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# Hierarchical Transformation-Based Image Coding Using Wavelet Shrinkage

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***Abstract:***
*Images acquired through modern sensors may be contaminated by a variety of noise sources. Noise refers to stochastic variations as opposed to deterministic distortions such as shading or lack of focus. It was assumed for this section that we are dealing with images formed from light using modern electro optics. In this paper, a wavelet-based multi-scale peak signal to noise ratio (PSNR) scheme for image de-noising is proposed, and the determination of the wavelet basis with respect to the proposed scheme is also discussed. Compared to other wavelet-based representation the proposed Moving window based wavelet shrinkage based method is more efficient for a wide range of contour features such as junctions, corners and ridges, especially at low bit rates.*

***Key words:*** *Directional filtering, Haar wavelet transform, PSNR, Soft thresholding, Wavelets*

## 1.Introduction

The discovery of orientation selective neural cells [1]–[3] in the mammal visual system have, undoubtedly, substantiated the perceptual importance of edges or contour information [4], [5] in the human vision. It is well known that image compression techniques based on cosine [6] and wavelet [7] transform coding produce visual artifacts [8], [9] at very lowbit rates. These artifacts are generally observed along high contrast regions such as object edges, which could severely distort structural information and, thus, impairing visual cognitive tasks.

In recent years, there is a growing body of work on image coding techniques that attempt to preserve edge information through methods that can be broadly categorized into edge-adaptive decomposition [10]–[13], directional filtering [14]–[15], and segmentation-based approximation [16]–[18]. The last one is related to a particular approach in image coding by separating the edge/contour and texture information and coding them independently. This idea originated in the 1980s during the development of second-generation image coding techniques [19], [20] that attempts to imitate some of the functions in the human visual system in order to obtain high compression. An advantage of such an approach is that the cross-distortion between edge and texture information can be avoided. At very low bit rates, images can be reconstructed from only contour information at a relatively low coding cost. This results in artificial graphic-like images which could still be both visually pleasing and cognitively meaningful.

In this paper, a wavelet-based multi-scale peak signal to noise ratio (PSNR) scheme for image de-noising is proposed, and the determination of the wavelet basis with respect to the proposed scheme is also discussed. Compared to other wavelet-based representation the proposed Moving window based wavelet shrinkage based method is more efficient for a wide range of contour features.

## 2.Wavelet Analysis

### *2.1.Fourier Analysis*

Signal analysts already have at their disposal an impressive arsenal of tools. Perhaps the most well known of these is Fourier analysis, which breaks down a signal into constituent sinusoids of different frequencies. Another way to think of Fourier analysis is as a mathematical technique for transforming our view of the signal from time-based to frequency-based. For many signals, Fourier analysis is extremely useful because the signal's frequency content is of great importance. Fourier analysis has a serious drawback. In transforming to the frequency domain, time information is lost. When looking at a Fourier transform of a signal, it is impossible to tell when a particular event took place. If the signal properties do not change much over time that is, if it is what is called a stationary signal this drawback is not very important. However, most interesting signals contain numerous nonstationary or transitory characteristics: drift, trends, abrupt changes, and beginnings and ends of events. These characteristics are often the most important part of the signal, and Fourier analysis is not suited to detecting them.

*2.2.Wavelet Analysis*

Wavelet analysis represents the next logical step: a windowing technique with variable-sized regions. Wavelet analysis allows the use of long time intervals where more precise low-frequency information, and shorter regions where high-frequency information is obtained.

One major advantage afforded by wavelets is the ability to perform local analysis that is, to analyse a localized area of a larger signal. Wavelet analysis is capable of revealing aspects of data that other signal analysis techniques miss aspects like trends, breakdown points, discontinuities in higher derivatives, and self-similarity. Furthermore, because it affords a different view of data than those presented by traditional techniques, wavelet analysis can often compress or de-noise a signal without appreciable degradation.

