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Semipseudo Symmetric Ideals in Partially Ordered Ternary Semigroups

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Abstract:
In this paper the terms pseudo symmetric ideals, semipseudo symmetric ideals of po ternary semigroups. It is proved that every pseudo symmetric ideal of po ternary semigroup is a semipseudo symmetric ideal. It is also proved that every semiprime ideal P minimal relative to containing a semipseudo symmetric ideal A of a po ternary semigroup is completely semiprime. If A is a semipseudo symmetric ideal of a po ternary semigroup T. Then (1) $A_1 =$ the intersection of all completely prime ideals of T containing A. 2) A_1^1 = the intersection of all minimal completely prime ideals of T containing A. 3) A_1^{11} = the minimal completely semiprime ideal of T relative to containing A.4) $A_2 = \{x \in T : x^n \in A$ for some odd natural number n}5) A_3 = the intersection of all prime ideals of T containing A.6) A_3^1 = the intersection of all minimal prime ideals of T containing A.7) A_3^{11} = the minimal semiprime ideals of relative to containing A.8) $A_4 = \{x \in T : \langle x \rangle^n \subseteq A$ for some odd natural number n } are equivalent. If A is an ideal in a po ternary semigroup then it is proved that (1) A is completely semiprime, A is semiprime and pseudo symmetric. A is semiprime and semipseudo symmetric are equivalent and (2) A is completely prime; A is prime and pseudo symmetric. A is prime and semipseudo symmetric are also equivalent. If M is maximal ideal of a po ternary semigroup T with $M_4 \neq T$ then it is proved that M is completely prime, M is completely semiprime, M is pseudo symmetric and M is semipseudo symmetric are equivalent.

***Key words**: pseudo symmetric, semipseudo symmetric, completely prime, prime, completely semiprime, semiprime.*

1. Introduction

Ramakotaiah and Anjaneyalu [1] introduced the notions of pseudo symmetric ideals in semigroups. Pseudo symmetric semigroups and exhibit some examples and some classes of pseudo symmetric semigroups .Krishna Murthy and Arul Dass [11] introduced the notions of pseudo symmetric Γ ideals in Γ semigroups. Sarala, Anjaneyulu and Madhusudhana Rao [21] introduce and made a study on pseudo symmetric ideals in ternary semigroups.

2. Preliminaries

DEFINITION 2.1: A ternary semigroup T is said to be a **partially ordered ternary semigroup** if T is a partially ordered set such that $a \leq b \Rightarrow [a\, a_1\, a_2] \leq [b\, a_1\, a_2]$, $[a_1\, a\, a_2] \leq [a_1\, b\, a_2]$, $[a_1\, a_2\, a] \leq [a_1\, a_2\, b]$ for all $a, b, a_1, a_2 \in$ T.

NOTE 2.2: A partially ordered ternary semigroup is also called as po ternary semigroup or ordered ternary semigroup.

NOTATION 2.3: Let T be a po ternary semigroup and S be a non-empty subset of T. If *H* is a non-empty subset of S, we denote $\{s \in S : s \leq h$ for some $h \in H\}$ by $(H]_T$.

NOTATION 2.4: Let T be a po ternary semigroup and *S* be a non-empty subset of T. If *H* is a non-empty subset of *S*, we denote $\{s \in S: h \leq s$ for some $h \in H\}$ by $[H)_T$.

DEFINITION 2.5: Let T be a po ternary semigroup. A nonempty subset *S* of *T* is said to be a **po ternary subsemigroup** of T if *i)* $abc \in S$ for all $a, b, c \in S$ *ii)* $t \in T$; $s \in S, t \leq S \Rightarrow t \in S$

NOTE 2.6: A non-empty subset *S* of a po ternary semigroup *T* is a po ternary subsemigroup of *T* if and only if i) $SSS \subseteq S$ ii) $(S] = S$.

EXAMPLE 2.7: Let *Z* be the set of all intergers. Define multiplication on *Z* by $[xyz] = \min\{x, y, z\}$ for all $x, y, z \in Z$. Then *Z* is po ternary semigroup. Let Z^- be the set of all negative integers. Then Z^- is a po ternary subsemigroup of Z.

EXAMPLE2.8: Let $T=[0,1]$.Then *T* is a po ternary semigroup under the usual multiplication and usual order relation .Let $S=[0,\frac{1}{2}]$. Then *S* is a po ternary subsemigroup of *T*.

