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Resolution Of Two Point Objects With Primary Spherical Aberration Under Incoherent Illumination

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Abstract:

The influence of primary spherical aberration on the resolution of two point objects in the case of apodised rotationally symmetric optical systems has been studied by applying the modified Sparrow criterion introduced by Asakura to suit the case of unequally bright object points. The results are presented in terms of Rayleigh and Sparrow limits obtained for coherent, partially coherent and incoherent illuminations. It is found that the chosen Hanning amplitude filter is effective in increasing the resolving power of the aberrated optical imaging systems.

Key words: Rotationally Symmetric Systems, Primary Spherical Aberration, Hanning amplitude filters.

1. Introduction

The study of two-point resolution forms an interesting area when the two point objects are situated within a close vicinity of each other. The performance of optical imaging systems can be judged depending upon its ability in resolving closely situated object points. There are a number of quality criteria in the field of image science; two-point resolution criterion is one of the most important and most basic measures proposed for judging the quality of optical imaging systems [1]. Rayleigh [2] criterion states that the two point sources are just resolved if the central maximum of one falls on the first minimum of irradiance produced by the other point source. Sparrow [3] proposed an alternative criterion according to which the two points are just resolved if the second derivative of the resultant image intensity distribution vanishes at the point midway between the respective Gaussian image points. Its practical applicability is limited by the fact that in its original context it is stated for two equally bright incoherent object points. Asakura [4] has modified the Sparrow criterion to be applicable for the more realistic case of unequally bright object points. In the present study this modified Sparrow criterion has been applied to optical imaging systems apodised by the Hanning amplitude filters. It is a known observable fact that the image produced by any physically feasible optical imaging system is not the perfect replica of the input object, but the diffraction pattern of non-zero dimensions. If two such image diffraction patterns overlap, it will be very difficult to detect the presence of two objects. Hence, it is desirable to find a means of enhancing the resolving power of the optical imaging system in identifying two object points for a given situation.

2. Theory

Following Hopkins and Barham [5], the expression for the total image intensity distribution in the image plane of an apodized optical system, as a function of the reduced co-ordinate Z , is given by,

$$I(z) = |A(Z+B)|^2 + \alpha |A(Z-B)|^2 + 2\sqrt{\alpha} \gamma(Z_0) |A(Z+B)| |A(Z-B)| \quad (1)$$

where $2B=Z_0$ is the separation between the two object points, α is the ratio of their intensities, $\gamma(Z_0)$ is the degree of spatial coherence of the illumination. Z is the dimensionless diffraction variable. $\alpha = 1$ gives the case of equal intensities while $\alpha \neq 1$ corresponds to unequally bright object points. The coherent and the incoherent extremes of illuminations are given by $\gamma = 1$ and $\gamma = 0$, respectively; whereas $0 < \gamma < 1$ is for the partially coherent illumination. The two object points are situated with a separation Z_0 at the same distance $Z_0/2$ on either side of the optical axis. $I(Z)$ is image intensity distribution as a function of Z , which is measured from

the axis of the optical system. $A(Z+B)$ and $A(Z-B)$ are the normalized amplitude point spread functions for circular apertures and are given by the following expressions:

$$A(Z+B) = 2 \int_0^1 J_0[(Z+B)r] e^{-i\phi_s} r dr \quad (2)$$

$$A(Z-B) = 2 \int_0^1 J_0[(Z-B)r] e^{-i\phi_s} r dr \quad (3)$$

J_0 being the Bessel function of first kind and zero order, r is the normalized distance of a general point on the exit pupil varying from 0 to 1. In the present case, as the Hanning amplitude filter is used for apodizing the optical system, the above Eqs. (2) And (3) become,

$$A(Z+B) = 2 \int_0^1 f(r) J_0[(Z+B)r] e^{-i\phi_s} r dr \quad (4)$$

$$A(Z-B) = 2 \int_0^1 f(r) J_0[(Z-B)r] e^{-i\phi_s} r dr \quad (5)$$

where $f(r)$ is the pupil function of the Hanning amplitude filter [6] and is given by

$$f(r) = \text{Cos}(\pi \beta r) \quad (6)$$

The variable β is the apodization parameter controlling the degree of non-uniformity of the transmission over the exit pupil in the range of $0 \leq \beta \leq 1$. For $\beta = 0$, $f(r) = 1$ which implies the uniform transmittance over the exit pupil. Thereby, for $\beta = 0$ the filter function becomes an Airy pupil. Φ_s if the primary spherical aberration parameter. For chosen apodizer the amplitude transmittance decreases monotonically from the centre towards the edges of the pupil. Higher spatial frequency components of the object are diffracted by a larger angle and hence these go predominantly through the edge of the aperture. As the pupil transmittance is decreased at the edges as compared to that of the centre, due to apodization, which results in the reduction in the higher spatial frequency components in the image. This manifests as partial or complete suppression of the undesired optical side lobes, which consequently enhances imaging characteristics. However, for annular apertures the expressions for the amplitude PSF can be expressed as

$$A(Z \pm B) = 2 \int_{\epsilon}^1 f(r) J_0[(Z \pm B)r] e^{-i\phi_s} r dr \quad (7)$$