A wavelet is a waveform of effectively limited duration that has an average value of zero. Compare wavelets with sine waves, which are the basis of Fourier analysis. Sinusoids do not have limited duration they extend from minus to plus infinity. And where sinusoids are smooth and predictable, wavelets tend to be irregular and asymmetric. Fourier analysis consists of breaking up a signal into sine waves of various frequencies. Similarly, wavelet analysis is the breaking up of a signal into shifted and scaled versions of the original (or mother) wavelet. Signals with sharp changes might be better analysed with an irregular wavelet than with a smooth sinusoid. It also makes sense that local features can be described better with wavelets that have local extent.

The fundamental idea behind wavelets is to analyze according to scale. Indeed, some researchers in the wavelet field feel that, by using wavelets, one is adopting a whole new mindset or perspective in processing data. Wavelets are functions that satisfy certain mathematical requirements and are used in representing data or other functions. This idea is not new. Approximation using superposition of functions has existed since the early 1800's when Joseph Fourier discovered that he could superpose sines and cosines to represent other functions. However, in wavelet analysis, the scale that we use to look at data plays a special role. Wavelet algorithms process data at different scales or resolutions. If a signal with a large "window" is viewed, gross features can be noticed. Similarly, if a signal with a small "window" is viewed, only small features can be noticed. The result in wavelet analysis is to see both the forest and the trees. This makes wavelets interesting and useful. For many decades, scientists have wanted more appropriate functions than the sines and cosines which comprise the bases of Fourier analysis, to approximate choppy signals (1). By their definition, these functions are non-local (and stretch out to infinity). They therefore do poor job in approximating sharp spikes. But with wavelet analysis, we can use approximating functions that are contained neatly in finite domain. Wavelets are well-suited for approximating data with sharp discontinuities.

The wavelet analysis procedure is to adopt a wavelet prototype function, called an analyzing wavelet or mother wavelet. Temporal analysis is performed with a contracted, high-frequency version of the prototype wavelet, while frequency analysis is performed with a dilated, low-frequency version of the same wavelet. Because the original signal or function can be represented in terms of a wavelet expansion (using coefficients in a linear combination of the wavelet functions), data operations can be performed using just the corresponding wavelet coefficients. And if the best wavelets are chosen to adapt for speech data, or truncate the coefficients below a threshold, then the speech data is sparsely represented. This sparse coding makes wavelets an excellent tool in the field of data compression.

The idea behind image denoising using wavelets is primarily linked to the relative scarceness of the wavelet domain representation for the signal. Wavelets concentrate speech information (energy and perception) into a few neighboring coefficients. Therefore as a result of taking the wavelet transform of a image, many coefficients will either be zero or have negligible magnitudes.

A major drawback of Fourier analysis is that in transforming to the frequency domain, the time domain information is lost. When looking at the Fourier transform of a signal, it is impossible to tell when a particular event took place. In an effort to correct this deficiency, Dennis Gabor (1946) adapted the Fourier transform to analyse only a small section of the signal at a time . a technique called windowing the signal. Gabor's adaptation, called the Windowed Fourier Transform (WFT) gives information about signals simultaneously in the time domain and in the frequency domain. To illustrate the time-frequency resolution differences between the Fourier transform and the wavelet transform consider the following figures.

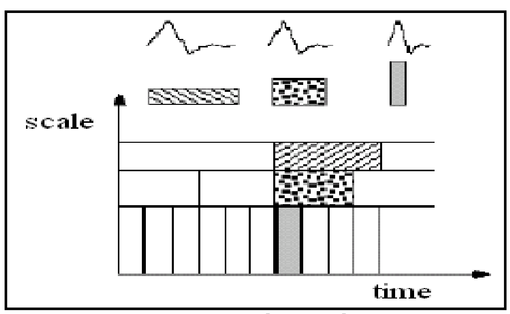



*Figure 1: Wavelet Resolution*

Fig.1 shows a time-scale view for wavelet analysis rather than a time frequency region. Scale is inversely related to frequency. A low-scale compressed wavelet with rapidly changing details corresponds to a high frequency. A high-scale stretched wavelet that is slowly changing has a low frequency.