DEFINITION 2.9: A nonempty subset *A* of po ternary semigroup *T* is said to be a **po left ternary ideal** or **po left ideal** of *T* if i) $b,c \in T, a \in A \Rightarrow bca \in A$ ii) $a \in A$ and $t \in T$ such that $t \leq a \Rightarrow t \in A$.

NOTE 2.10: A nonempty subset *A* of po ternary semigroup *T* is a po left ternary ideal of *T* if and only if i) $TTA \subseteq A$ ii) $(A] \subseteq A$.

DEFINITION 2.11: A nonempty subset *A* of po ternary semigroup *T* is said to be a **po lateral ternary ideal** or **po lateral ideal** of T if i) $b, c \in T, a \in A \Rightarrow bac \in A$ ii) $a \in A$ and $t \in T$ such that $t \leq a \Rightarrow t \in A$.

NOTE 2.12: A nonempty subset *A* of po ternary semigroup *T* is a po lateral ternary ideal of *T* if and only if i) $TAT \subseteq A$ ii) $(A] \subseteq A$.

DEFINITION 2.13: A nonempty subset *A* of po ternary semigroup *T* is said to be a **po right ternary ideal** or **po right ideal** of *T* if i) $b, c \in T, a \in A \Rightarrow abc \in A$ ii) $a \in A$ and $t \in T$ such that $t \leq a \Rightarrow t \in A$.

NOTE 2.14: A nonempty subset *A* of po ternary semigroup *T* is a po right ternary ideal *T* if and only if i) $ATT \subseteq A$ ii) $(A] \subseteq A$.

DEFINITION 2.15: A non-empty subset A of po ternary semigroup T is said to be **po two sided ternary ideal** or **po two sided ideal** of T if i) $b, c \in T, a \in A \Rightarrow bca \in A, abc \in A$
ii) $a \in A$ and $t \in T$ such that $t \leq a \Rightarrow t \in A$.

NOTE 2.16: A nonempty subset *A* of po ternary semigroup *T* is a po two sided ternary ideal of *T* if and only if i) $TTA \subseteq A$; $ATT \subseteq A$ ii) $(A] \subseteq A$.

NOTE 2.17: A nonempty subset *A* of po ternary semigroup *T* is a po two sided ideal of *T* if and only if it is both a po left ideal and a po right ideal of *T*.

DEFINITION 2.18: A nonempty subset *A* of po ternary semigroup *T* is said to be a **po ternary ideal** or **po ideal** of *T* if i) $b, c \in T, a \in A \Rightarrow bca \in A, bac \in A, abc \in A$
ii) $a \in A$ and $t \in T$ such that $t \leq a \Rightarrow t \in A$.

NOTE 2.19: A nonempty subset *A* of po ternary semigroup *T* is a po ideal of *T* if and only if i) $TTA \subseteq A$; $ATT \subseteq A$, $TAT \subseteq A$ ii) $(A] \subseteq A$.

NOTE 2.20: A nonempty subset A of po ternary semigroup *T* is a po ideal of *T* if and only if it is a left po ideal, lateral po ideal, and right po ideal of *T*.

EXAMPLE 2.21: Let *N* be the set of all natural numbers. Define the ternary operation from $N \times N \times N \rightarrow N$ as $(a, b, c) = a.b.c$ where '. ' is usual multiplication and ordered relation $\leq$ on *N*. Then *N* is a po ternary semigroup and $A = 3N$ is a po ideal of the po ternary semigroup *N*.

DEFINITION 2.22: A po ternary semigroup *T* is said to be a **commutative** provided for all $a,b,c \in T$ we have i) $abc = bca = cab = bac = cba = acb$
ii) $a \in A$ and $t \in T$ such that $t \leq a \Rightarrow t \in A$.

DEFINITION 2.23: A po ternary semigroup *T* is said to be a **quasi commutative** provided i) for each $a,b,c \in T$ there exist natural number '*n*' such that
$abc = b^n ac = bca = c^n ba = cab = a^n cb = acb$.

DEFINITION 2.24: An element *a* of a po ternary semigroup *T* is said to be a **left identity** of *T* provided $aat = t$ and $t \leq a$ for all $t \in T$.

NOTE 2.25: Left identity element *a* of a po ternary semigroup *T* is also called as a left unital element.