Where ϵ is the central obscuration parameter whose value specifies the amount of central obscuration. In the presence of central obscuration, the expression for the total image intensity distribution is given by

$$I(Z) = |A(Z+B)|^2 + \alpha |A(Z-B)|^2 + 2\sqrt{\alpha} \gamma |A(Z+B)| |A(Z-B)| \quad (8)$$

The modified Sparrow criterion introduced by Asakura [4] states that, “the resolution is retained when the second derivative of the image intensity distribution vanishes at a certain point ($Z = Z_0^1$) between two Gaussian image points, with the condition that this point Z_0^1 should be a solution for the first derivative of the image intensity distribution becoming zero”. This can be expressed mathematically as

$$\left. \frac{\partial^2 I(Z)}{\partial Z^2} \right|_{Z=Z_0^1} = 0 \quad (9)$$

3.Results And Interpretations

Expressions (8) and (9) have been used to compute the image intensity distribution of the two point objects and the Sparrow and Rayleigh limits of the optical systems with circular and annular apertures apodised with the chosen amplitude filter. The results have been compared with that of the Airy case. Figure 1 depicts the intensity distribution curves of the circular aperture for Airy pupil function. In this case the two point objects are of equal intensity under incoherent illuminated and the optical system is subjected to a higher order of primary spherical aberration. The resolution has been observed when the two object points are placed at a distance of about $z_0 = 3.5$ dimensionless diffraction units. With the employment of Hanning amplitude filter, the resolving power of the optical imaging system is considerably enhanced and there appears to be a clear resolution of two point objects when compared to that of Airy case. Fig. 2 represents the case with the central obscuration parameter ' $\epsilon = 0.4$ ' and when the two object points are having the intensity ratio to be ' $c = 0.6$ ' under the conditions of optical system being highly apodised with Hanning amplitude filter suffering with primary spherical aberration. It is clear from the profile of the intensity distribution curves that the two points are visibly resolved even for $z_0 = 3.0$ and its limit of resolution in Sparrow sense is even lower than this value. Hence, the employed filter is very effective in resolving the two point objects.