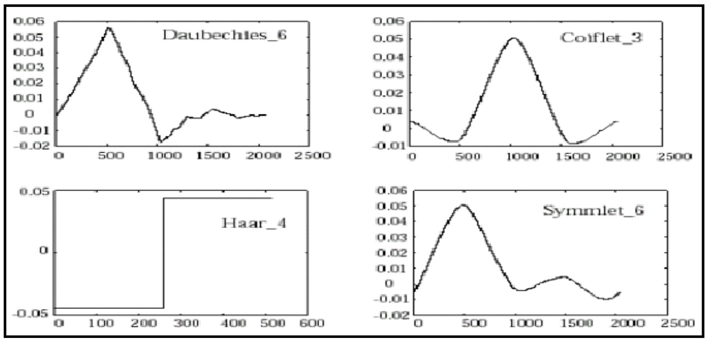
The figure below illustrates four different types of wavelet basis functions.

The different families make trade-offs between how compactly the basis functions are localized in space and how smooth they are. Within each family of wavelets are wavelet subclasses distinguished by the number of filter coefficients and the level of iteration. Wavelets are most often classified within a family by the number of vanishing moments. This is an extra set of mathematical relationships for the coefficients that must be satisfied. The extent of compactness of signals depends on the number of vanishing moments of the wavelet function used.

*2.3.Wavelet Transform Based Denoising:*

The general wavelet denosing procedure is as follows:

- Apply wavelet transform to the noisy signal to produce the noisy wavelet coefficients.
- Select appropriate threshold limit at each level and threshold method (hard or soft thresholding) to best remove the noises.
- Inverse wavelet transforms of the thresholded wavelet coefficients to obtain a denoised signal.




*Figyre 2: Different wavelet Families*

2.3.1.<u>Wavelet selection</u>

To best characterize the noisy signal, we should select our "mother wavelet" carefully to better approximate and capture the transient spikes of the original signal. "Mother wavelet" will not only determine how well we estimate the original signal in terms of the shape, but also, it will affect the frequency spectrum of the denoised signal. The choice of mother wavelet can be based on eyeball inspection of the PD spikes, or it can be selected based on correlation γ between the signal of interest and the wavelet-denoised signal, or based on the cumulative energy over some interval where PD spikes occur.

$$\gamma = \frac{\sum(X-\overline{X})(Y-\overline{Y})}{\sqrt{\sum(X-\overline{X})^2(Y-\overline{Y})^2}}$$

Where $\overline{X}$ and $\overline{Y}$ are the mean value of set X and Y, respectively.

$$E = \sum X^2$$

where E is the energy and X is the signal vector.

We choose to select the mother wavelet based on the last two methods: correlation between two signals and cumulative energy over some interval of noise occurrence. We found that the two methods give us a very similar outcome.

*2.4.Multilevel Decomposition*

From the previous section, we have known that the wavelet transform is constituted by different levels. The maximum level to apply the wavelet transform depends on how many data points contain in a data set, since there is a down-sampling by 2 operation from one level to the next one. In our experience, one factor that affects the number of level we can reach to achieve the satisfactory noise removal results is the signal-to-noise ratio (SNR) in the original signal.

The decomposition process can be iterated, with successive approximations being decomposed in turn, so that one signal is broken down into many lower resolution components. This is called the wavelet decomposition tree and this is shown in fig.3.

The wavelet decomposition of the signal s analyzed at level j has the following structure [cAj, cDj ...].The approximation coefficients cAj (low frequency information) and the detail coefficients cDj (high frequency information). Looking at a signals wavelet decomposition tree can reveal valuable information. Since the analysis process is iterative, in theory it can be continued indefinitely. In reality, the decomposition can only proceed until the vector consists of a single sample. Normally, however there is little or no advantage gained in decomposing a signal beyond a certain level. The selection of the optimal decomposition level in the hierarchy depends on the nature of the signal being analyzed or some other suitable criterion, such as the low-pass filter cut-off.