DEFINITION 2.26: An element *a* of a po ternary semigroup *T* is said to be a **right identity** of *T* provided $taa = t$ and $t \leq a$ for all $t \in T$.

NOTE 2.27: Right identity element *a* of a po ternary semigroup *T* is also called as right unital element.

DEFINITION 2.28: An element *a* of a po ternary semigroup *T* is said to be a **lateral identity** of T provided $ata = t$ and $t \leq a$ for all $t \in T$.

NOTE 2.29: Lateral identity element *a* of a po ternary semigroup *T* is also called as a lateral unital element.

DEFINITION 2.30: An element *a* of a po ternary semigroup *T* is said to be a **two sided identity** of *T* provided $aat = taa = t$ and $t \leq a$ for all $t \in T$.

NOTE 2.31: Two- sided identity element of a ternary semigroup *T* is also called as a
bi-unital element.

DEFINITION 2.32: An element *a* of a po ternary semigroup *T* is said to be an **identity provided** $aat = taa = ata = t$ and $t \leq a$ for all $t \in T$.

NOTE 2.33: An identity element of *a* po ternary semigroup *T* is also called as a unital element.

NOTE 2.34: An element *a* of a po ternary semigroup *T* is said to be an identity of *T* then
a is a left identity, lateral identity and right identity of *T*.

NOTATION 2.35: let *T* be *a* po ternary semigroup. if *T* has an identity, Let $T^1 = T$ and if *T* does not have an identity, let T^1 be the po ternary semigroup *T* with an identity adjoined usually denoted by the symbol 1.

Definition 2.36: An ideal A of a po ternary semigroup T is said to be a **trivial ideal** provide T\A is singleton.

Definition 2.37: An ideal *A* of a po ternary semigroup *T* is said to be a **completely prime ideal** provided $x, y, z \in T$ and $xyz \in A$ implies either $x \in A$ or $y \in A$ or $z \in A$

Definition 2.38: An ideal *A* of a po ternary semigroup *T* is said to be a **completely semiprime ideal** provided $x \in T$, $x^n \in A$ for some odd natural number $n > 1$ implies $x \in A$.

Definition 2.39: An ideal *A* of a po ternary semigroup *T* is said to be a **prime ideal** of *T* provided are x, y, z are ideals of *T* and $XYZ \subseteq A \Rightarrow X \subseteq A$ or $Y \subseteq A$ or $Z \subseteq A$.

Definition 2.40: An ideal *A* of a po ternary semigroup *T* is said to be a **semiprime ideal** provided *x* is an ideal of *T* and $X^n \subseteq A$ implies $X \subseteq A$ for some odd natural number *n*

Theorem 2.41: Let *A* be any pseudo symmetric ideal in a po ternary semigroup *T* and $a_1 a_2, \ldots\ldots a_n \in$ T where *n* is an odd natural number. Then $a_1 a_2 \ldots\ldots a_n \in A$ if and only if $< a_1 >< a_2 > \ldots . < a_n > \subseteq A$.

Corollary 2.42: Let *A* be a pseudo symmetric ideal in a po ternary semigroup *T*, then for any odd natural number n, $a^n \in A$ if and only if $< a >^n \subseteq A$

Theorem 2.43: Let *A* be a prime ideal of a po ternary semigroup *T*. If *A* is completely semiprime ideal of *T* then *A* is compleltely prime.

Theorem 2.44: Every completely semiprime ideal of a po ternary semigroup is semiprime.

Theorem 2.45: An ideal *A* of a po ternary semigroup *T* is semiprime if and only if *X* is an ideal of *T*, $X^3 \subseteq A$ implies $X \subseteq A$

Theorem 2.46: Every completely prime ideal of a po ternary semigroup is prime.

Theorem 2.47: Every prime ideal of a po ternary semigroup is semiprime.

Notation 2.48: If *A* is an ideal of a po ternary semigroup *T*, then we associate the following four type of sets.

$A_1 =$ The intersection of all completely prime ideals of *T* containing *A*.

$A_2 = \{x \in T : x^n \in A$ for some odd natural number $n\}$

$A_3 =$ The intersection of all prime ideals of *T* containing *A*

$A_4 = \{ x \in T : < x >^n \subseteq A$ for some odd natural number $n\}$

Theorem 2.49: If *A* is an ideal of a po ternary semigroup *T*, then $A \subseteq A_4 \subseteq A_3 \subseteq A_2 \subseteq A_1$

Corollary 2.50: If an ideal *A* of a po ternary semigroup *T* is completely semiprime then x,y,z, $\in T$, $xyz \in A \Rightarrow < x >< y >< z > \subseteq A$.

