to observe relationship between them. The study should be replicated in order banks to determine if the findings will be collaborated.

Key words: *Model; Optimal; Loan; Portfolio; Simplex method; Algorithm*

1.Introduction

Many Ghanaians either in the formal sector or the informal sector go for loans for various reasons some being investment in businesses or their wards education. Some also take loans to purchase individual properties such as cars and houses. Most people rely on Banks for Loans. Due to pitiable distribution of funds by the majority of banks to prospective loan seekers the banks are not able to exploit their profits. Because of this, monies that can be used for social services in the community in which they operate go into bad debt.

The idea of rural banking system was introduced into the country by the Central Bank of Ghana in 1976. From the Association of Rural Banks (1992), these are some the aims of Rural Banks are:

- to stimulate banking habits among rural dwellers;
- to mobilize resources locked up in the rural areas into the banking systems to facilitate development; and
- to identify feasible industry in their respective areas for venture and progress."

Due to these liberal policies of the Bank of Ghana many banks across Africa are opening branches in Ghana, this has also facilitated the opening of a lot of Rural Banks across the country. Currently there are over One Hundred and Thirty (130) Rural Banks in the country.

Rural Banks are banks recognized to offer services for the rural communities in which they are situated. They are owned, managed and most at times patronized by the local people. A number of these banks also run agencies to provide banking services for communities that are positioned far from the bank's facilities.

Savings mobilized through rural banks are invested in small-scale agricultural activities, cottage industries, transportation and trading. Rural banks also provide commercial banking services such as giving loans to people within the community in which they operate.

A loan is a kind of debt similar to all debt instruments. It entails the redeployment of monetary assets over time with agreement between the lender and the borrower. In a loan, the borrower initially receives an amount of money, called the principal, from the lender, and is required to pay back an equal amount of money to the lender at afterward.

Usually, the money is paid back in regular and equal installments. The loan is generally provided at a cost, kwon as interest on the debt, which provides a motivation for the lender to engage in the loan. In a authorized loan, each of these obligations and limitations is enforced by contract, which can also place the borrower under additional restrictions known as loan covenants. Performing as a provider of loans is one of the key responsibilities for financial institutions.

1.1.Problem Statement

The institution of Banks is one of the fastest growing institutions in the Ghana which has a tremendous impact on the economy and the society. Among other things banks also give loans prospective loan seekers. Outdated and ineffective loan policies can

contribute to a series of troubles. Introduction of a loan that is not adequately addressed in the written loan policy can create a diversity of challenges for the lending staff and involve risks that management did not foresee.
If lending authorities loan limitation are not revised when circumstances change, a Bank could be operating within guidelines that are too restrictive or too lenient. If guidelines do not comply with current laws and rules, lending decisions may not reflect best fifteen (15) practices or regulatory requirements. A loan policy that does not anticipate risks can lead to asset quality problems and poor earnings.
The bank might run at a lost or even collapse if they are not able to retrieve all the loans they give out. Due to this, a more scientific approach must be employed by banks to ensure adequate, effective and efficient distribution of funds they have available for loans to ensure constant growth of these banks. When banks run efficiently they are able to allocate a larger amount of its funds for social services in the community in which they operate.
Due to poor allocation of funds some rural banks record marginal profits with some running at a lost. The problem of the paper is to develop a linear model subject to some constraints for a newly established rural bank at Duayaw - Nkwanta named Atweaban Rural Bank to enable them disburse their funds allocated for loans optimally leading to maximization of profits.
The proposed model is going to help banks to efficiently distribute the funds they have available for loan in order to maximize their profit. The proposed model will also help decision makers at the Bank to formulate prudent and effective loan policies.

1.2.Objective
The main objective of this paper is to contribute to the body of knowledge in the area of optimal loan allocation using linear programming by exploring ways of disbursing funds allocated for loans effectively and efficiently in order to obtain the expected profit.

1.3.Justification
Very few empirical works exist in literature. Also in the knowledge of the authors no such empirical work has been done in the study area and as such the paper fills in the gap.