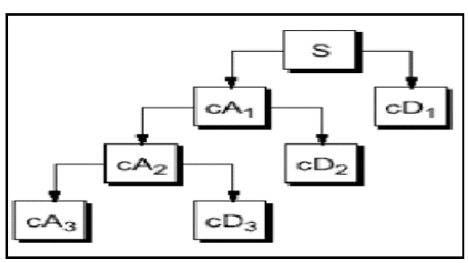



*Figure 3: Decomposition of DWT Coefficients*

Starting with a discrete input signal vector s, the first stage of the FWT algorithm decomposes the signal into two sets of coefficients. These are the approximation coefficients cA1 (low frequency information) and the detail coefficients cD1 (high frequency information), as shown in the fig.4.

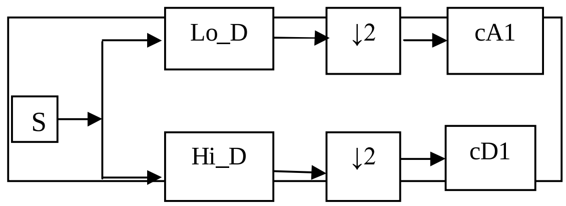



*Figure 4: Signal decomposition*

The coefficient vectors are obtained by convolving s with the low-pass filter Lo_D for approximation and with the high-pass filter Hi_D for details. This filtering operation is then followed by down sampling by a factor of 2.

*2.5.Signal Reconstruction*

The original signal can be reconstructed or synthesized using the inverse discrete wavelet transform (IDWT). The synthesis starts with the approximation and detail coefficients cAj and cDj, and then reconstructs cAj-1 by up sampling by a factor of 2 and filtering with the reconstruction filters as shown in fig. 5.

The reconstruction filters are designed in such a way to cancel out the effects of aliasing introduced in the wavelet decomposition phase. The reconstruction filters (Lo_R and Hi_R) together with the low and high pass decomposition filters, forms a system known as quadrature mirror filters (QMF). For a multilevel analysis, the reconstruction process can itself be iterated producing successive approximations at finer resolutions and finally synthesizing the original signal.

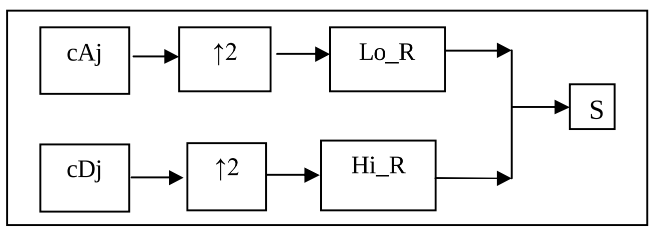



*Figure 5: Signal reconstruction*

2.5.1.Performance Measures

A number of quantitative parameters can be used to evaluate the performance of the wavelet based image denoising algorithm, in terms of reconstructed signal quality after denoising. The following parameters are compared:

- Signal to Noise Ratio (SNR).
- Peak Signal to Noise Ratio (PSNR).

The results obtained for the above quantities are calculated using the following formulas:

Signal to Noise Ratio:-

$$\mathrm{SNR} = 10\log_{10}\left(\frac{\sigma_x^2}{\sigma_e^2}\right)$$

$\sigma_x^2$ is the mean square of image signal.

$\sigma_e^2$is the mean square difference between original and reconstructed signal.

Peak Signal to Noise Ratio:-

$$\mathrm{SNR} = 10\log_{10}\frac{NX^2}{||x - r||^2}$$

N is the length of reconstructed signal, X is the maximum absolute square value of the signal x and $\|x - r\|^2$ is the energy of the difference between the original and reconstructed signals.