3 Semipseudo Symmetric Ideals

DEFINITION 3.1: An ideal *A* in of a po ternary semigroup *T* is said to be a **semipseudo symmetric** provided for any odd natural numbers *n* , $x \in T$, $x^n \in A \Rightarrow < \mathrm{x} >^n \subseteq \mathrm{A}$

DEFINITION 3.2: An ideal *A* of a Po ternary semigroup *T* is said to be a **Pseudo symmetric** provided *x ,y , z* ∈ *T;* $xyz \in A$ implies $xsytz \in A$ *for all s , t* $\in T$.

THEOREM 3.3: Every Pseudo symmetric ideal of a po ternary semigroup is a semipseudo symmetric ideal.

Proof: Let *A* be a pseudo symmetric ideal of a po ternary semigroup *T*.

Let x $\in T$ and $x^n \in A$ for some odd natural numbers *n*.

By corollary 2.42 $x^n \in A \Rightarrow < x >^n \subseteq A$

Therefore *A* is a semipseudo symmetric ideal.

Note 3.4: The converse of theorem 3.3 is not true i.e a semipseudo symmetric ideal of a po ternary semigroup need not be a pseudo symmetric ideal.

Example 3.5: Let *T* be a free po ternary semigroup over the alphabet *{a ,b ,c ,d ,e}*. Let *A* = < *abc*> ∪ < *bca*> ∪ <çab>. Since abc $\in A$ and adbec $\notin A$, A is not pseudo symmetric.

Suppose $x^n \in A$ for some odd natural numbers *n*. Now the word 'x' contains abc or bcause or cab and hence <x>$^n \subseteq A$.

Therefore $x^n \in A$ for some odd natural number 'n'. $\Rightarrow < x >^n \subseteq A$. Therefore *A* is a semipseudo symmetric ideal.

THEOREM 3.6: Every semiprime ideal *P* minimal relative to containing a semipseudo symmetric ideal *A* in a po ternary semigroup *T* is completely semiprime.

Proof: Write $S=\{x^n : n \in T \setminus P$ for any odd natural numbers n }.

First we show that $A \cap S = \phi$ If $A \cap S \neq \phi$, then there exist an element $x \in T \setminus P$ such that $x^n \in A$ where *n* is an odd natural number. Since *A* is a semipesudo symmetric ideal, $< x >^n \subseteq A \subseteq p \Rightarrow < x >^n \subseteq p \Rightarrow x \in p$. It is a contradiction thus $A \cap S = \phi$ consider the set $\sum = \{B : B$ is an ideal in *T* containing *A* such that $B \cap S = \phi\}$. Since $A \in \sum$, $\sum$ is nonempty. Now $\sum$ is a poset under set inclusion and satisfies the hypothesis of Zorn's lemma. Thus by zorn's lemma $\sum$ contains a maximal element, say *M*.

Suppose $< a >^3 \subseteq$ M and $a \notin M$. Then $M \cup < a >$ is an ideal containing *A*. Since *M* is maximal in $\sum$, we have (M ∪ <a>) $\cap S \neq \phi$.

Then there exists $x \in T \setminus P$ such that $x^n \in < a > \cap S$ for some odd natural number *n*.

Therefore $x^{3n} \in < a >^3 \cap S \subseteq M \cap S \Rightarrow x^{3n} \in M \cap S$. It is a contradiction. Therefore *M* is a semiprime ideal containing *A*

Now $A \subseteq M \subseteq T \setminus S \subseteq P$ Since P is a minimal semiprime ideal relative to containing A. We have $M = T \setminus S = P$. Let $x \in S ; x^m \in P$ suppose if possible $x \notin p$

Now $x \notin P \Rightarrow x \in S \Rightarrow x^m \in S$. It is a contradiction. Therefore $x \in P$ Hence *P* is a completely semiprime ideal.

COROLLARY 3.7: Every prime ideal *P* in a po ternary semigroup *T* minimal relative to containing a semipseudo symmetric ideal *A* is completelyprime.

Proof: Since every prime ideal is a semiprime ideal, by Theorem 3.6, we have *P* is a completely semiprime ideal and by Theorem 2.43, P is a completelyprime ideal.