1.4..Limitations And Scope

The findings are limited by the use of secondary data and the errors in the use of secondary data, such as error in variables and omission variables. The development of the model is based on Linear programming method alone. The findings may lack external validity since data from only one bank is used. Other variables that affect the cost of loans such as exchange rate are not included in the model. Other lending activities of the banks such as lease were also not considered in the model.

2.Literature Review
In this section of the work, other people's works of various fields of research using linear programming programs are considered.
Stewart et al., (2008) examined the numerical implementation of a linear programming (LP) formulation of stochastic control problems involving singular stochastic processes. The decision maker has the ability to influence a diffusion process through the selection of its drift rate (a control that acts absolutely continuously in time) and may also decide to instantaneously move the process to some other level (a singular control). The first goal of the paper is to show that linear programming provides a viable approach to solving singular control problems. A second goal is the determination of the absolutely continuous control from the LP results and is intimately tied to the particular numerical implementation.
Consider a linear-programming problem in which the "right-hand side" is a random vector whose expected value is given and where the anticipated objective function value is to be minimized. In order to find an approximate solution, the right – hand side has to be replaced by the anticipated value and solving the resulting linear programming problem. Mandansky (1960) gave conditions in order to equate the anticipated value of the objective function for the optimal solution and the value of the objective function for the approximate solution; bounds on these values were also given. In addition, the relation between this problem and any other associated problem, wherever identification is made to the "right-hand side" and solves the (nonstochastic) linear programming problem based on this observation, was discussed.
Greenberg et al., (1986) developed a framework for model formulation and analysis to support operations and management of large-scale linear programs from the combined capabilities of camps and analyze. Both the systems were reviewed briefly and the interface which integrates the two systems was then described. The model formulation, matrix generation, and model management capability of camps and the complementary model and solution analysis capability of analyze were presented within a unified framework. Relevant generic functions were highlighted, and an example was presented in detail to illustrate the level of integration achieved in the current prototype system.
Church et al., (1963) used linear programming procedures with the aid of an electronic computer to formulate fattening rations for weaned calves. Rations were formulated using digestible energy or estimated net energy, crude protein, crude fiber, calcium and other elements. The formulation of rations on digestible energy bases was specified as 1.24, 1.36 or 1.48 megcal. Per lb. of feed, and 0.581, 0.638 or 0.694 megcal. Per lb were estimated net ernergy. Specifications for crude protein (11.5%), calcium (0.75%), phosphorus (0.50%) and salt (0.50%) were the same for each ration. Crude fiber was restricted to a maximum of 15% and a minimum of 8%. Minimum and/or maximum specifications were used for several feedstuffs; alfalfa meal (5 and 15%), beet pulp (min. 10%) and molasses (5 and 10%).
Sinha et al., (2003) proposed a modified fuzzy programming method to handle higher level multi-level decentralized programming problems (ML (D) PPs). They presented a simple and practical method to solve the same. Their technique overcomes the prejudice inbuilt in selecting the acceptance values and the link functions. They considered a linear ML (D) PP and applied linear programming (LP) for the optimization of the system in a supervised search procedure, supervised by the advanced solver. The advanced solver provides the preferred values of the decision variables under his control to enable the lesser solver to

explore for his optimum in a narrower feasible space. The basic idea is to lessen the sufficient space of a decision variable at each level until a satisfactory point is sought at the last level.

Wu et al., (2000) proposed a neural network model for linear programming that is designed to optimize radiotherapy treatment planning (RTP). This kind of neural network can be easily implemented by using a kind of `neural' electronic system in order to obtain an optimization solution in real time.

Jianq et al., (2004) proposed a novel linear programming based technique to guess subjective movement from two images. The projected technique always finds the global optimal solution of the linearized movement estimation energy function and thus is much extra vigorous than traditional movement estimation schemes. To further reduce the complexity of even a complexity-reduced pure linear programming method they presented a two-phase scheme for estimating the dense motion field.

Vimonsatit et al., (2003) proposed a linear programming (LP) formulation for the evaluation of the plastic limit temperature of flexibly connected steel frames exposed to fire. From the models framework with piecewise linearized yield surfaces, the formulation was derived based on the lower-bound theorem in plastic theory, which lead to a compact matrix form of an LP problem. The plastic limit temperature was determined when the equilibrium and yield conditions were satisfied. The plastic mechanism can be checked from the dual solutions in the final simplex tableau of the primal LP solutions.