**3.Wavelet Transform**

*3.1.Haar Wavelet Transform*

To give the definition of Haar wavelet-based M-channel filter bank, the concept of discrete M–channel filter bank is introduced. The discrete M-channel filter bank is described in fig.6 in which the input signal x (n) is split by a low-pass filter $H_0(Z)$and high pass filters$H_k(Z)$, k=1…M-1 into the reference signal $x_0(n)$and the detail signals$x_k(n)$, k=1…M-1, each of which is decimated by a factor of M. For reconstruction, interpolation by a factor of M is performed, followed by reconstruction filters $G_k(Z)$ , k=0, 1 …M-1.

The M-channel orthogonal filter bank was constructed for a number of communication applications such as sub band coders for speech signals .However it is interesting to apply a multi-channel filter bank to the de-noising of signals. The key of this problem is to find such a multi-channel filter bank whose high-pass filters and low-pass filter are good at edge detecting and noise smoothing respectively. It is known that the Haar wavelet has the most compact spatial support of all wavelets and is also an optimal edge matching filter. Thus, based on the Haar wavelet and a simple shift operation, M-channel filter bank can be designed. Its high-pass and low-pass filters are defined as

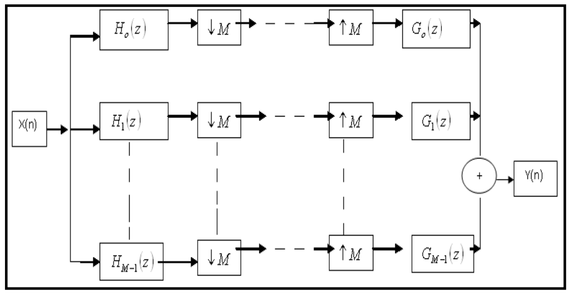



*Figure 6: Structure Of Discrete M-Channel Filter Bank*

$$H_k(z) = \frac{1}{M}\left(z^{-k+1} - z^{-k}\right) \quad k = 1, \ldots.. M-1$$

and

$$H_0(Z) = \frac{1}{M}\left(1 + z^{-1} + \cdots + z^{-M+1}\right)$$

To obtain the reconstruction filters $\mathbf{G_k(Z)}$, k=0, 1…M, we need to express the M-channel analysis filters as a matrix notation. Let

$$\begin{bmatrix} H_1(Z) \\ H_2(Z) \\ \vdots \\ H_0(Z) \end{bmatrix} = H \begin{bmatrix} 1 \\ z^{-1} \\ \vdots \\ z^{-M+1} \end{bmatrix}$$

$$\begin{bmatrix} H_1(Z) \\ H_2(Z) \\ \vdots \\ H_0(Z) \end{bmatrix} = \frac{1}{M} \begin{bmatrix} 1 & -1 & 0 & \ldots & 0 \\ 0 & 1 & -1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & 1 & \ldots & 1 \end{bmatrix} \begin{bmatrix} 1 \\ z^{-1} \\ \vdots \\ z^{-M+1} \end{bmatrix}$$

The inverse matrix of H can be written as

$$H^{-1} = \begin{bmatrix} M-1 & M-2 & \ldots & 1 & 1 \\ -1 & M-2 & \ldots & 1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -1 & -2 & \ldots & -M+1 & 1 \end{bmatrix}$$

Then the reconstruction filters, $G_k(z)$, k=0,1,…………M, are given by

$$\begin{bmatrix} G_1(Z) \\ G_2(Z) \\ \vdots \\ G_0(Z) \end{bmatrix} = \begin{bmatrix} M-1 & M-2 & \ldots & 1 & 1 \\ -1 & M-2 & \ldots & 1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -1 & -2 & \ldots & -M+1 & 1 \end{bmatrix}^{T} \begin{bmatrix} z^{-M+1} \\ z^{-M+2} \\ \vdots \\ 1 \end{bmatrix}$$

Where T denotes the transpose of matrix.

The span of the low-pass filter $H_0(z)$ is just M. After decimating with a factor of M, the noise in the decimated reference signal will preserve its independent property. This property is very useful for some de-noising operators that ask the noise to be white in the different scales of wavelet transform domain. The design of the Haar wavelet-based M-channel filter bank (HMF) is finished. When M>2, the HMF is non-orthogonal.