COROLLARY 3.8: Every prime ideal minimal relative to containing a pseudo symmetric ideals A is a po ternary semigroup *T* is completelyprime.

Proof: Let *P* be a prime ideal containing a pseudo symmetric ideal *A* of a po ternary semigroup *T*. By theorem 3.3, every pseudo symmetric ideals is a semipseudo symmetric ideal, by corollary 3.7, *P* is a completelyprime ideal of *T*.

THEOREM 3.9: If *A* is an ideal in a po ternary semigroup *T*, then the following are equivalent.

1) A is completely semiprime
2) A is semiprime and pseudo symmetric
3) A is semiprime and semipseudo symmetric.

Proof: (1) $\Rightarrow (2)$: Suppose *A* is completely semiprime ideal of *T* By theorem 2.44, *A* is a semiprime ideals of *T* Let x, y, z $\in T$ and xyz $\in A$

$$(yzx)^3 = (yzx)(yzx)(yzx) = yz(xyz)(xyz)x \in A; therefore\,(yzx)^3 \in A,$$

A is completely semiprime ideal $\Rightarrow yzx \in A$

Similarly $(zxy)^3 = (zxy)(zxy)(zxy) = z(xyz)(xyz)xy \in A; therefore (zxy)^3 \in A,$

A is completely semiprimeideal $\Rightarrow zxy \in A$

If s,t $\in T^1$ then $(xsytz)^3 = (xsytz)(xsytz)(xsytz) = xsyt[zx(syt)(zxs)y]tz \in A$

$(xsytz)^3 \in A$, A is completely semiprime ideal $\Rightarrow xsytz \in A$. Therefore A is a pseudo symmetric ideal.

(2) $\Rightarrow (3)$: Suppose *A* is semiprime and pseudo symmetric .By theorem 3.3, *A* is a semipseudo symmetric ideal. Hence *A* is a semiprime and semipseudo symmetric.

(3) $\Rightarrow (1)$: Suppose *A* is semiprime and semipseudo symmetric .Let $x \in T, x^3 \in A$, Since *A* is semipseudo symmetric, $x \in T, x^3 \in A \Rightarrow < x >^3 \subseteq A$. Since *A* is semiprime, by Theorem 2.45, $< x >^3 \subseteq A \Rightarrow < x > \subseteq A$. Therefore A is completely semiprime.

DEFINITION 3.10: An element a of a po ternary semigroup T is said to be **semisimple** provided $a \in \langle a \rangle^3$, that is $\langle a \rangle^3 = \langle a \rangle$

DEFINITION 3.11: A po ternary semigroup T is said to be a **semisimple po ternary semigroup** provided every element in T is semisimple.

THEOREM 3.12: If A is an ideal of a semisimple po ternary semigroup T, then the following are equivalent
1) A is completely semiprime
2) A is pseudo symmetric
3) A is semipseudo symmetric.
Proof: (1) $\Rightarrow (2)$: Suppose that A is completely semiprime. By theorem 3.9, A is pseudo symmetric.
(2)$\Rightarrow (3)$: Suppose that A is pseudo symmetric. By theorem 3.9, A is semipseudo symmetric
(3)$\Rightarrow (1)$: Suppose that A is semipseudo symmetric. Let $x \in T, x^3 \in A,$ Since A is semipseudo symmetric $x^3 \in A \Rightarrow \langle x \rangle^3 \subseteq A.$ Since T is semisimple, x is a semisimple element, therefore $x \in \langle x \rangle^3 \subseteq A.$ Thus A is completely semiprime.

THEOREM 3.13: If A is an ideal of po ternary semigroup T, then the following are equivalent.
1) A is completely semiprime
2) A is prime and pseudo symmetric
3) A is prime and semipseudo symmetric.
Proof: (1)$\Rightarrow (2)$: Suppose that A is completely prime.
By theorem 2:46, A is prime Let $x, y, z \in T$ and $xyz \in A$
$xyz \in A;$ A is completely prime $\Rightarrow x \in A \; or \; y \in A \; or \; z \in A \Rightarrow xsytz \in A \; for \; all \; s, t \in T$
Therefore A is pseudo symmetric and prime.
(2) $\Rightarrow (3)$: Suppose that A is prime and pseudo symmetric.
Since A is pseudo symmetric, by Theorem 3.9, A is semi pseudo symmetric
(3) $\Rightarrow (1)$: Suppose A is prime and semipseudo symmetric.
Since A is prime by theorem 2.47, A is semiprime.