Kas et al., (1996) studied linear inverse problems where a vector with positive components was chosen from a feasible set defined by linear constraints. The problem requires the minimization of a certain function which is a determiner of distance from a known estimate. An explicit and perfect dual of the resulting programming problem was shown, the corresponding duality theorem and optimality criteria were proven, and an algorithm solution was proposed.

Nace et al., (2006) introduced the lexicographically minimum load linear programming problem, and they provided a polynomial approach followed by the proof of precision. The problem has applications in several areas wherever it is desirable to achieve an equitable distribution or sharing of resources.

Konickova (2006) said a linear programming problem whose coefficients are prescribed by intervals is called strongly unbounded if every linear programming problem attained by putting in the coefficients in these intervals is unbounded. In the main result of the paper a necessary and sufficient condition for strong unboundedness of an interval linear programming problem was described.

A linear programming problem in an inequality form having a bounded solution is solved error-free using an algorithm that sorts the inequalities, removes the redundant ones, and uses the p-adic arithmetic. Lakshmikantham et al., (1997).

Biswal et al., (1998) developed a technique to solve probability problems which involve linear programming. This is done first by obtaining the probability density function (p.d.f.) of the linear combination of number of independent exponential random variables. The resultant constraints are then changed to the deterministic constraints using the p.d.f. The model is then solved using any technique which is not applicable to linear programming.

Frangioni et al., (2009) discussed a general framework for algorithms which can rely on a polar reformulation of the problem. Moreover, exploit an approximated vision in order to verify overall optimality. Consequently, approximate optimality conditions were introduced and bounds on the quality of the approximate global optimal solution were obtained. A thorough analysis of properties which guarantee convergence was carried out; two families of conditions were introduced which lead to design six implementable algorithms, whose convergence can be proved within a unified framework.

Cherubini et al., (2009) described an optimization model which targets at minimizing the maximum link utilization of IP networks system with the use of the conventional IGP protocols and the more sophisticated MPLS-TE technology. The survivability of the network was taken into account in the optimization process implementing the path restoration scheme.

They applied this approach to a network design problem, comparing the recently developed algorithm with those based on both the standard continuous relaxation and the two usual variants of PR relaxation.

Harlan et al., (1983) reported on the solution to optimality of ten large-scale zero-one linear programming problems. All problem data come from real-world industrial applications and are characterized by sparse constraint matrices with rational data. Most of the related problems have no apparent special structure; the remainder show structural characteristics that their computational procedures do not exploit directly.

This popular technique has almost attained an industry standard level and usually helps to set up proper models promptly. Nevertheless, there is a challenge which hides in the actual implementation which still needs tailoring to the particular application. Their work is based on practical data from a German in-plant railroad.

3.Research Methodology

The paper is based on explanatory quantitative research design using secondary data from Atweaban Rural Bank at Duayaw Nkwanta in the Tano North District of the Brong Ahafo Region, Ghana. The bank was selected using convenience sample method. Secondary data collected were type of loans with their interest rates and the probability of bad debt as estimated from past years. Types of secondary data obtained are shown in Table 1.

Type of Loan	Interest rate	Probability of bad debt
Salary	0.28	0.02
Commercial	0.3	0.12
Farm	0.3	0.2
Funeral	0.4	0.01
Susu	0.3	0.03

Table 1: Loans Available To The Atweaban Rural Bank
Source: Research's Fieldwork (2012).

Bad debts are assumed unrecoverable and hence produce no interest revenue. For policy reasons, there are limits on how the bank allocates the funds. The competition with other banking institutions in the area requires that the bank:

- Allocate at least 50% of the total funds to salary loan and commercial loan.
- To optimize profit salary loan must be at least greater than 50% of the farm loan, funeral loan and susu loan.
- The sum of Salary loan and susu loan must be at least greater than 50% of commercial loan, farm loan and funeral loan.
- The sum of farm loan and funeral loan must be at least 25% of the total funds.
- The sum of commercial and farm loans must be at least 29% of the total funds.
- Allocate at least 5% of the total funds to farm loan.
- The bank also stated that the total ratio for the bad debt on all loans may not exceed 0.05.