*3.2.Double Haar Wavelet Transfor*

The HMF with M=3 is called the DHWT. About the DHWT, we have

$$H = \frac{1}{3}\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix}, H^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ -1 & -2 & 1 \end{bmatrix}$$

Suppose x (n), x (n-1) and x (n-2) are the samples of the input signal at a given time. The estimate of the center pixel x (n-1) is especially important for some applications of image processing. According to the definition of DHWT we have the reconstructed

$$x(n-1) = x_0(n) - x_1(n) + x_2(n)$$

3.2.1.<u>DHWT For Image Denoising</u>

The double Haar wavelet is as well as the Haar wavelet in edge detection, but better in denoising for its improved low-pass filter. Particularly, the short support makes it very suitable for a moving window based multiscale analysis. Similar to the two-channel orthogonal wavelet transform, the DHWT can be extended to 2-D for image denoising. Let $x_0(m,n)$ be an image of N×N pixels. The steps of the 2-D discrete double Haar wavelet transform are defined by the following steps.

- In the horizontal direction, the original image $x_0(m,n)$ is filtered by the filters $H_0(Z)$, $H_1(Z)$and $H_2(Z)$ respectively. Three image $x'_{00}(m,n)$, $x'_{01}(m,n)$and $x'_{02}(m,n)$ are produced.
- In the vertical direction, the three images$x'_{00}(m,n)$, $x'_{01}(m,n)$and $x'_{02}(m,n)$ are filtered by the filters$H_0(Z)$, $H_1(Z)$and $H_2(Z)$respectively. This gives nine images $x^{n}_{0j}(m,n)$, $0 \le j \le 8$.
- Down-sampling the images $x''_{0j}(m,n)$, $0 \le j \le 8$, with an interval of three, we obtain nine subimages $x_{0j}(m,n)$, $0 \le j \le 8$.
- Steps 1)–3) can be repeated on the subimage $x_{00}(m,n)$ so as to get the other subimages in the next scale.

Soft thresholding (ST) is one of the well known noise smoothers in wavelet domain. Here, we first investigate the effectiveness of the Haar wavelet based ST and the DHWT-based LS in image denoising. The reason for selecting the Haar wavelet is that it is a linear phase filter and almost has the same filter support length as the double Haar wavelets.

Unfortunately, the DHWT-based ST may produce “mosaic” which can be observed .The reason for this is that too much details of image are lost. To improve it, we may change the value of threshold, or replace the double Haar wavelet with a smoother wavelet. However, considering that a moving window based mean filter does not produce mosaic, we may embed the DHWT in a moving window so as to delete the mosaic.

*3.3.Moving Window Based Double Haar Wavelet Transform*

For this purpose, a $3^n$ X $3^n$ rectangular window should be defined first. Suppose that, at a given time, the center of this window is at the pixel position (i,j) of an image. By applying the 2-D DHWT-based ST to the pixels within the window, we obtain the estimate of pixel x (i, j) with the n number of decompositions. Then, we move the center of the window to the neighbor pixel so as to compute the next one estimate by a new cycle of the denoising operation. This procedure continues until all of the estimates of the pixels in the image are obtained. For simplicity, the moving window based denoising in wavelet domain is called MWD. Since a pixel of an image bears no relation to the pixels far from it in distance, a small size of the moving window will be good enough for the MWD.

To explore the difference between the traditional denoising in wavelet domain and the MWD, an example is given below. Fig.7(a) shows nine neighbor pixels of an image. Based on the 2-D DHWT of the image, those pixels may use the same wavelet coefficients to obtain their estimates. In the worst case of denoising, all of the wavelet coefficients are set to zero. Thus, by reconstruction, the estimates of the nine pixels are equal to the average of their grays, which just forms a “mosaic.” To describe the process of MWD, Fig. 7(b) shows that the center of a 3X3 moving window W is at the pixel e of the image at a given time. By applying the 2-D DHWT-based ST to the nine pixels within the W, the estimate of the pixel e is obtained. Then, the center of the window is shifted by one pixel to the neighbor pixel h shown in Fig. 7(c). By applying the same denoising operation to the pixels within the W at the new position, the estimate of the pixel h is subsequently obtained. In the worst case, the estimates of the pixels e and h are the average of the grays of the nine pixels in their respective windows, which just forms a 3 X 3 moving window-based mean filtering.