Since A is semiprime and semipseudo symmetric, by theorem 3.9, A is completely semiprime. Since A is prime and completely semiprime by theorem 2.43, A is completely prime.

Theorem 3.14: Let A be a semipseudo symmetric ideal of a po ternary semigroup T. Then the following are equivalent.
1) A_1 = The intersection of all completely prime ideals of T containing A.
2) A_1^1 = The intersection of all minimal completely prime ideals of T containing A
3) A_1^{11} = The minimal completely semiprime ideal of T relative to containing A.
4) $A_2 = \{ x \in T : x^n \in A$ for some odd natural number $n \}$.
5) A_3 = The intersection of all prime ideals of T containing A.
6) A_3^1 = The intersection of all minimal prime ideals of T containing A.
7) A_3^{11} = The minimal semiprime ideal of T relative to containing A.
8) $A_4 = \{ x \in T : \langle x \rangle^n \subseteq A$ for some odd natural number $n \}$
Proof: Since completely prime ideals containing A and minimal completely prime ideals containing A and minimal completely semiprime ideals relative to containing A are coincide, it follows that $A_1 = A_1^1 = A_1^{11}$. Since prime ideals containing A and minimal prime ideals containing A and the minimal semiprime ideals relative to containing A are coincide, it follows that $A_3 = A_3^1 = A_3^{11}$ .Since A is semipseudo symmetric ideal, we have $A_2 = A_4$ .Now by theorem 3.6, we have $A_1^{11} = A_3^{11}$ .Therefore $A_1 = A_1^1 = A_1^{11} = A_3 = A_3^1 = A_3^{11} \; and \; A_2 = A_4$

Hence the given conditions are equivalent.

Theorem 3.15: If *M* is a maximal ideal of a po ternary semigroup *T* with $M_4 \neq T,$ then the following are equivalent.
1) M is completely prime.
2) M is completely semiprime.
3) M is pseudo symmetric.
4) M is semipseudo symmetric.
Proof: (1) $\Rightarrow$ (2). Suppose that *M* is completely prime.
By theorem2.46, *M* is completely semiprime.
(2) $\Rightarrow$ (3).Suppose that *M* is completely semiprime ideal of the po ternary semigroup T.
By theorem 3.9, *M* is pseudo symmetric.
(3) $\Rightarrow$ (4).Suppose that *M* is pseudo symmetric. By theorem 3.12, *M* is semipseudo symmetric.
(4) $\Rightarrow$ (1):Suppose that M is semipseudo symmetric.
By theorem 3.14, $M \subseteq M_4 \subseteq T$ Since *M* is maximal ideal and $M_4 \neq T,$ it implies that $M = M_4$
Let $x \in T,\ x^3 \in M,$ Since *M* is semipseudo symmetric, $<x>^3 \subseteq M.$ Then $x \in M_4 = M$
$\therefore$ *M* is completely semiprime.
Let $x, y, z \in T; xyz \in M$. Since *M* is completely semiprime, by corollary 2.50
$xyz \in M \Rightarrow <x><y><z> \subseteq M$
Suppose if possible $x \notin M, y \notin M, z \notin M$ Then $MU<x>,\ MU<y>, M<z>$ are ideals of T
and $M \cup <x>,\ M \cup <y>, M \cup <z> = T.$ Since M is maximal,
$y, z \in M \cup <x>; x, z \in M \cup <y>; and\ x, y \in M \cup <z> \Rightarrow y, z \in <x>;\ x,\ z \in <y>;\ x, y \in <z>$
$\Rightarrow <x> = <y> = <z>$
Now $<x><y><z> \subseteq M \Rightarrow <x><y><z> = <x>^3 \subseteq M \Rightarrow x^3 \in M \Rightarrow x \in M$

It is a contradiction. Therefore either $x \in M\ or\ y \in M\ or\ z \in M$
$\therefore$ M is completely prime.

4. Conclusion
Anjaneyulu .A initiated the study of pseudo symmetric ideals in semigroups. Madhusudhana Rao.D initiated the study of theory of Γ ideals in Γ -semigroups. Sarala.Y initiated the study of theory of ideals in ternary semigroups and hence the study of ideals in semigroups, Γ semigroups and po Γ semigroups creates a platform for the pseudo symmetric ideals in po ternary semigroups.

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