Data obtained were analysed using Simplex Algorithm. In order for the bank to maximize their profit, the proposed model will be based strictly on the Bank's Loan Policy and its previous history on loan disbursement. The model will be solved using the Simplex Algorithm.

The Linear Programming model has three basic components, that is the objective function which is to be optimized (Maximized or minimized), the constraints or limitation and the non negativity constraint.

In general the Linear Programming model can be formulated as follows

Let $x_1, x_2, ..., x_n$ be n decision variables with m constraints, then

The objective function:

Maximize or Minimized

$$Z = \sum_{j=1}^{n} c_j x_j \qquad (1)$$

Subject to the m constraints

$$\sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} x_j \ (\leq = \geq) \ b_i \qquad (2)$$

The Non negativity constraints

$$x_j \geq 0$$

$$i = 1, 2, 3...., \text{m and j} = 1, 2, 3, ..., \text{n}$$

The Simplex method is an iterative procedure for solving Linear Programming Problems in a finite number of steps. This method provides an algorithm which consists of moving from one vertex of the region of feasible solution to another in such a manner that the value of the objective function at the succeeding vertex is less or more than the previous vertex. This procedure is repeated and since the number of vertices is finite, the method leads to an optimal vertex in a finite number of steps or it may indicate the existences of unbounded solution.

3.1.Model

A banking institution, Atweaban Rural Bank, is in the process of formulating a loan policy involving a total of GH¢120,000. Being a full-service facility, the bank is obligated to grant loans to different clientele

The variables of the model can be defined as follows:

$x_1 =$ Salary loan (in thousands of cedis)

$x_2 =$ Commercial loan

$x_3 =$ Farm loan

$x_4 =$ Funeral loan

$x_5 =$ Susu loan

The objective of the Atweaban Rural Bank is to maximize its net returns which comprised of the difference between the income from interest and lost funds due to dad debts.

Objective function:

$$\begin{aligned} Maximize\ Z = &0.28(0.98x_1) + 0.3(0.88x_2) + 0.3(0.8x_3) + 0.4(0.99x_4) + 0.3(0.97x_5) \\ &- 0.02x_1 - 0.12x_2 - 0.2x_3 - 0.01x_4 - 0.03x_5 \end{aligned} \qquad (3)$$

$$\boxed{Max.\ Z = 0.2544x_1 + 0.144x_2 + 0.04x_3 + 0.386x_4 + 0.261x_5} \quad (4)$$

The problem has nine constrains:

1. Limit on total funds available
   $x_1 + x_2 + x_3 + x_4 + x_5 \le 120000$
2. Limit on Salary and Commercial loans
   $x_1 + x_2 \ge 0.5 \times 120000$

   $x_1 + x_2 \ge 60000$
3. Limit on salary loan compare to farm, funeral and susu loans
   $x_1 \ge 0.5(x_3 + x_4 + x_5)$

   $x_1 - 0.5x_3 - 0.5x_4 - 0.5x_5 \ge 0$
4. Limit on Salary and Susu loans compare to commercial, farm and funeral loans
   $x_1 + x_5 \ge 0.5(x_2 + x_3 + x_4)$

   $x_1 - 0.5x_2 - 0.5x_3 - 0.5x_4 + x_5 \ge 0$
5. Limit on farm and funeral loans
   $x_3 + x_4 \ge 0.25 \times 120000$

   $x_3 + x_4 \ge 30000$

6. Limit on commercial and farm loans

$x_2 + x_3 \ge 0.29 \times 120000$

$x_2 + x_3 \ge 34800$

7. Limit on farm loan

$x_3 \ge 0.05 \times 120000$

$x_3 \ge 6000$

8. Limit on bad debts

$$\frac{0.02x_1 + 0.12x_2 + 0.2x_3 + 0.01x_4 + 0.03x_5}{x_1 + x_2 + x_3 + x_4 + x_5} \le 0.05$$

$$-0.03x_1 + 0.07x_2 + 0.15x_3 - 0.04x_4 - 0.02x_5 \le 0$$

9. Non-negativity

$x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0,\ x_4 \ge 0,\ x_5 \ge 0.$

The following is the output returned by the management Scientist solver for the above model.