Compared to the traditional denoising in wavelet transform domain, the MWD uses different wavelet coefficients for different pixels to obtain the estimates. Thus, the estimate error of the wavelet coefficients for a pixel does not influence the estimates of the other pixels in the image.

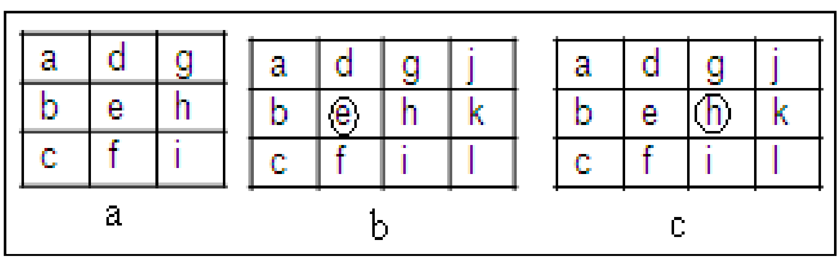


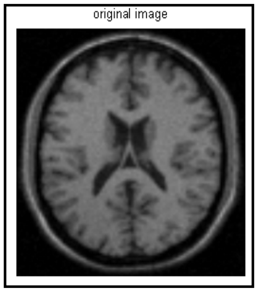

*Figure7: (a) Part Of An Image; (b) 3X3 Moving Window Centered At The Pixel E (c) By Shifting One Pixel, The 3X3 Moving Window Is Centered At The Pixel H*

*3.4.Moving Window Based Wavelet Shrinkage*

The wavelet shrinkage [21] is a signal denoising technique based on the idea of thresholding the wavelet coefficients. Wavelet coefficients having small absolute value are considered to encode mostly noise and very fine details of the signal. In contrast, the important information is encoded by the coefficients having large absolute value. Removing the small absolute value coefficients and then reconstructing the signal should produce signal with lesser amount of noise. The wavelet shrinkage approach can be summarized as follows:

- Apply the wavelet transform to the signal.
- Estimate a threshold value.
- Remove (zero out) the coefficients that are smaller than the threshold.
- Reconstruct the Signal (apply the inverse wavelet transform).

**4.Results**



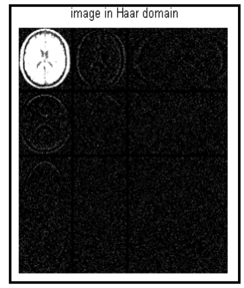
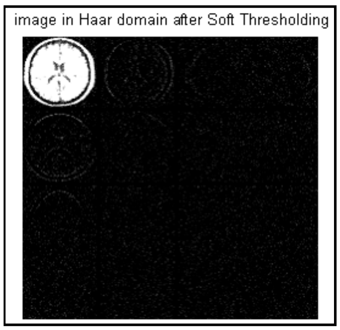
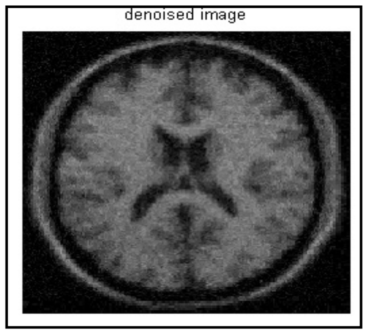



*Figure 8: Original Image &*
*Figure 9: Noisy Image PSNR= 12.4473db, Threshold=3.2925 &*
*Figure 10: Harr Transform Image*