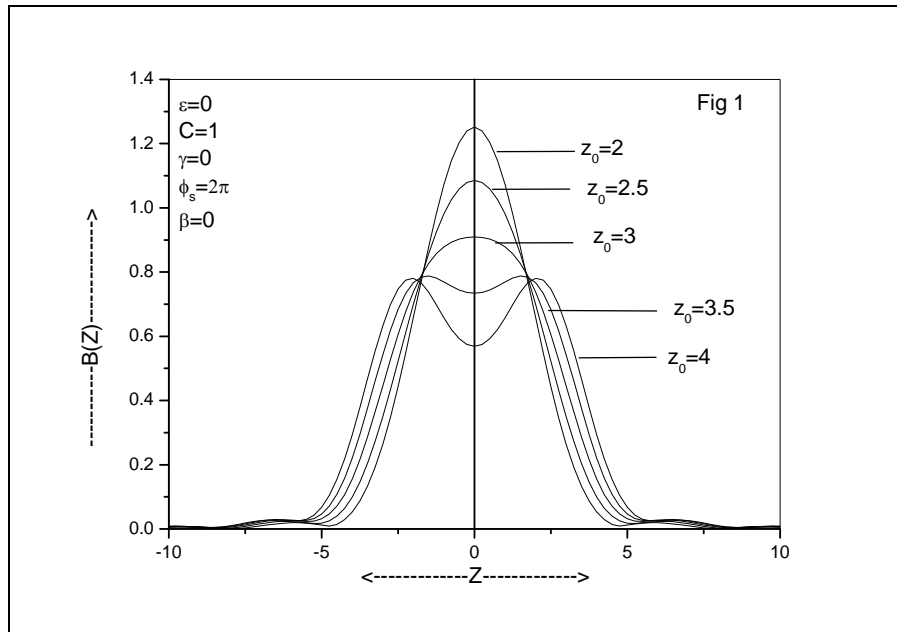


Figure 1

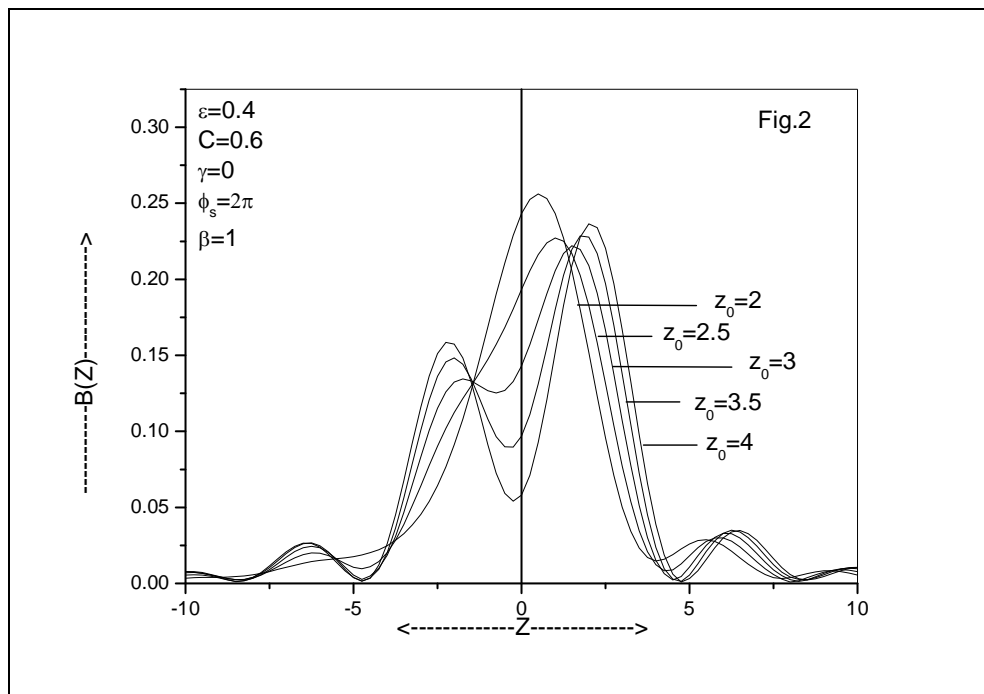


Figure 2

$\Phi_s = 2\pi$	$\beta = 0$				$\beta = 1$				
	C=0.2		C=1		C=0.2		C=1		
	γ	SL	RL	SL	RL	SL	RL	SL	RL
$\epsilon = 0$	0.0	3.9470	4.7410	2.9700	3.8910	2.1010	3.2010	2.3940	2.8660
	0.2	4.3840	5.1730	3.3160	4.2560	2.3010	3.2710	2.6840	3.1570
	0.4	4.8200	5.5570	3.6610	4.5940	2.5010	3.4010	2.9790	3.4400
	0.6	5.2480	5.8000	4.0180	4.9060	2.6320	3.5280	3.2940	3.7270
	0.8	5.4280	5.8360	4.4040	5.1840	2.7040	3.7940	3.6560	_____
	1.0	5.2880	6.2000	4.8600	5.4110	2.9010	4.9910	_____	_____
$\epsilon = 0.4$	0.0	3.2640	3.9240	2.8020	3.5380	2.5010	3.1560	2.5300	3.1070
	0.2	3.6920	4.3160	3.1340	3.8800	2.8290	3.5140	2.8310	3.4110
	0.4	4.1120	4.6960	3.4680	4.2040	3.2710	3.8470	3.1360	3.7010
	0.6	4.5560	5.0790	3.8180	4.5170	3.6970	4.1820	3.4560	3.9870
	0.8	5.0300	5.3680	4.2070	4.8220	4.1630	4.5460	3.8150	4.2750
	1.0	4.8500	6.4000	4.6980	5.0970	4.2090	5.4540	4.2760	4.5530
$\epsilon = 0.8$	0.0	2.3010	2.8580	2.3900	2.9100	2.3010	2.8450	2.3810	2.8990
	0.2	2.5010	3.2030	2.6750	3.1970	2.5010	3.1900	2.6650	3.1840
	0.4	2.8370	3.5210	2.9630	3.4710	2.8200	3.5060	2.9520	3.4570
	0.6	3.3200	3.8350	3.2670	3.7410	3.3060	3.8190	3.2550	3.7270
	0.8	3.7840	4.1730	3.6080	4.0169	3.7680	4.1560	3.5950	4.0019
	1.0	3.9340	4.9200	5.3350	_____	3.9130	4.8960	5.3150	_____

Table 1: Sparrow And Rayleigh Limits

Table-1 lists the computed values of Sparrow and Rayleigh limits for Airy ($\beta = 0$) and Apodised ($\beta = 1$) apertures. The limit of resolution happen to decrease thereby enhancing the resolving power of the optical imaging system with central obscuration parameter (ϵ). Resolution limit happen to decrease with intensity ratio (c) thereby enhancing the resolving power of the optical imaging system. However, this limit tends to show an increasing trend with coherence parameter (γ). The same trend can be observed in the case of apodised aperture with further reduced limit of resolution values.

4. Conclusion

The process of apodising the optical system with Hanning amplitude filter, suppresses fully or partially the optical side-lobes of the individual point spread functions. The apodisation parameter is effective in shaping the point spread function so that there is an improvement of resolution of two point objects rendering the optical system to be more effective in their resolution.. Hence, the Hanning amplitude filters are effective in enhancing the resolving power of the given optical imaging system.

5. References

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