Maximize $Z = 0.2544x_1 + 0.144x_2 + 0.04x_3 + 0.386x_4 + 0.261x_5$

Subject to:

1) $x_1 + x_2 + x_3 + x_4 + x_5 \le 120000$
2) $x_1 + x_2 \ge 60000$

3) $x_1 - 0.5x_3 - 0.5x_4 - 0.5x_5 \geq 0$

4) $x_1 - 0.5x_2 - 0.5x_3 - 0.5x_4 + x_5 \geq 0$

5) $x_3 + x_4 \geq 30000$

6) $x_2 + x_3 \geq 34800$

7) $x_3 \geq 6000$

8) $-0.03x_1 + 0.07x_2 + 0.15x_3 - 0.04x_4 - 0.02x_5 \leq 0$

4.Empirical Results And Discussions

The results based on the estimated model are provided in this section of the paper. The Optimal Solutions, Objective Coefficient Ranges and Right Hand Side Ranges are provided.

4.1.Optimal Solution

Objective Function = 32068.4800

Variable	Value	Reduced costs
x_1	31200.0000	0.0000
x_2	28800.0000	0.0000
x_3	6000.0000	0.0000
x_4	45200.0000	0.0000
x_5	8800.0000	0.0000

Constraint	Slack/Surplus	Dual prices
1	0.0000	0.3443
2	1200.0000	0.0000
3	0.0000	-0.0833
4	21200.0000	0.0000
5	0.0000	-0.2354
6	0.0000	-0.1106
7	0.0000	-0.0066
8	4.0000	0.0000

Objective coefficient ranges

Variable	Lower limit	Current value	Upper limit
x_1	0.0190	0.2544	0.2610
x_2	0.0334	0.1440	0.3794
x_3	No lower limit	0.0400	0.1506
x_4	0.2610	0.3860	No upper limit
x_5	0.2544	0.2610	0.3860

Right hand side ranges

Constraint	Lower limit	Current value	Upper limit
1	119880.0000	120000.0000	122400.0000
2	No lower limit	0.0000	1200.0000
3	-13200.0000	0.0000	300.0000
4	No lower limit	30000.0000	51200.0000
5	26000.0000	34800.0000	34833.3333
6	4800.0000	6000.0000	6057.1429
7	59600.0000	60000.0000	68800.0000
8	-4.0000	0.0000	No upper limit

4.2.Discussion Of Results

There are several things to observe about this output data. The reduced costs for x_1, x_2, x_3, x_4 *and* x_5 are zero. This is because the reduced costs are the objective function coefficients of the original variables, and since x_1, x_2, x_3, x_4 *and* x_5 are basic at the optimum, their objective function coefficients must be zero when the tableau is put into proper form.

This is always true, either the variable is zero (non-basic), or the reduced cost or dual price is zero. It is also seen that the pattern holds for the slack and surplus variables too. The dual prices for constraints (1), (3), (5), (6) and (7) are nonzero at the optimum because they correspond to the five active constraints at the optimum, hence their slack variables are non-basic (value is zero), so the dual prices can be positive value.

Whenever the variable and the related reduced cost or dual prices are zero, then we have either degeneracy if the variable is basic or multiple optima if the variable is non-basic. It must be noted that the optimal solution with $x_1 =$ GH¢31200, $x_2 =$ GH¢28800, $x_3 =$GH¢6000, $x_4 =$GH¢45200 and $x_5 =$ GH¢8800 show that the bank should allocate funds to all the types of loans since none of them has a value of zero.

4.3.Conclusion And Policy Implications

We conclude that we have completed our main objectives as stated in Section 1.3. We have successfully implemented linear programming models. Under specific model constraints, the model is able to optimally disburse loans for the Bank's Loan Portfolio.

If the bank adapts the model they can make an annual profit of GH¢32068.4800 on loans alone. Hence we recommend that our model be used in disbursing loans in other to achieve dramatic increase in the profit margin of the Bank. Future studies should include more Banks to assess if these findings will be supported. Future models should also include exchange rate and lease which have not been considered in our model.

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