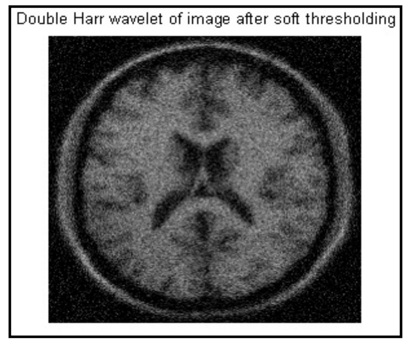
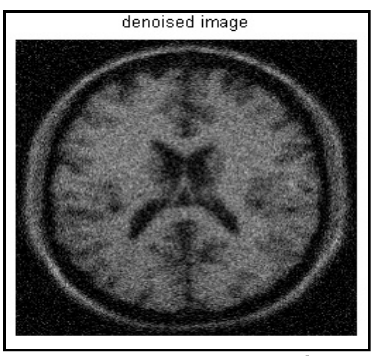
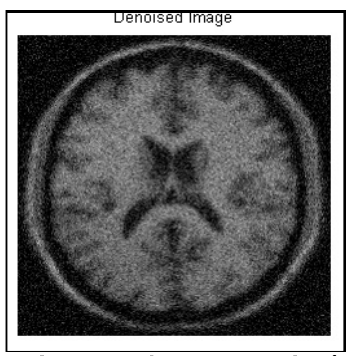



*Figure 11: Image In Haar Wavelet Domain After Soft Thresholding. Threshold=3.29% &*
*Figure 12: Denoised Image &*
*Figure 13: Image In DHWT Domain After Soft Thresholding*





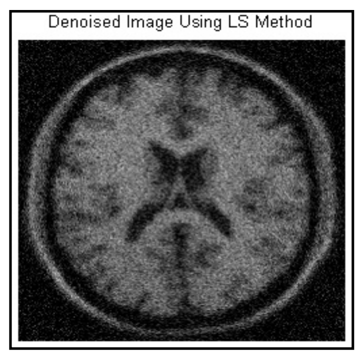



*Figure 14: The Denoised Image With DHWT Based Soft Thresholding With PSNR=28.4529db &*
*Figure 15: The Denoised Image With PSNR=33.5878db &*
*Figure 16: The Denoised Image With PSNR=39.1201Db*

| S.no | Image denoising methods | PSNR(dB) |
|---|---|---|
| 1 | Haar wavelet transform | 21.3655 |
| 2 | Double Haar wavelet transform | 28.4529 |
| 3 | Moving window based Double Haar wavelet transform | 33.5878 |
| 4 | Moving window based wavelet shrinkage | 39.1201 |

*Table 1: Comparisons Of Psnr For Various Image Denoising Methods*

**5.Conclusion**

Image denoising methods and edge detection using Haar wavelet transforms is implemented in this thesis. Based on the Haar wavelet, a class of multi-channel nonorthogonal filter bank is designed. As a special case of this filter bank, a double Haar wavelet transform is introduced. After that the double Haar wavelet transform is embedded in a moving window. In the moving window, based on the double Haar wavelet a multi-channel filter bank, the wavelet shrinkage is developed. These methods are suitable for image denoising and edge detection.

Soft thresholding (ST) is one of the well known noise smoothers in wavelet domain. Here, Haar wavelet based ST and the DHWT- based ST is applied on image for denoising .From the results it is observed that the PSNR of DHWT- based ST is better than the Haar wavelet based ST. Unfortunately, the DHWT- based ST may produce mosaic which can be observed. The reason for this is that too much details of image are lost. Thus moving window based double haar wavelet transform was developed to avoid mosaic. After that moving window based wavelet shrinkage was developed. It was concluded that moving window based wavelet shrinkage has high PSNR value than the moving window based DHWT. Compared to the traditional denoising in wavelet transform domain, the moving window uses different wavelet coefficients for different pixels to obtain the estimates. Thus, the estimate error of the wavelet coefficients for a pixel does not influence the estimates of the other pixels in the image.

**6.References**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